```
%% HW3 Q1
load fiberpaper.dat
Y = fiberpaper(:, 1 : 4);
X = fiberpaper(:, [7 5 6]);
alpha = 0.01;
X1 = X(1, :)'; X4 = X(4, :)';

X_1 = X(setdiff(1:size(X,1),1),:); Y_1 = Y(setdiff(1:size(Y,1),1),:);
u1 = lrt_env(X_1, Y_1, alpha);
ModelOutput1 = env(X_1,Y_1, u1);
p1env = predict_env(ModelOutput1, X1, 'prediction');
p1penv = predict_env2(X_1, Y_1, X1, 'prediction');
[p1env.value p1env.SE p1penv.value p1penv.SE p1env.SE./p1penv.SE]

X_4 = X(setdiff(1:size(X,1),4),:); Y_4 = Y(setdiff(1:size(Y,1),4),:);
u4 = lrt_env(X_4, Y_4, alpha);
ModelOutput4 = env(X_4,Y_4, u4);
p4env = predict_env(ModelOutput4, X4, 'prediction');
p4penv = predict_env2(X_4, Y_4, X4, 'prediction');
[p4env.value p4env.SE p4penv.value p4penv.SE p4env.SE./p4penv.SE]

% OUTPUT
%
% u =
%
%      2
%
%
% ans =
%
%    21.0006    2.6388   21.0352    2.6254    1.0051
%     7.0854    0.7162    7.0385    0.6966    1.0282
%     5.3011    1.2500    5.2725    1.2356    1.0116
%     0.8613    0.5833    0.8633    0.5711    1.0212
%
%
% u =
%
%      2
%
%
% ans =
%
%    21.8770    2.5521   21.8708    2.5608    0.9966
%     7.3094    0.6952    7.3202    0.6792    1.0236
%     5.7082    1.2082    5.7123    1.2017    1.0054
%     1.0508    0.5639    1.0571    0.5584    1.0099
```

Problem 3.4.

When *S* is the parameter of interest:

1. u is selected to be 1, 1 and 0 by AIC, BIC and LRT at $\alpha = 0.01$, respectively. Hence we build our partial envelope model with $u = 1$.
2. The OLS and partial envelope estimator of the coefficient vector are close to each other (in the first output matrix, cols 1 and 2).
3. Only one of the envelope coefficient/SE ratios is > 2 (column 4), hence significant at 95% level.
4. Partial envelope gives gains in SE for all components of the coefficient matrix, although they are at max about 2-fold (col 5).
5. In the $\hat{\Gamma}$ matrix (which is a vector here, col 6), all the elements are small in absolute value compared to the 4th one. This means that **variation in Ozone levels is a big part of the material variation in solar radiation**, and variations in other variables are mostly immaterial.

```
%% HW3 Q2
load Ozone.txt
Y = Ozone(:,3:7); X = Ozone(:,1:2);
ols = fit_OLS(X,Y); olsSE = sqrt(diag(ols.SigmaOLS,0)/ols.n);
alpha = 0.01;

%% parameter of interest is solar radiation
XS.X1 = X(:,2); XS.X2 = X(:,1);
[modelselectaic(XS, Y, 'penv') modelselectbic(XS, Y, 'penv')
modelselectlrt(XS, Y, alpha, 'penv')] %% OUTPUT: 1 1 0
uS = 1;
penvS = penv(XS, Y, uS);
[ols.betaOLS(:,2) penvS.beta1 penvS.asySE ...
sqrt(ols.n)*penvS.beta1./penvS.asySE penvS.ratio penvS.Gamma]

% coefficient estiamtes by OLS and coeff estimates, asymptotic SE, ratio
statistic, SE
% ratio and Gamma for partial envelope
%     0.0117    0.0097    0.0384    1.6379    1.8033   -0.0988
%    -0.0064   -0.0033    0.0236   -0.9146    2.5548    0.0340
%     0.0205    0.0153    0.1041    0.9540    1.8561   -0.1562
%     0.0952    0.0963    0.2953    2.1135    1.0067   -0.9817
%     0.0027    0.0031    0.0167    1.1986    2.3644   -0.0315
```

When *W* is the parameter of interest:

1. Here also u is selected to be 1, 1 and 0 by AIC, BIC and LRT at $\alpha = 0.01$, respectively. Hence we take $u = 1$ for our `penv` model.
2. The OLS and partial estimator of the coefficient vector are very different, especially for NO_2 and Ozone levels (in the first output matrix, cols 1 and 2).
3. All the envelope coefficient/SE ratios are > 2 (col 4) in absolute value.
4. Partial envelope gives gains in SE over OLS for all components of the coefficient matrix, and the gains are massive for, yet again, NO_2 and Ozone levels (col 5).
5. In the $\hat{\Gamma}$ matrix (which is a vector here, col 6), elements corresponding to these two variables are smaller in absolute value than other elements.
6. Compared to OLS the much smaller coefficients and standard errors, and also the corresponding elements in $\hat{\Gamma}$ for NO_2 and Ozone means that according to the partial envelope analysis, **variations in these two variables are mostly immaterial in wind speed variations**.

```
%% parameter of interest is wind speed
XW.X1 = X(:,1); XW.X2 = X(:,2);
[modelselectaic(XW, Y, 'penv') modelselectbic(XW, Y, 'penv')
modelselectlrt(XW, Y, alpha, 'penv')] %% OUTPUT: 1 1 0
uW = 1;
penvW = penv(XW, Y, uW);
[ols.betaOLS(:,1) penvW.beta1 penvW.asySE ...
sqrt(ols.n)*penvW.beta1./penvW.asySE penvW.ratio penvW.Gamma]

% coefficient estiamtes by OLS and coeff estimates, asymptotic SE, ratio
statistic, SE
% ratio and Gamma for partial envelope
%    -0.1382    0.0710    0.1674    2.7481    4.5328    0.4504
%    -0.1925   -0.0749    0.1687   -2.8771    3.9233   -0.4754
%    -0.2113   -0.0166    0.0410   -2.6197   51.6941   -0.1051
%    -0.7868   -0.0106    0.0256   -2.6786  127.1239   -0.0673
%     0.0713    0.1175    0.2437    3.1235    1.7782    0.7454
```