Q1

a)

$$Q = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0 & 0 & 0.25 & 0.75 \\ 0 & 0 & 0.75 & 0.25 \end{bmatrix} \rightarrow \text{transition matrix}$$

b) ~~Transient~~ Recurrent stages → (1 and 2) ; (3 and 4)

they are recurrent stages as if we enter any one of them we cannot leave them. i.e. after some point we have to come to the starting stage.

There are no transient stages as there exists no such state such that upon entering it, we will not come back.

c) We know that, $\pi Q = \pi$ and $\sum_{i=1}^{4} \pi_i = 1$

1) let $\pi_3 = \pi_4 = 0$, $\pi_1 + \pi_2 = 1$

$$\left.\begin{aligned} \pi_1 &= 0.5\pi_1 + 0.25\pi_2 \\ \pi_2 &= 0.5\pi_1 + 0.75\pi_2 \end{aligned}\right] \rightarrow \pi_1 = 0.5$$

$\therefore \pi_1 = 1/3$ & $\pi_2 = 2/3$

1st stationary distribution $= \left(\frac{1}{3}, \frac{2}{3}, 0, 0\right)$

2). Now, Let $\pi_1 = \pi_2 = 0$, $\pi_3 + \pi_4 = 1$

$$\left.\begin{aligned} \pi_3 &= 0.25\pi_3 + 0.75\pi_4 \\ \pi_4 &= 0.75\pi_3 + 0.25\pi_4 \end{aligned}\right] \rightarrow \pi_3$$

$\therefore \pi_3 = \pi_4 = 1/2$

2^{nd} stationary state - $(0, 0, \frac{1}{2}, \frac{1}{2})$

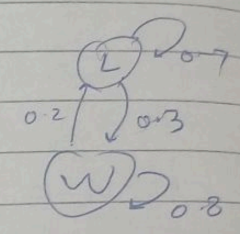
Q2

	Next win	Next Lose
Win	0.8	0.2
Lose	0.3	0.7

W_π = long run proportion of games after a win
L_π = long run proportion of games after a loss

a) steady state → $W_\pi + L_\pi = 1$ & $\pi Q = \pi$

$$W_\pi = 0.8 W_\pi + 0.3 L_\pi$$
$$L_\pi = 0.2 W_\pi + 0.7 L_\pi$$



$$\therefore W_\pi = 1.5 L_\pi$$

$$1.5 L_\pi + L_\pi = 1$$
$$L_\pi = 2/5 = 0.4$$

$$W_\pi = 0.6 \ \& \ L_\pi = 0.4$$

$\therefore$ Long run probability of win is 60%.

b)

	dinner	No dinner
win	0.7	0.3
Lose	0.2	0.8

$$\text{dinner} = 0.7 \times W_\pi + 0.2 L_\pi$$
$$= 0.7 \times 0.6 + 0.2 \times 0.4$$
$$= 0.42 + 0.08 = 0.5$$

$\therefore$ 50% chance

c) Expected no. of games for a dinner = $1/P(d) = \frac{1}{0.5}$

$= 2$

$\therefore$ 2 games.

Q5

a) $P(UP) = 0.1 \quad P(same) = 0.85 \quad P(down) = 0.05$

Expected change (drift) $= (1 \times P(UP)) + (-1 \times P(down)) + (0 \times P(same))$

$= 0.1 - 0.05$

$= +0.05$ ticks

as the drift is +ve, therefore the stock price has a tendency to move up over time. Therefore, probability of returning to a specific level is less than 0.

Hence it is not recurrent but transient.

b) Stationary distribution exists only if there is +ve recurrence.

As the chain is not recurrent the price-level cannot follow stationary distribution

(No stationary distribution)

Date ___/___/___

3\. a) let transition matrix for cat be.

$$P_c = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$$

let stationary distribution be $\pi_c = [\pi_{c_1}, \pi_{c_2}]$

$\therefore \pi_c P_c = \pi_c$ and $\pi_{c_1} + \pi_{c_2} = 1$

solving.

$$\pi_c = [0.5, 0.5]$$

for mouse,

$$P_M = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$$

and $\pi_M = [\pi_{M_1}, \pi_{M_2}]$

solving $\pi_M P_M = \pi_M$ & $\pi_{M_1} + \pi_{M_2} = 1$

$$\pi_M = [0.6, 0.4]$$

b) Cat in 1, mouse in 1 $\Rightarrow$ state 1
" " ", " " 2 $\Rightarrow$ " 2
" " 2, " " 1 $\Rightarrow$ " 3
" " ", " " 2 $\Rightarrow$ " 4

let Z_N be the current state of cat & mouse at time n
since they move independant of each other and their ~~and~~ " ~~current~~ movement

depends only on their current state.

∴ the state at time $n+1$ depends only at time n.

∴ $Z_0, Z_1, Z_2 \ldots$ is a Markov chain.

4(a) This is a ~~finite~~, markov chain i.e a unique stationary distribution exists.

The board has 64 squares but edges & corner have fewer legal moves.

square type	count	no. of moves.
corner	4	3
edge.	24	5
inside	36	8

$\pi(i,j) \propto$ no. of moves fro i, j

for normalization, ~~total weight~~
total weight $= 4 \times 3 + 24 \times 5 + 36 \times 8$
$= 420$

∴ stationary probability

$\frac{3}{420} = \frac{1}{140}$ for each corn.

$\frac{5}{420} = \frac{1}{84}$ for each edge.

$\frac{8}{420} = \frac{2}{105}$ " " inside

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