Q2

As discussed in the class, we will use the method of Langrange multipliers to solve the question (2)

The constraints are $f^Te = 0$, $f^Tf = 1$

we write,

$$J(f) = f^TCf - \lambda(f^Tf - 1) - \alpha(f^Te)$$

Taking derivative w.r.t f^T, we get

$$\frac{\partial}{\partial f^T} J(f) = Cf - \lambda f - \alpha e$$

to find an optimum, we set the derivative to 0.

$$\therefore (C - \lambda)f - \alpha e = 0$$

$$Cf - \lambda f - \alpha e = 0$$

multiplying by e^T

$$e^TCf - \lambda e^Tf - \alpha e^Te = 0$$

from maximizing e^TCe we get $\lambda_1 \rightarrow$ maximum eigen value when e corrosponds to it.

$$\therefore e^TCe = \lambda_1$$

$$\Rightarrow Ce - \lambda_1 e = 0 \quad - \{\lambda_1 \text{ is eigen value}\}$$

$$\therefore Ce = \lambda_1 e \Rightarrow e^TC^T = \lambda_1 e^T$$

C is symmetric $\Rightarrow C^T = C$

$$\therefore e^TC = \lambda_1 e^T$$

$$\therefore \lambda_1 e^Tf - \lambda e^Tf - \alpha e^Te = 0$$

$e^T f = 0$ - constraint

$\therefore \alpha = 0$

Hence we have.

$$Cf - \lambda f = 0 \quad \text{i.e.} \quad f^T C f = \lambda$$

Similar to $e^T C e$ we need to choose maximum λ for maximizing $f^T C f$, but as e corresponds to largest value, f should correspond to the next largest value