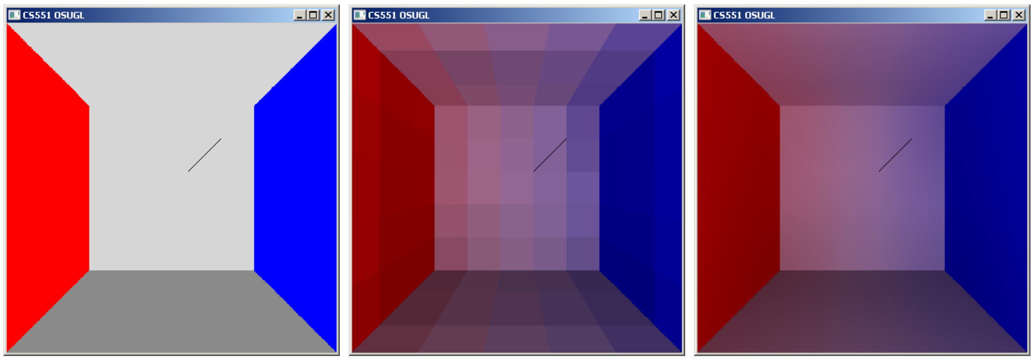
Radiosity



(a) (b) (c)

Figure 1: Cornell Box: (a) Simple Rendering, (b) Rendering with Radiosity, (c) Radiosity with Smoothing

Abstract

Radiosity is a global illumination technique that models diffuse-diffuse interactions between surfaces. We implemented this algorithm in an attempt to understand it in depth and tested its behavior under simple scenarios. In this paper we report the results from these experiments, the challenges we faced in implementing it, and our perception of its strengths and weaknesses.

CR Categories: I.3.5 [Computer Graphics]: Rendering Equation—Radiosity;

Keywords: radiosity, diffusion, rendering.

1 Introduction

Radiosity was introduced to model light interactions between diffuse surfaces [Goral et al. 1984]. This presents one perspective on how to solve the rendering equation [Kajiya 1986]. Due to its simplifying assumptions, the original radiosity algorithm was limited to diffuse-diffuse interactions and was unable to account for other forms such as specular-specular and specular-diffuse. Despite this limitation, radiosity made it possible to render certain aspects of natural lighting (like *color bleeding*) that previously had not been possible using other techniques like Phong shading and ray tracing.

2 Previous Work

Radiosity was first introduced to computer graphics by [Goral et al. 1984]. Figure 1 shows the classic setup described in that paper. Here the eye is placed just in front of the wall which emits light. The colors for each face are shown in Figure 1(a). The notable aspect in this scene is *color bleeding* – the red and blue colors from side faces tinge the other grey and white faces.

One major challenge in radiosity is the computation of geometric properties called *form factors* between each element in the scene. To compute these somewhat efficiently, [Cohen and Greenberg 1985] introduced the *hemi-cube* approximation that can handle occluding objects in complex scenes. This is still computationally expensive and further refinements continue to be made ([Cohen et al. 1988]).

Radiosity has been extended by [Immel et al. 1986] to render some types of diffuse-specular interactions. However, it is common to use hybrid techniques where radiosity is used in conjunction with other methods (like ray-tracing) to render all desired forms of light interactions ([Wallace et al. 1987]).

3 Radiosity Theory

Radiosity, usually denoted by B, is defined as the energy per unit area leaving a surface patch per unit time and is the sum of the emitted and the reflected energy [Watt 2000].

$$B_iA_i = E_iA_i + \rho_i \sum_{j=1}^{n} B_jF_{ji}A_j \tag{1}$$

where

$$\begin{aligned}
B_i &= \text{radiosity of patch } i, \\
E_i &= \text{emission of patch } i \\
A_i &= \text{area of patch } i \\
A_j &= \text{area of patch } j \\
\rho_i &= \text{reflectivity of patch } i \\
F_{ji} &= \textit{form factor} \text{ from } j \text{ to } i \\
n &= \text{number of discrete patches}
\end{aligned}$$

