1) a) Implied volatility via black-scholes.

we solve for σ in

$$C_{BS}\left(S=38,\ K=35,\ r=0.06,\ T=\frac{4}{12},\ \sigma\right)=4.20$$

~~for~~, Trial σ values,

σ	C_{BS}
0.21	4.180
0.22	4.239

Interpolating, $\sigma = 0.21 + \dfrac{4.2-4.18}{4.239-4.18} \times 0.01$

$\therefore \sigma = 0.213$

$\therefore$ implied volatility = 21.3%

b) Price of the European put ($\sigma = 0.28$)

$C_{BS} \approx 4.625$

$$P = Ke^{-rT}N(-d_2) - SN(-d_1)$$

$$P_{BS(\sigma=0.28)} \approx 0.93$$

$\therefore \boxed{P \approx 0.93}$

c)

c) Real-option decision

- Intrinsic value: if firm paid 35m$ setup now and commercialized immediately the "in-the-money" ~~payoff~~ payoff would be

$$S - k = 38 - 35 = 3 \text{ million}$$

- Option to delay value: treating the launch opportunity as a european call (exercise at $T = 7/12$) the BS value at $\sigma = 0.28$ is

$$c_{BS} \approx 4.625 \text{ million}$$

which exceeds the immediate exercise payoff.

∴ The extra "time value" (~1.625 million) reflects the value of waiting for more information on the stochastic revenue.

Since the option value exceeds the intrinsic value it is optimal to retain the launch option rather than commit to the project immediately.

Date ___ / ___ / ___

Q2. $S_0 = \$100$ $K = \$105$ $T = 10$ days

Part-A : Discrete Binomial Model

(a) Stock ends in profit only if $S_T > K = 105$.

$\Rightarrow$ # up moves $> (5+10)/2 = 7.5$

$\therefore$ min up moves required $= 8$

$$P(\text{in money}) = P(8,9,10 \text{ up moves}) = \frac{{}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}}{2^{10}} = \frac{45+10+1}{2^{10}}$$

$$= 56/1024 = \underline{0.0546875}$$

(b) $E[X] = 0 \times P(1) + 0 \times P(2) + \cdots + 0 \times P(7) + 1 \times P(8) + 3 \times P(9) + 5 \times P(10)$ (where X = no. of up movements)

$$= \frac{{}^{10}C_8}{2^{10}} + \frac{3 \times {}^{10}C_9}{2^{10}} + \frac{5 \times {}^{10}C_{10}}{2^{10}} = \frac{45+30+5}{1024}$$

$$= 0.078125$$

(c) Without discounting, fair value = expected payoff

$\therefore$ fair value $= \$0.078125$

Part-B : Continuous Normal Distribution Model

(a) Expected absolute daily move $= E[|X|] = 1$

$$E[|X|] = \sigma\sqrt{2/\pi} \Rightarrow \sigma = \sqrt{\frac{\pi}{2}} = 1.253$$

$\therefore$ daily SD $\sigma = 1.253$

SD over 10 days $= \sigma\sqrt{10} = 3.963$

(b) PDF of normal distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)}$

$$\Rightarrow f_{S_T}(S) = \frac{1}{3.963\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{(S-100)^2}{3.963^2}\right]} = \frac{1}{9.934} e^{\frac{-(S-100)^2}{31.41}}$$

$$\Rightarrow E[\max(S_T-105,0)] = \int_{105}^{\infty} (S-105) \times \frac{1}{9.934} \exp\left[\frac{-(S-100)^2}{31.41}\right] dS$$

(c) Using python code, payoff $= 0.1955$

Date ___/___/_____

Part - C : Uniform Distribution Model

(a) Let $X \sim U[a,b]$ and $E[|X|] = 1$

Dist. is symmetric about zero, so $a = -c$, $b = c$

$$\Rightarrow E[|X|] = \frac{1}{2c}\int_{-c}^{c} |x|\,dx$$

$$= \frac{1}{2c}\left[-\int_{-c}^{0} x\,dx + \int_{0}^{c} x\,dx\right]$$

$$= \frac{1}{2c}\left[\frac{c^2}{2} + \frac{c^2}{2}\right] = \frac{c}{2}$$

Since $E[|X|] = 1$, $\frac{c}{2} = 1 \Rightarrow c = 2$

$\therefore [a,b] = \underline{[-2,2]}$

(b) Uniform model : Each day, price moves randomly between -2 to $+2$, and after 10 days, final price is sum of 10 independent moves.

Binomial model : Each day, price moves by exactly $+1$ or -1. Final price can only take discrete values.

Normal model : Each day, price is drawn from a normal distribution with mean 0, SD st $E[|X|] = 1$. And after 10 days, final price may allow for any real value, though extreme values have very low probability.

(c) Simulation method:

- Use a high number of simulations, eg. $N = 100000$
- For each simulation:
  - Generate 10 independent random moves in $U[-2,2]$
  - Sum the 10 moves to get final change over 10 days
  - Compute S_T, then payoff $(\max(S_T - K, 0))$
  - Average of the payoffs will give the fair value.

(Monte Carlo - Method)