3)a) $P\left(\frac{A \cap B}{C}\right) = P\left(\frac{A}{B \cap C}\right) P\left(\frac{B}{C}\right)$ ← To prove

$$P\left(\frac{A \cap B}{C}\right) = \frac{P(A \cap B \cap C)}{P(C)}$$

$$P\left(\frac{A}{B \cap C}\right) \times P\left(\frac{B}{C}\right) = \frac{P(A \cap B \cap C)}{P(B \cap C)} \times \frac{P(B \cap C)}{P(C)}$$

$$= \frac{P(A \cap B \cap C)}{P(C)}$$

∴ a) is true

3)b) $P\left(\frac{A \cap B}{C}\right) = P\left(\frac{A}{C}\right) P\left(\frac{B}{C}\right)$ for independent events A & B

Counter example, $\Omega = \{1, 2, 3, 4\}$

let, $A = \{1, 2\}$, $B = \{1, 3\}$, $C = \{1, 4\}$

$P\left(\frac{A \cap B}{C}\right) = \frac{P(A \cap B \cap C)}{P(C)}$, $P\left(\frac{A}{C}\right) P\left(\frac{B}{C}\right) = \frac{P(A \cap C)\, P(B \cap C)}{[P(C)]^2}$

~~PCA~~ $A \cap B \cap C = \{1\}$ ⇒ $P(A \cap B \cap C) = \frac{1}{4}$

$P(C) = \frac{1}{2}$

& $\cancel{P\left(\frac{A}{C}\right)}$ $P(A \cap C) = P(B \cap C) = \frac{1}{4}$

∴ ~~P~~ LHS ⇒ $\cancel{\frac{1}{4}}$ $\frac{1}{2}$

RHS ⇒ $\frac{1}{4}$

∴ b is false.

c) if $P\left(\frac{A}{D \cap B^c}\right) > P\left(\frac{A}{D \cap B}\right)$ & $P\left(\frac{A}{D^c \cap B^c}\right) > P\left(\frac{A}{D^c \cap B}\right)$

then, $P\left(\frac{A}{B}\right)$ must be greater than $P\left(\frac{A}{B^c}\right)$

Let, $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8\}$

$D = \{\omega_1, \omega_2, \omega_3, \omega_4\}$

$D^c = \{\omega_5, \omega_6, \omega_7, \omega_8\}$

$B = \{\omega_1, \omega_3, \omega_5, \omega_7\}$

$B^c = \{\omega_2, \omega_4, \omega_6, \omega_8\}$

$A = \{\omega_1, \omega_4, \omega_6, \omega_7\}$

$P\left(\frac{A}{D \cap B^c}\right) > P\left(\frac{A}{D \cap B}\right)$

$P\left(\frac{A}{D^c \cap B^c}\right) > P\left(\frac{A}{D^c \cap B}\right)$

now, $P\left(\frac{A}{B}\right)$ & $P\left(\frac{A}{B^c}\right)$

$\frac{P(A \cap B)}{P(B)}$ $\qquad \frac{P(A \cap B^c)}{P(B^c)}$

$A \cap B = \phi \Rightarrow P(A \cap B) = 0$

$A \cap B^c = \{\omega_4, \omega_6\} \Rightarrow P = \frac{2}{8}$

$P(B^c) = \frac{4}{8}$

$\therefore P\left(\frac{A}{B}\right) = 0 < \frac{1}{2} = P\left(\frac{A}{B^c}\right)$

$\therefore$ (c) is false.

7) a) let originator be O

then O tells to someone else

$\therefore$ here probablity is 1 that it has not returned to the originator

Now at rest $r-1$ times the person can choose anyone apart from ~~times~~ themselves & O in ${}^{n-1}C_1 = n-1$ ways from $n-1$ people.

$\therefore$ probablity $= \frac{n-1}{n}$

$\therefore$ required probablity $= \left(\frac{n-1}{n}\right)^{r-1}$

b) when it is told first time, $P = 1$

for second time, $P = \left(\frac{n-1}{n}\right) \times 1$

" third time, $P = \underset{3}{\left(\frac{n-2}{n}\right)} \times \underset{2}{\left(\frac{n-1}{n}\right)} \times \underset{1}{1}$

$\therefore$ in r steps $\Rightarrow P = \prod_{k=0}^{k=r-1} \frac{n-k}{n}$

$$P = \frac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{n^r}$$

7) To group of N people.

a) 0 chooses N people who cannot be 0

$\therefore$ P(not choosing 0) = 1

b) at each step, choose N from $n-1$

$\therefore$ P at each step $= \dfrac{{}^{n-1}C_N}{{}^{n}C_N}$

$\therefore$ P required $= \left(\dfrac{(n-1)!}{N!\,(n-1-N)!} \times \dfrac{N!\,(n-N)!}{n!} \right)^{r-1}$

$= \left(\dfrac{n-N}{n} \right)^{r-1}$

b) at step 1, 0 is ~~telling~~ choosing N from n people.

$\therefore$ P(choosing new people) = 1

Now $1+N$ people have heard.

at step 2, people who haven't heard $= (1+n)-(N+1)$
$= n-N$

$\therefore$ P(no repeat) $= \dfrac{{}^{n-N}C_N}{{}^{n}C_N}$

similarly at step 3, $\dfrac{{}^{n-2N}C_N}{{}^{n}C_N}$

$\therefore$ $\boxed{\text{required } P = \prod_{j=0}^{1} \dfrac{{}^{n-jN}C_N}{{}^{n}C_N}}$