Regional Geoid Modelling

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***Abstract*— We estimated a regional geoid over a $1^\circ \times 1^\circ$ area, in New Mexico, USA using the classical remove-restore approach to determine geoid using classical gravity anomalies. The overall process consisted of reduction of normal gravity, free air correction, atmospheric attraction correction, long wavelength gravity anomaly and terrain correction from observed air-borne gravity values and subsequent restoration of long-wavelength undulation and indirect effect to the calculated undulation from Bruns formula. The results we received were in agreement with the global geoid computed from EGM2008. It is evident that air-borne gravimetry could be used to develop high resolution regional geoid models.**

I. INTRODUCTION

Physical geodesy's main objective is to accurately determine the geoid to the level of cm, matching GPS height determination accuracy. The geoid is an equipotential surface of the Earth's gravity field, it approximately coincides with the mean-sea level. Due to the Earth's rotation and inhomogeneities in its masses, the geoid is irregular. Its shape is described by its undulation height N with respect to a global reference ellipsoid. Geoid undulation varies from +85m near Iceland to -106 in Southern India. A geoid model and its temporal fluctuations enable the study and explanation of a variety of natural phenomena, including the interior structure of the Earth and ongoing physical processes like hydrological mass variations and glacier melt, isostasy, etc.

The geoid height can be determined from measurements of gravity acceleration on the Earth, and in order to obtain global coverage gravity surveys, dedicated satellite missions such as CHAMP, GRACE, and GOCE have been conducted. These satellite-based gravity observations have a spatial resolution of 100 km, but we require much high-resolution geoid for practical applications. For obtaining high-resolution observations, airborne gravity surveys are carried out, improving accuracy as well as spatial resolution. But these airborne gravity data for the entire globe is not available, so airborne gravity data are used for obtaining high-resolution regional geoids [8].

In order to obtain geoid undulations using gravity data, traditionally remove-restore method [1] is used. In the remove-restore method, the effect of the topographic-isostatic masses is removed from the source gravitational data and then undulations due to each of the components i.e. long wavelength, small wavelength, and indirect effects due to terrain are restored to the resulting geoidal heights. And in order to obtain the undulation due to small wavelength gravity anomaly is obtained using strokes integral, but since its computed over the entire globe and the data for entire globe is not available this this truncation of the area cause an error in the computation of geoid. For handling, this truncation of the Stokes global integral into a limited spherical cap radius a modifying the Stokes' kernel are often used [2, 5].

II. METHODOLOGY

A. Gravity Corrections

Following reductions have been performed on gravity field to convert it to faye gravity anomalies.

1) The Normal gravity values

$$\gamma = \frac{a\gamma_a \cos^2\varphi + b\gamma_b \sin^2\varphi}{\sqrt{a\cos^2\varphi + b\sin^2\varphi}} \tag{1}$$

where a and b are the semi-major and semi-minor axes of the reference ellipsoid, γ_a and γ_b are the normal gravity values at the equator and at the poles, respectively and φ is the geodetic/ellipsoidal latitude. For WGS84 ellipsoid a = 6378137 meters, b = 6356752.3142 meters γ_a = 978032.53359 m/s^2 and γ_b= 983218.49378 m/s^2.

2) Free air disturbance given by

$$\delta g_{FA} = \frac{2\gamma}{a}(1 + f + m - 2f\sin^2(\phi))H - \frac{3\gamma_a H}{a^2} \tag{2}$$

where $f = (a-b)/a, m$ is the ration the gravitational and centrifugal forces at the equator and H is the orthometric height in meters.

