Q2

For 1D case,

$$g = h * f$$ ← original image.

g ↑ Gradient image

h ↑ convolution kernel to represent gradient operation

Taking fourier transform, The convolution will get converted to multiplication.

$$\therefore G = HF$$

$$F = G/H$$

we shall get original image after taking inverse fourier transform of (G/H).

$h \rightarrow$ Gradient kernel $\Longrightarrow H \rightarrow$ High pass filter

At low frequencies High pass filters tend to zero and hence the calculation of F will be problamatic. even if H is not exactly equal to zero but very close to it, it will add up noise to f.

for 2D case,

We have been given images's gradient in both X and Y direction,

we can write,

$$\cancel{G_x(x,y)} \quad g_x(x,y) = (h_x * f)(x,y)$$

$$g_y(x,y) = (h_y * f)(x,y)$$

Similar to 1D case, taking fourier transforms,

$$G_x(u,v) = H_x(u,v)\, F(u,v)$$

$$G_y(u,v) = H_y(u,v)\, F(u,v)$$

$$\therefore F(u,v) = \frac{G_x(u,v)}{H_x(u,v)}, \quad F(u,v) = \frac{G_y(u,v)}{H_y(u,v)}$$

$H_x(u,v)$, $H_y(u,v)$ are high pass filters as in part (1) i.e. 1D case. at ~~u →0~~ low u, $H_x(u,v)$ will tend to zero and at low v $H_y(u,v)$ will tend to zero. we might be able to calculate F if any one of u, v is large enough to have $H \not\to 0$. but if both of them are tending to zero i.e. have small values, the noise will get amplified.

i.e if u is low, v is high,

$$f = \mathcal{F}^{-1}(F(u,v)) = \mathcal{F}^{-1}\left(\frac{G_y}{H_y}\right)$$

if u is high, v is low

$$f = \mathcal{F}^{-1}(F(u,v)) = \mathcal{F}^{-1}\left(\frac{G_x}{H_x}\right)$$