

Comments on the manuscript "Robust estimation of principal components from depth-based multivariate rank covariance matrix" by Majumdar and Chatterjee

A scatter matrix functional based on a chosen depth functional and the spatial sign is proposed. The connections to competing, similar rank-based scatter matrices and M-functionals in the literature are not discussed in a sufficient way. The proofs also need re-examination. See the comments below.

1. I think that the concepts of spatial sign and spatial sign covariance matrix were first introduced by Möttönen and Oja (1995).
2. Condition (P2*) is not clear to me. Is $M_D(F) = \sup_{\mathbf{x} \in \mathbb{R}^p} D(\mathbf{x}, F_{\mathbf{x}}) < \infty$?
3. What is the strange term 'htped'? Is it an acronym of something? Do you really need this general concept? Could one simply define $\tilde{D}(\mathbf{x}, F) = M_D(F) - D(\mathbf{x}, F)$?
4. The multivariate rank as defined on page 8 looks like a mixture of several approaches in the literature. Several competing concepts of multivariate rank found in the literature such as the marginal rank function (Puri and Sen, 1971), spatial rank function (Möttönen and Oja, 1995), the rank function by Hallin and Paindaveine (2002) and Monge-Kantorovich rank (Chernozhukov et al, 2015) should be discussed.
5. The matrix $Cov(\tilde{\mathbf{X}})$ looks like a one-step M-functional. The connections between the proposed rank covariance matrix and the (one-step) M-estimates should be clarified. The role and choice of location functional $\mu(F)$ should be carefully discussed. Note that also the depth function centers the data with respect to another (?) location functional which may cause problems.
6. I cannot follow the steps in the proof of Lemma 4.1. Is the convergence $D_{\mathbf{X}}^n(\mathbf{x}) \rightarrow D_{\mathbf{X}}(\mathbf{x})$ pointwise or uniform? The steps leading to (11) seem wrong to me. I think that (11) is true with probability $\min(1 - \delta, \delta)$? Also I cannot see why $A_k(B_k + C_k) \rightarrow_P 0$.

References

- Chernozhukov et al. (2015). Monge-Kantorovich Depths, Quantiles, Ranks and Signs. arXiv:1412.8434 [math.ST]
- Hallin, M., and Paindaveine, D. (2002). Optimal tests for multivariate location based on interdirections and pseudo-Mahalanobis ranks, *The Annals of Statistics* 30, 1103-1133.
- Möttönen, J. and Oja, H. (1995). Multivariate spatial sign and rank methods. *Journal of Nonparametric Statistics* 5, 201-213.
- Puri, M.L. and Sen, P.K. (1971) *Nonparametric Methods in Multivariate Analysis*. Wiley, New York.