The free air gravity anomaly can be calculated as

$$\Delta g = (g_{observed} + \delta g_{FA}) - \gamma \tag{3}$$

3) The effect of a global gravity field model, a long-wavelength gravity anomaly (Δg_{GGM}). After it's removal the resultant reduced gravity anomaly only contains the medium and small wavelengths. For our study we use GGM05 global gravity field model.

$$\Delta g_{smw} = \Delta g - \Delta g_{GGM} \tag{4}$$

4) Correction to account for the gravitational attraction of the atmosphere given by,

$$\begin{aligned}\delta g_{atm} = 0.871 - 1.0298\times10^{-4}H + 5.3105\times10^{-9}H^2 \\ - 2.1642\times10^{-13}H^3 + 9.5246\times10^{-18}H^4 \\ - 2.2411\times10^{-22}H^5\end{aligned} \tag{5}$$

5) The terrain correction or the gravitational effect of topography [3]. At a given point P, the terrain correction

could be evaluated using:

$$\delta g_T = \frac{G\rho}{2}\left[H_P^2\left(\mathcal{F}^{-1}\left(\mathcal{F}(1)\mathcal{F}\left(\frac{1}{r_Q^3}\right)\right)\right)\right.$$
$$+\left(\mathcal{F}^{-1}\left(\mathcal{F}(H_Q^3)\mathcal{F}\left(\frac{1}{r_Q^3}\right)\right)\right)$$
$$\left.-2H_P\left(\mathcal{F}^{-1}\left(\mathcal{F}(H_Q)\mathcal{F}\left(\frac{1}{r_Q^3}\right)\right)\right)\right] \quad (6)$$

Where H_p and H_Q are the heights of the evaluation and data points respectively and

$$r = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2 + (z_P - z_Q)^2}$$

. $\mathcal{F}$ and $\mathcal{F}^{-1}$ are the forward (analysis) and the inverse (synthesis) fourier transforms.
Faye Anomaly is calculated by removing δg_{atm} and δg_T from the small wavelength gravity anomaly.

$$\Delta g_{faye} = \Delta g_{smw} - \delta g_{atm} - \delta g_T \quad (7)$$

B. Disturbing Potential

The disturbing potential is the product of small wavelength geoid undulation and normal gravity values.

$$N_r = \frac{T_r}{\gamma} \quad (8)$$

It can be evaluated using stokes integral, where $S(\psi)$ is the stokes kernel.

$$T_r = \frac{R}{4\pi}\left[\left(\mathcal{F}^{-1}\left(\mathcal{F}(\Delta g_{Faye})\mathcal{F}\left(S(\psi)\right)\right)\right)\right] \quad (9)$$

In closed from, the stokes kernel is given by the formula

$$S(\psi) = \frac{1}{\sin\left(\frac{\psi}{2}\right)} - 6\sin\left(\frac{\psi}{2}\right) + 1 - 5\cos\psi$$
$$- 3\cos\psi\ln\left(\sin\left(\frac{\psi}{2}\right) + \sin^2\left(\frac{\psi}{2}\right)\right) \quad (10)$$

$S(\psi)$ is the unmodified stokes kernel. ψ is the spherical distance, it is computed as a function of the spherical coordinates (latitude φ_P and longitude λ_P) of the computation point and the coordinates (φ_Q, λ_Q) of the data points.

$$\sin^2(\psi/2) = \sin^2\left(\frac{\varphi_P - \varphi_Q}{2}\right)$$
$$+ \sin^2\left(\frac{\lambda_P - \lambda_Q}{2}\right)\cos\varphi_P\cos\varphi_Q \quad (11)$$

C. Restoring the Geoid

By using the Brun's formula (8) we calculate the small wavelength geoid undualtion. Next we calculate the co-geoid by adding long wavelength undulations corresponding to the GGM05 global gravity field model.

$$N_{cogeoid} = N_r + N_{GGM} \quad (12)$$

The geoid can be computed by adding the indirect effect of the corresponding gravity terrain correction (δg_T) to the co-geoid.

$$N_{geoid} = N_{cogeoid} + \delta N_{indirecct} \quad (13)$$

$$\delta N_{indirect} = -\frac{G\rho}{\gamma}\left[\pi H_P^2 + \frac{R^2}{6}\left[\mathcal{F}^{-1}\left(\mathcal{F}(H_Q^3)\mathcal{F}\left(\frac{1}{r_Q^3}\right)\right)\right.\right.$$
$$\left.- H_P^3\mathcal{F}^{-1}\left(\mathcal{F}(1)\mathcal{F}\left(\frac{1}{r_Q^3}\right)\right)\right]$$
$$- \frac{3R^2}{40}\left[\mathcal{F}^{-1}\left(\mathcal{F}(H_Q^5)\mathcal{F}\left(\frac{1}{r_Q^5}\right)\right)\right.$$
$$\left.\left.- H_P^5\mathcal{F}^{-1}\left(\mathcal{F}(1)\mathcal{F}\left(\frac{1}{r_Q^5}\right)\right)\right]\right] \quad (14)$$

