CS 663

Assignment 5

Qs 5 - Report

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Verifying the results

The spike for the Image J restoration plot occurs at (31, 231). This is interpreted as (31, -71) by applying a wrap-around on the image of size 300 * 300 for translation. The initial translation applied was (-30, 70).
Hence this new translation will restore Image I from Image J

Similarly, in case of noisy images,we see a spike at (31, 231). This again is interpreted as (31, -71) by applying a wrap-around on the image of size 300 * 300 for translation. But since the original images were noisy, the spike is not clear but surrounded by other non-zero frequencies.

The logarithm plots of the Fourier magnitudes is a constant of value =log(2) because the result of the cross-power spectrum is a complex number of unit magnitude always.

Analysis of time complexities

For an Image of size N * N, this method involves first step, the calculation of Fourier transforms using FFT [time complexity of each being of O(N log(N))] followed by a conjugation [O(N)] & vectorized pointwise multiplication & division [O(1)]. Thus, the overall time complexity is **O(N log N).**
If we use pixel-wise image comparison for an N * N image, the time complexity of predicting the translation would be O(N^2).

Rotation Correction mentioned in the paper

If f2(x,y) is a rotated version of f1(x, y) [with a rotation of θ o],, doing a Fourier Transform in the cartesian coordinates would yield

$$F2(u, v) = F1(u\cos(\theta o) + v\sin(\theta o), -u\sin(\theta o) + v\cos(\theta o)).$$

The magnitudes for both are same. So, we can use the same concept of cross-power spectrum as before by converting the rotation by θ o into a translation. In the polar coordinates, the rotation would become a translation. So we convert the images into polar coordinates & take their Fourier Transform.

$$f2(r, \theta) = f1(r, \theta - \theta o)$$

$$F2(m, n) = \exp(-2\pi j(n.\theta o)) * F1(m, n)$$

Thus, cross-power spectrum of F1(m, n) & F2(m, n) would yield $\exp(2\pi j(n.\theta o))$, using which we can calculate the rotation.
Any translation in x & y would lead to a change in r by ro, such that the cross power spectrum would yield $\exp(2\pi j(m.ro + n.\theta o))$. Hence, displacement & rotation can be figured out. The exact (x, y) translations can be figured out using the original cross-power spectrum in the cartesian coordinates.

* All the Images and plots are included in the MATLAB publish myMainScript.html file