# Tutorial-4

1) $T(n) = 3T(n/2) + n^2$

Ans) $a = 3, b = 2 \quad , \quad f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

Comparing $n^{\log_2 3}$ and $n^2$

$n^{\log_2 3} < n^2 \quad (case\ 3)$

∴ According to Master's Theorem

$$T(n) = \Theta(n^2)$$

2) $T(n) = 4T(n/2) + n^2$

$a = 4, b = 2$

$n^{\log_b a} = n^{\log_2 4} \implies n^2 = f(n) \quad (case\ 2)$

∴ according to Master theorem $T(n) = \Theta(n^2 \log n)$

3) $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n \quad (case\ 3)$

∴ According to Master Theorem. $T(n) = \Theta(2^n)$

Ques 4) $T(n) = 2^n T(n/2) + n^n$

∴ Master's Theorem is not applicable as $a$ is function of $n$.

Ques 5) $T(n) = 16T(n/4) + n$

Ans)
$a = 16, b = 4$ , $f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > f(n)$ (Case 1)

$T(n) = \theta(n^2)$

Ques 6) $T(n) = 2T(n/2) + n \log n$

Ans)
$a = 2, b = 2$ , $f(n) = n \log n$

$n^{\log_b a} = n^{\log_2 2} = n$

Now $f(n) > n$

∴ According to Master Theorem $T(n) = \theta(n \log n)$

Ques 7) $T(n) = 2T(n/2) + \dfrac{n}{\log n}$

Ans)
$a = 2, b = 2$ , $f(n) = \dfrac{n}{\log n}$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > f(n)$

∴ According to Master Theorem $T(n) = \theta(n)$

Ques 8) $T(n) = 2T(n/4) + n^{0.5}$

Ans) $a = 2, b = 4$, $f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

∴ According Master Theorem $T(n) = \theta(n^{0.5})$

Ques 9) $T(n) = 0.5 T(n/2) + \frac{1}{n}$

Master's theorem is not applicable. as $a < 1$.

Ques 10) $T(n) = 16T(n/4) + n!$

Ans)
$$a = 16, \quad b = 4, \quad f(n) = n!$$
$$n^{\log_b a} = n^{\log_4 16} = n^2$$
$$n^2 < n!$$
$$\therefore \text{According to Master's}, \quad T(n) = \Theta(n!)$$

Ques 11) $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

Ans)
$$a = 4, \quad b = 2, \quad f(n) = \log n$$
$$n^{\log_b a} = n^{\log_2 4} = n^2$$
$$n^2 > f(n)$$
$$\therefore \text{According to Master's}, \quad T(n) = \Theta(n^2)$$

Ques 12) $T(n) = \text{sqrt}(n) \, T(n/2) + \log n$

Ans)
$\therefore$ Master's theorem not applicable as $a$ is not constant.

Ques 13) $T(n) = 3T(n/2) + n$

$$a = 3, \quad b = 2, \quad f(n) = n$$
$$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$$
$$n^{1.58} > f(n)$$
$$\therefore \text{According to Master's Theorem}, \quad T(n) = O(n^{\log_2 3})$$

Ques 14) $T(n) = 3T(n/3) + \sqrt{n}$

Ans) $a = 3, b = 3, f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

∴ According to Master's Theorem, $T(n) = \Theta(n)$

Ques 15) $T(n) = 4T(n/2) + cn$

Ans) $a = 4, b = 2, f(n) = c*n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c*n$

∴ According to Master's theorem, $T(n) = \Theta(n^2)$

Ques 16) $T(n) = 3T(n/4) + n\log n$

Ans) $a = 3, b = 4, f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

$T(n) = \Theta(n\log n)$

Ques 17) $T(n) = 3T(n/3) + n^2 \log n$

$a = 6, b = 3, f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2 \log n$

∴ $T(n) = \Theta(n^2 \log n)$

Ques 19) $T(n) = 4T(n/2) + n/\log n$

Ans) $a = 4, b = 2$ , $f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n$

$\therefore T(n) = \Theta(n^2)$

Ques 20) $T(n) = 64T(n/8) - n^2 \log n$

Masters theorem not applicable as $f(n)$ is not increasing function.

Ques 21) $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3, f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^7 < n^2$

$\therefore T(n) = \Theta(n^2)$

Ques 22) $T(n) = T(n/2) + n(2 - \cos n)$

Ans) Master's theorem isn't applicable since regularity condition is isolated in case 3.