III. DATA

Air-borne gravity and elevation data provided by NOAA's National Geodetic survey ,"Gravity for the Redefinition of the American Vertical Datum," or GRAV-D was used [4]. The study area was $1° \times 1°$ ranging between $32.5°N - 33.5°N$ and $105°W - 106°W$, lying in New Mexico, USA. The study area consisted of 21,346 data points.
We used long wavelength undulation and long wavelength gravity anomaly from the GGM05C model, made available by ICGEM [6]. Height data from SRTM Digital Elevation Model, provided by CGIAR-CSI [**9**] was used to calculate terrain correction and indirect effect due to terrain correction.

IV. IMPLEMENTATION

A. Reduction

The observed gravity values from our data are reduced to faye anomalies in following steps:

1) A rough estimate of orthometric heights is made at all 21,346 data points by adding long wavelength undulation (N_{GGM}) from GGM05 model to elevation at those points.
2) Normal gravity and free air correction computed using (1) and (2) respectively for all data points. These are removed from observed gravity values to give free air gravity anomaly Δg.
3) Long wavelength gravity anomaly Δg_{GGM} from the GGM05 model and correction due to atmospheric attraction δg_{atm} are reduced from free air gravity anomaly.
4) The resultant reduced gravity Δg_{smw}^{atm} at 21,346 data points is interpolated to a $0.01° \times 0.01°$ grid.
5) Terrain correction [7] is calculated using heights from SRTM over our study area with a resolution of 3

