Image Processing - Ass 1

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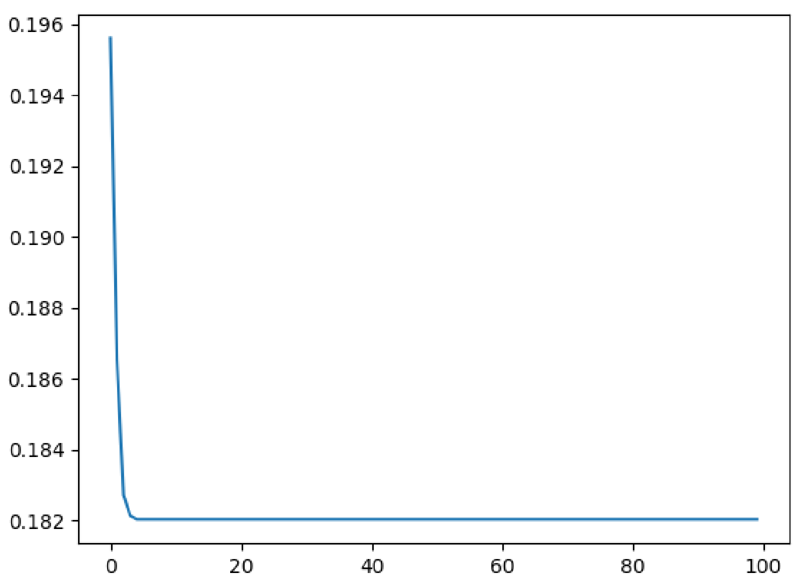
1

1.1 b)

Transition levels [-2.31146537 -0.71605884 0.31251945 1.22769064 2.82184381]
Representation levels [-1.33005093 -0.10206675 0.72710565 1.72u27562]

1.2 c)



1.3 d)

On repeating the experiment with random initializations, the algorithm converges to the same value of MSE and similar representation and transition levels. This implies that the initialization of transition levels doesn't affect the final values of the quantizer for high number of iterations.

1.4 e)

For a signal with actual value x from distribution p(x) and quantized signal x' with values $i_1, i_1 ... i_L$ and interval edges $m_1, m_1..m_{L+1}$, error ϵ is given as

$$\epsilon = E[(x - x')^2] = \int_{m_1}^{m_{L+1}} (x - x')^2 p(x)dx = \sum_{k=1}^{L} \int_{m_k}^{m_{k+1}} (x - i_k)^2 p(x)dx$$

Using conditions $\frac{\partial \epsilon}{\partial m_k} = 0$ and $\frac{\partial \epsilon}{\partial i_k} = 0$,

$$m_k = \frac{i_k + i_{k-1}}{2}$$

and

$$i_k = \frac{\int_{m_k}^{m_{k+1}} xp(x)dx}{\int_{m_k}^{m_{k+1}} p(x)dx}$$

for uniform distribution, $p(x) = \frac{1}{m_{L+1} - m_1} \forall x \in (m_1, m_{L+1})$ Putting in the formulae obtained above,

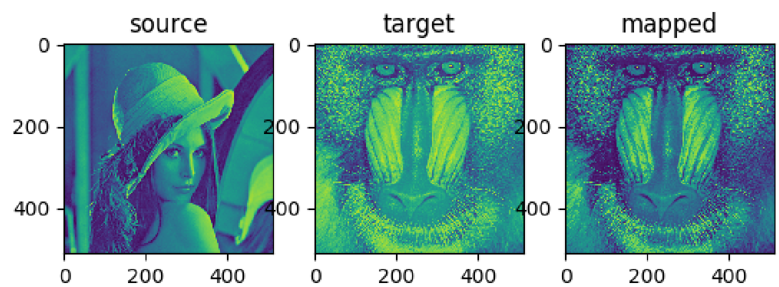
$$i_k = \frac{m_{k+1}^2 - m_k^2}{2(m_{k+1} - m_k)} = \frac{m_{k+1} + m_k}{2}$$

