CSCI 6461: Computer System Architecture – Homework 1

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Case Study 1: Chip Fabrication Cost

1.1.a)

To find: Yield for Phoenix Chip

Given:

Phoenix Die Size: $200mm^2 = 2cm^2$

Defect Rate: 0.04 per cm^2

N: 14

Die Yield = $Wafer\ yield * \frac{1}{(1+Defect\ Rate*Die\ size)^N}$

Let's assume wafer yield to be 100% that is 1.

Die Yield = $1 * 1/(1+0.04*2)^{14}$ = 0.3404

The yield for Phoenix chip is 34.04%.

1.1.b)

To evaluate: Why does Phoenix have a higher defect rate than BlueDragon?

As the technology becomes current and advanced, more complex inventions are made. Chip and processor size are reduced year on year with each new version. With each new chip being launched, number transistors are increased, whereas the size of the chip is reduced.

Here we see that, BlueDragon chip has fewer transistors and larger manufacturing size than Phoenix chip. Also, Phoenix chip has a greater number of cores per chip than BlueDragon. Therefore, we can assume that Phoenix is a relatively current or new chip in the market. And as BlueDragon has been around for longer, the manufacturing process would have gone through modifications, tuning and updates to make the process better and efficient and hence lower defect rate.

1.2.a)

Dies per wafer = $\frac{\pi*\left(\frac{Wafer\ Diameter}{2}\right)^2}{Die\ area} - \frac{\pi*Wafer\ Diameter}{\sqrt{(2*Die\ area)}} = \frac{\pi*\left(\frac{450}{2}\right)^2}{200} - \frac{\pi*450}{\sqrt{(2*200)}}$ = 724 Phoenix chips

Yield of Phoenix chip = $Wafer\ yield * \frac{1}{(1+Defect\ Rate*Die\ size)^N}$

Let's assume wafer yield to be 100% that is 1.

Die Yield = $1 * 1/(1 + 0.04 * 2)^{14}$ = 0.3406

The yield for Phoenix chip is 34.06%.

Defect free Phoenix chip for each wafer = Total number of chips * Die Yield

Defect free chip = 724 * 0.3406= 246 chips

If profit for each Phoenix chip is $30, then for 246 chips the profit = 246 * $30 = $7380

1.2.b)

Dies per wafer = $\frac{\pi*\left(\frac{Wafer\ Diameter}{2}\right)^2}{Die\ area} - \frac{\pi*Wafer\ Diameter}{\sqrt{(2*Die\ area)}} = \frac{\pi*\left(\frac{450}{2}\right)^2}{120} - \frac{\pi*450}{\sqrt{(2*120)}}$ = 1234 RedDragon chips

Yield of RedDragon chip = $Wafer\ yield * \frac{1}{(1+Defect\ Rate*Die\ size)^N}$

Let's assume wafer yield to be 100% that is 1.

Die Yield = $1 * 1/(1 + 0.04 * 1.2)^{14}$ = 0.5187

The yield for RedDragon chip is 51.87%.

Defect free RedDragon chip for each wafer = Total number of chips * Die Yield

Defect free chip = 1234 * 0.5187 = 640 chips

If profit for each RedDragon chip is $15, then for 640 chips the profit = 640 * $15 = $9600

1.2.c)

For per wafer of 450 mm, 1234 RedDragon chips can be manufactured.

Similarly, 724 Phoenix chips can be manufactured for each 450mm wafer.

To satisfy 50000 RedDragon demand we would require (50000/1234) i.e., 40.51 wafers or 41 wafers.

Similarly, for 25000 Phoenix chips demand, we require (25000/724) i.e., 30.53 wafers or 31 wafers. As the facility can only fabricate 70 wafers a month, to have a lucrative outcome or to make maximum profit, we must make 40 wafers for RedDragon chip and 30 wafers for Phoenix chips. Since, per RedDragon wafer profit is greater than per Phoenix wafer.