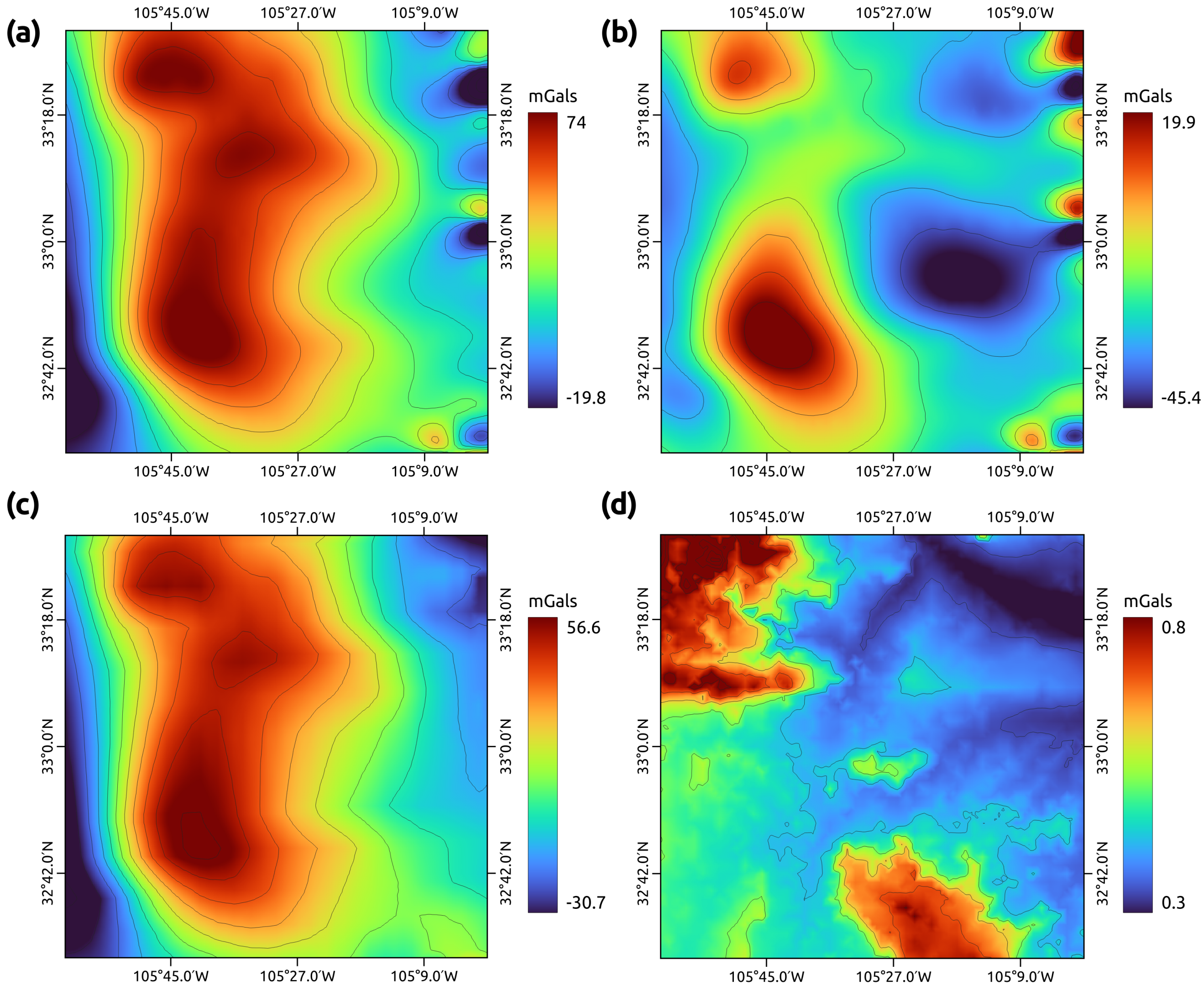

Fig. 1: 0.01° grids of **(a)** Free air gravity anomaly Δg_{FA} **(b)** sma wavelength gravity anomaly Δg_{smw}^{atm} **(c)** Faye gravity anomaly Δg_{faye} **(d)** Terrain Correction δg_T

arcseconds.

$$\delta g_T = \frac{G\rho}{2}\Bigg[H^2\left(\mathcal{F}^{-1}\left(\mathcal{F}(1)\mathcal{F}\left(\frac{W}{r^3}\right)\right)\right) + \left(\mathcal{F}^{-1}\left(\mathcal{F}(H^3)\mathcal{F}\left(\frac{W}{r^3}\right)\right)\right) - 2H\left(\mathcal{F}^{-1}\left(\mathcal{F}(H)\mathcal{F}\left(\frac{W}{r^3}\right)\right)\right)\Bigg] \tag{15}$$

where H are the heights from DEM, r is the euclidean distance and W is the kernel weighing function. r is calculated with respect to the center point of the grid. For our study, we have taken W = 1. A grid of resolution 0.01° is extracted and δg_T is removed from the Δg_{smw}^{atm} to give Δg_{faye}.

B. Restoration

Now the reduced faye gravity anomaly are used to calculate geoid.

1) Disturbing potential is calculated using (9) and (10) with a resolution of 0.01°. It is used to calculate small wave
2) Co-geoid is estimated by adding long wavelength undulation N_{CGM} from the GGM05 model to the grid.
3) Indirect correction is calculated at a resolution of 0.01° and added to the co-geoid to obtain Geoid. For our study the kernel weighing factor W = 1.

$$\delta N_{indirect} = -\frac{G\rho}{\gamma}\Bigg[\pi H^2 + \frac{R^2}{6}\Bigg[\mathcal{F}^{-1}\left(\mathcal{F}(H^3)\mathcal{F}\left(\frac{W}{r^3}\right)\right) - H^3\mathcal{F}^{-1}\left(\mathcal{F}(1)\mathcal{F}\left(\frac{W}{r^3}\right)\right)\Bigg] - \frac{3R^2}{40}\Bigg[\mathcal{F}^{-1}\left(\mathcal{F}(H^5)\mathcal{F}\left(\frac{W}{r^5}\right)\right) - H^5\mathcal{F}^{-1}\left(\mathcal{F}(1)\mathcal{F}\left(\frac{W}{r^5}\right)\right)\Bigg]\Bigg] \tag{16}$$