or

$$m_k = \frac{m_{k+1} + m_{k-1}}{2}$$

$$m_{k+1} - m_k = m_k - m_{k-1} = i_k - i_{k-1} = const$$

2



3

For $i \in 1, 2, 3$ interpolating on x first

$$f'(x, y_i) = \frac{x_2 - x}{x_2 - x_1} f(x_1, y_i) + \frac{x - x_1}{x_2 - x_1} f(x_2, y_i)$$

$$f'(x, y_i) = \frac{x_3 - x}{x_3 - x_1} f(x_1, y_i) + \frac{x - x_1}{x_3 - x_1} f(x_3, y_i)$$

$$f'(x, y_i) = \frac{x_3 - x}{x_3 - x_2} f(x_2, y_i) + \frac{x - x_2}{x_3 - x_2} f(x_3, y_i)$$

Interpolating on y,

$$f'(x, y) = \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2)$$

$$f'(x, y) = \frac{y_3 - y}{y_3 - y_1} f(x, y_1) + \frac{y - y_1}{y_3 - y_1} f(x, y_3)$$

$$f'(x, y) = \frac{y_3 - y}{y_3 - y_2} f(x, y_2) + \frac{y - y_2}{y_3 - y_2} f(x, y_3)$$

Hence,

$$f'(x,y) = \frac{1}{(x_m - x_n)(y_k - y_l)}$$

Since f'(x,y) only depends on x,y, xy and constant terms,

$$f'(x,y) = a_0 + a_1 x + a_2 y + a_3 xy$$

For the 8 neighborhood pixels, we get a total of 8 equations, which create a linear system.

$$\begin{bmatrix} f(x_1,y_1) \\ \vdots \\ f(x_8,y_8) \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_8 & y_8 & x_8 y_8 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

There exists a unique solution for this system of equations if there are exactly 4 linearly independent equations out of these 8. If there are less than 4 linearly independent equations, infinitely many solutions exist. If more than 4 linearly independent equations are there, there is no solution.

4

Average of K observations can be obtained by

$$\hat{g}(x,y) = \frac{1}{K}\sum_{i=0}^{K} g_i(x,y) = f(x,y) + \frac{1}{K}\sum_{i=0}^{K} \eta_i(x,y)$$

Mean and variance of $\hat{g}(x,y)$ is given by

$$\begin{aligned}
\mu_{\hat{g}(x,y)} &= E[\hat{g}(x,y)] = f(x,y) + \frac{1}{K}\sum_{i=0}^{K} E[\eta_i(x,y)] \\
&= \left(f(x,y) + \frac{1}{K}\sum_{i=0}^{K} 0 \right) \quad = f(x,y) \\
\sigma^2_{\hat{g}(x,y)} &= E[(\hat{g}(x,y) - \mu_{\hat{g}(x,y)})^2] \\
&= \frac{1}{K^2}\sum_{i=0}^{K}\sum_{j=0}^{K} E\left[\eta_i(x,y) \cdot \eta_j(x,y)\right] \\
&= \frac{1}{K^2}\sum_{i=0}^{K} E\left[\eta_i(x,y)^2\right] \\
&= \frac{1}{K^2}\sum_{i=0}^{K} \sigma^2_{\eta(x,y)} \\
&= \sigma^2_{\hat{g}(x,y)} = \frac{\sigma^2_{\eta(x,y)}}{K}
\end{aligned}$$

5

Let axes (x, y) be rotated by angle θ to get (x', y')

$$x' = x\cos\theta + y\sin\theta$$
$$x = x'\cos\theta - y'\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

$$\frac{\partial}{\partial x'} = \cos\theta\frac{\partial}{\partial x} + \sin\theta\frac{\partial}{\partial y}$$
$$\frac{\partial}{\partial y'} = -\sin\theta\frac{\partial}{\partial x} + \cos\theta\frac{\partial}{\partial y}$$