1.3.a)

For single core Phoenix chip:

Defect Rate = 0.04 per cm^2

Die size = $\frac{200mm^2}{8} = \frac{2cm^2}{8}$ = 0.25cm^2

Yield of single core Phoenix chip = $Wafer\ yield * \frac{1}{(1+Defect\ Rate*Die\ size)^N}$

Let's assume wafer yield to be 100% that is 1.

Die Yield = $1 * 1/(1 + 0.04 * 0.25)^{14}$ = 0.8699 = 86.99%.

To establish a Phoenix chip as $Phoenix^4$ chip, any of the 4 to 7 core out of 8 must be defect-free. If we consider 8 cores in a chip as individual cores, then we need 4/8, 5/8, 6/8, and 7/8 cores to be defect-free.

For 4/8 defect-free cores, 4 cores must be defect-free, and 4 cores must be with defect. We know from previous step that the yield of single core is approximately 0.87. Similarly, we can derive the yield of cores with defect as (1-0.87) that is 0.13. Additionally, any of the 4 cores out of 8 can be defect-free, and combination can be derived as ${}^nC_r$, where n=total cores which is 8, r=no of defect free cores which is 4.

${}^8C_4 = \frac{8!}{4!(8-4)!} = \frac{8!}{4!4!}$ = 70 combinations.

Therefore, the probability of getting 4 defect-free cores is

= Combinations * (probability of defect free chip)$^{\text{no of defect free chip}}$ * (probability of with defect chip)$^{\text{no of with defect chip}}$

= 70 * $(0.87)^4$ * $(0.13)^4$ = 0.0114 = 1.14%

Similarly, for 5 defect-free cores, combinations = $^{8}C_{5}$ = 56.

Therefore, the probability of getting 5 defect-free cores is

= 56 * $(0.87)^{5}$ * $(0.13)^{3}$ = 0.0613 = 6.13%

Similarly, for 6 defect-free cores, combinations = $^{8}C_{6}$ = 28.

Therefore, the probability of getting 5 defect-free cores is

= 28 * $(0.87)^{6}$ * $(0.13)^{2}$ = 0.2051 = 20.51%

Similarly, for 7 defect-free cores, combinations = $^{8}C_{7}$ = 8.

Therefore, the probability of getting 5 defect-free cores is

= 8 * $(0.87)^{7}$ * $(0.13)^{1}$ = 0.3923 = 39.23%

Therefore, the yield of $Phoenix^{4}$ chip = 0.0114 + 0.0613 + 0.2051 + 0.3923 = 0.6701 = 67.01%

Similarly, for 2 defect-free cores, combinations = $^{8}C_{2}$ = 28.

Therefore, the probability of getting 5 defect-free cores is

= 28 * $(0.87)^{2}$ * $(0.13)^{6}$ = 0.0001 = 0.01%

Similarly, for 3 defect-free cores, combinations = $^{8}C_{3}$ = 56.

Therefore, the probability of getting 5 defect-free cores is

= 56 * $(0.87)^{3}$ * $(0.13)^{5}$ = 0.0013 = 0.13%

Therefore, the yield of $Phoenix^{2}$ chip = 0.0001 + 0.0013 = 0.0014 = 00.14%

Similarly, for 1 defect-free cores, combinations = $^{8}C_{1}$ = 8.

Therefore, the probability of getting 5 defect-free cores is

= 8 * $(0.87)^{1}$ * $(0.13)^{7}$ = 0.000004 = 0.0004%

1.3.b)

From all the four chips, it seems feasible and profitable to manufacture and sell single core Phoenix chip, as the yield is very high compared to rest four.

But on the other hand, the cost of single core Phoenix chip might be less as compared to others and their overall profitability might take a hit.

If we were to compare Phoenix^4 vs Phoenix^2 vs Phoenix^1, from these three, Phoenix^4 seems a better choice and worthwhile to package and sell due to its relatively high yield as compared to other two.