REPORT ON THE MANUSCRIPT
"ON WEIGHTED MULTIVARIATE SIGN FUNCTIONS"

The paper introduces generalizations to the spatial median that employ data-dependent weights, and applies proposed methods to robust scatter estimation of elliptically symmetric distributions as well as closely related problems. Presented ideas certainly deserve attention, however, I feel that the manuscript requires significant work. In particular, it was not clear to me that proposed estimators can be actually computed without knowledge of parameters of the distribution that are not available. I suggest that authors clarify this point early on.

Here are few more detailed comments:

(a) Do I understand correctly that your weights are not allowed to depend on q? Then your claim on top of page 3 that this is the same object that has been studied before (e.g., in [Kol97]) is incorrect. If it is allowed to depend on q, then please explain how your results are different from the existing literature.
(b) Regarding the proof of Theorem 2.1: if you claim that this is a new result, the proof can not be omitted completely: please include it in the supplementary material. If it is not new, please give a precise reference.
(c) I could not understand Corollary 2.2: is q_0 still a minimizer of $\Psi(q; 0, \mathbb{F})$? If so, how can the weight depend on this unknown quantity? Even if q_0 is a fixed vector, this still does not make sense to me. Additionally, please show that conditions A1-A3 hold in your examples.
(d) Page 3 states "We report only the case of the weighted spatial median for brevity.." In this case, it could be better to focus on the median everywhere and avoid mentioning general quantiles.
(e) Corollary 2.4 states "then $S(X; q_0)$ is uniform on the unit ball in $\mathbb{R}^p$.." – do you mean $S(Z; q_0)$? If X has arbitrary elliptically symmetric distribution, this is not true. Moreover, why is it interesting in this case to consider the case when $\mathbb{F}$ is the (unknown) distribution of X?
(f) Page 5: "..we propose using $W(x, \mathbb{F}_n)$..: please define what you mean by an empirical distribution function here, as X is a vector.
(g) The weight functions discussed in section 2.1 can only be computed if the distribution of X is known. Why would such functions be of any use in statistical context? If the unknown distribution is replaced by its empirical version, why would all the useful properties be preserved? I suggest explaining these points clearly in the beginning.
(h) Equation (5): is $\tilde{X}$ a statistic?
(i) On page 7, it is stated that "We now discuss the properties of the sample version $\hat{\tilde{\Sigma}}$ computed from X. In practice, we cannot obtain $W(x)?W(x, \mathbb{F})$, and consequently use $W(x, \mathbb{F}_n)$ instead.." – this is a good point, but I suggest discussing it earlier in the paper, as it affects many assumptions made prior to this statement.

Minor comments:

(a) Last paragraph of page 3: should $\hat{q}_n$ be $\hat{q}_{nW}$?

References

[Kol97] V. I. Koltchinskii. M-estimation, convexity and quantiles. *Ann. Statist.*, 25(2):435–477, 1997.