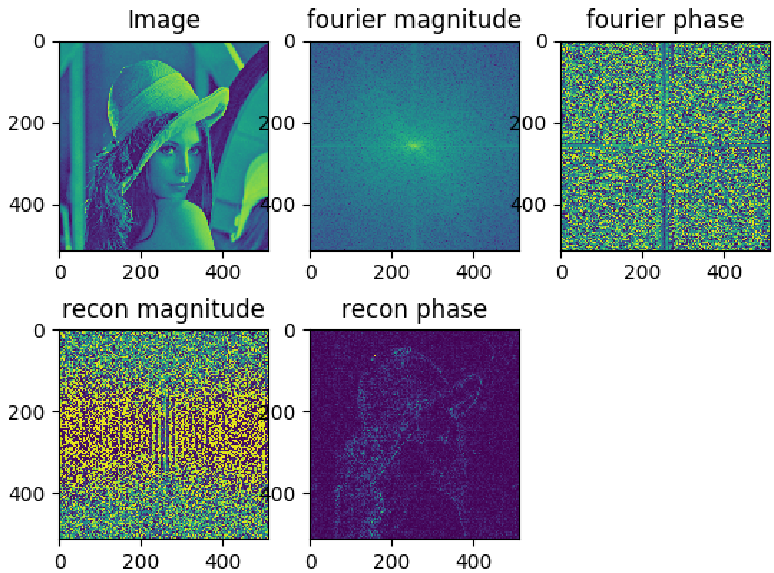
$$\frac{\partial^2}{\partial x'^2} = \cos\theta\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x'}\right)+\sin\theta\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x'}\right) = \cos^2\theta\frac{\partial^2}{\partial x^2}+\cos\theta\sin\theta\frac{\partial^2}{\partial x\partial y}+\sin\theta\cos\theta\frac{\partial^2}{\partial y\partial x}+\sin^2\theta\frac{\partial^2}{\partial y^2}$$
$$\frac{\partial^2}{\partial y'^2} = -\sin\theta\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y'}\right)+\cos\theta\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y'}\right) = \sin^2\theta\frac{\partial^2}{\partial x^2}-\sin\theta\cos\theta\frac{\partial^2}{\partial x\partial y}-\cos\theta\sin\theta\frac{\partial^2}{\partial y\partial x}+\cos^2\theta\frac{\partial^2}{\partial y^2}$$

Adding,

$$\frac{\partial^2}{\partial x'^2}+\frac{\partial^2}{\partial y'^2} = \frac{\partial^2}{\partial x^2}\left(\sin^2\theta + \cos^2\theta\right)+\frac{\partial^2}{\partial y^2}\left(\cos^2\theta + \sin^2\theta\right) = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} = Laplacian$$

Therefore, laplacian doesn't change with rotation of axes.

6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fourier magnitude
fourier phase
recon magnitude
recon phase
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400
0
200
400

7

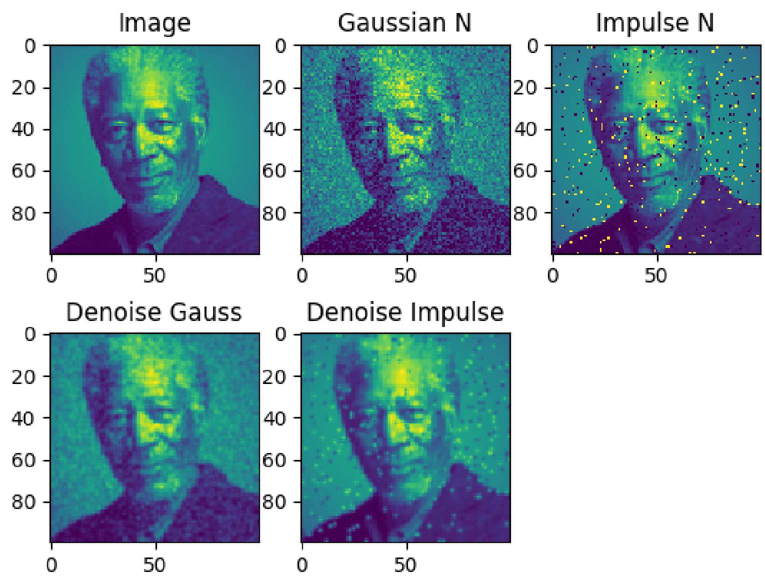
Image
Gaussian N
Impulse N
Denoise Gauss
Denoise Impulse
0
20
40
60
80
0
50