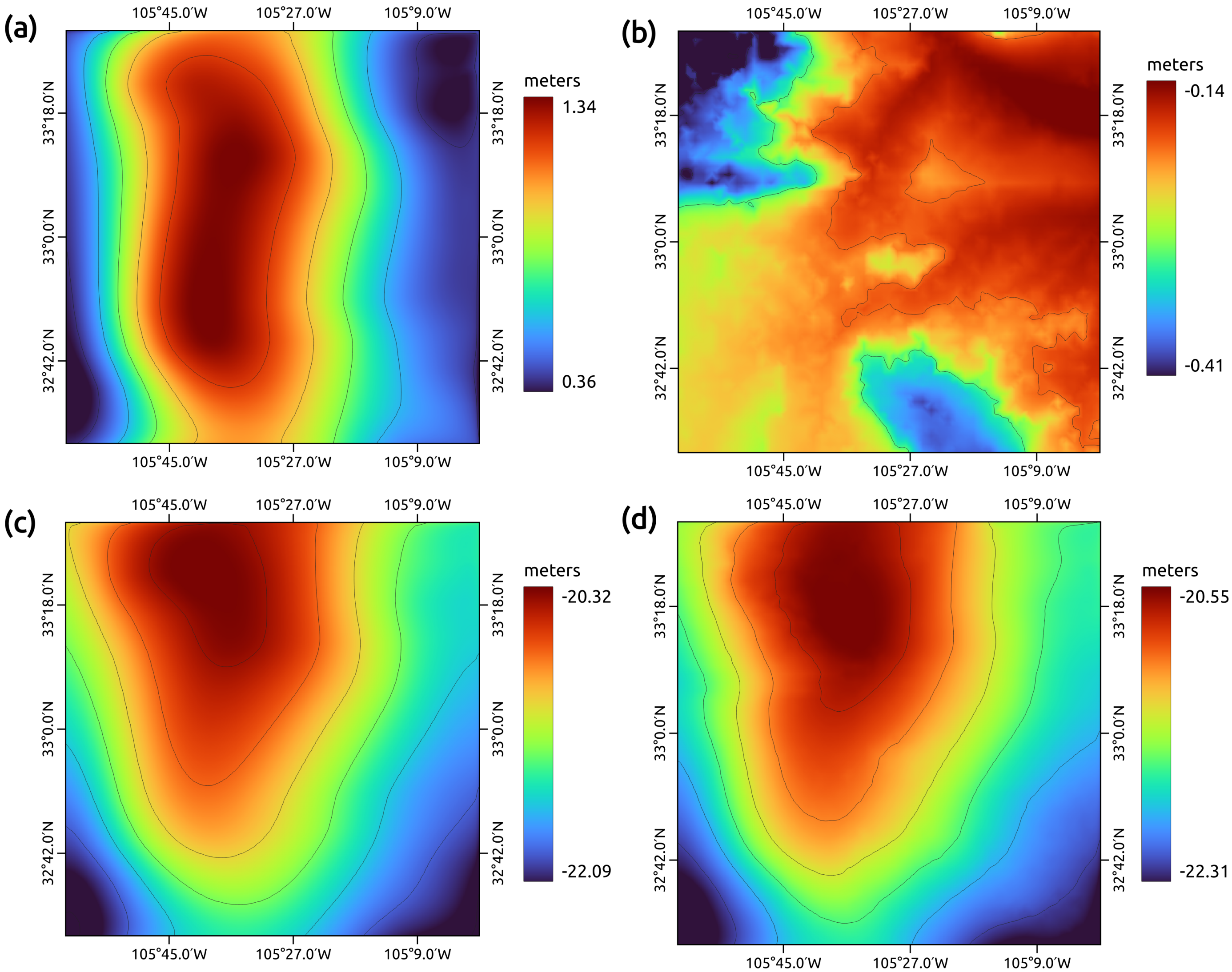

Fig. 2: 0.01° grids of **(a)** small wavelength geoid undulation N_r **(b)** Indirect effect $\delta N_{indirect}$ **(c)** Estimation of cogeoid $N_{cogeoid}$ and **(d)** Estimation of Geoid N_{geoid}

V. RESULTS AND DISCUSSIONS

We estimated regional geoid over a study area of $1^\circ \times 1^\circ$ having a resolution of 0.01°. The Geoid undulations in our study area range from -22.67 meters to -19.8 meters. An interesting observation is that the regions that had low values of terrain correction have high values of indirect effect and vice versa fig.1 and fig.2. The combined free-air and bouguer correction are either an under- or over-correction, depending on the algebraic sign of the geoid. This under- or over- correction gives rise to indirect effect, which needs to be restored for precise geoid calculation. Thus it behaves in the manner reflected in fig.1 and fig.2.
The absolute differences between EGM2008 global geoid (provided by ICGEM [6]) and the regional computed geoid are presented in fig.3. The residuals range between 0.03 to 0.79 meters. The two models show good agreement with each other, with higher differences over higher elevations.

VI. CONCLUSIONS

The resolution we received using air-borne gravity data was 0.6'x0.6'. Thus it is clear that air-borne gravimetry techniques could help develop high resolution geoid models and help in geoid improvements over a region. Further the use of a modified stokes kernel as presented in [2] and [5] could improve the estimation further. Presence of more dense data would help in further improvements.

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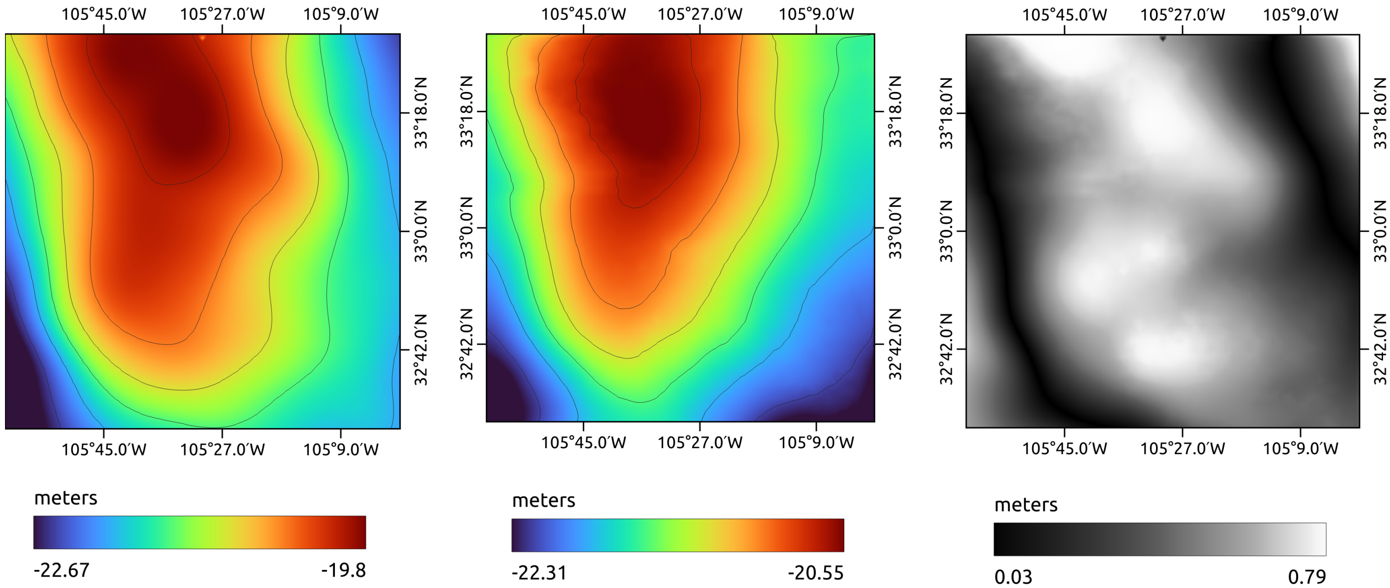

Fig. 3: From the left, Geoid over our study area **(1)** from EGM 2008, computed by ICGEM [6], **(2)** Computed using Remove Restore Approach N_{geoid}, **(3)** Absolute Difference between (1) and (2)

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