Stated in words, Equation 1 means the following – for a patch i:

$$\text{radiosity} \times \text{area} = \text{emitted energy} + \text{reflected energy}$$

The form factors follow the reciprocity relationship [Watt 2000]:

$$F_{ij}A_i = F_{ji}A_j \tag{2}$$

Now, dividing Equation 1 on both sides by A_i, we get the more familiar radiosity equation:

$$B_i = E_i + \rho_i \sum_{j=1}^{n} B_j F_{ij} \tag{3}$$

We can express Equation 3 in matrix notation as:

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} \tag{4}$$

In Equation 4 the only unknowns are the B_i's. We assume that the reflectances (ρ_i), emissions (E_i) and the form factors (F_{ij}) are either known or may be computed from the environment.

We should note that the left-most matrix in Equation 4 is diagonally dominant (when energy balance is satisfied). Therefore, we can use the iterative algorithm *Gauss-Seidel* (GS) and guarantee its convergence. This is the classic way of solving the radiosity equation. [Cohen et al. 1988] introduced the Progressive Refinement (PR) algorithm for solving Equation 4 that updates the variables in a way similar to how light travels through the scene. This makes the algorithm progress gracefully between iterations and allows the user to stop at any point the images look acceptable. Each iteration of PR updates the radiosity of only one patch and has $O(n)$ (linear) complexity in the number of patches. GS updates all patch radiosities in each iteration and has $O(n^2)$ complexity. In this project both algorithms (GS and PR) have been implemented.

An important implication of solving Equation 4 with GS is that if the environment has occlusions and we do not take them into account, then the energy balance is no longer satisfied and diagonal dominance is lost. The algorithm will likely fail to converge in such a case. Hence, occlusions *must* be accounted for.

Form factor F_{ij} is defined as the fraction of energy leaving patch j which arrives at patch i. This only depends upon the geometry of the environment. Computing this for a large and complex environment is a challenge because, first, no closed analytical form exists and as a result we have to employ numerical techniques. Second, it usually implies an $O(n^2)$ computational complexity and storage requirement.

For the geometry shown in Figure 2 the form factor can be computed by the double integral:

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} V(i,j) \frac{cos\phi_i cos\phi_j dA_j dA_i}{\pi r^2} \tag{5}$$

$V(i,j)$ in the above equation is the *visibility* term which is 1 when the patches are visible to each other and 0 otherwise.

Nusselt developed a geometric analog for the form factor which was shown to be equivalent to Equation 5. This analog forms the basis for the *hemi-cube* approximation [Cohen and Greenberg 1985].

The hemi-cube approach is probably the most common for estimating form factors in real applications as it has $O(n^2)$ complexity which includes the visibility (occlusion) test. The implementation

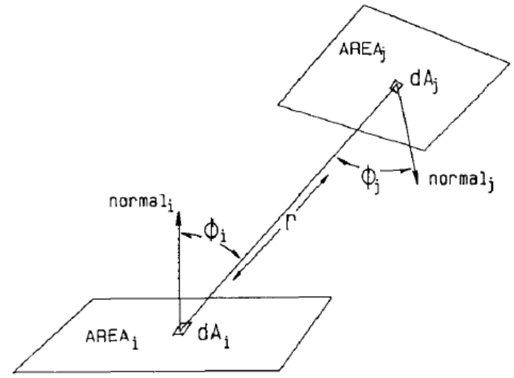


Figure 2: Form factor Geometry [Cohen 1985]

for this project estimates form factors using Equation 5 and numerical integration. To test for visibility, this implementation uses *ray-tracing* which has a higher computational complexity ($O(n^3)$) but is simpler to implement. For complex scenes having thousands of patches, spatial data structures (BSP-tree, Octree, etc.) would be essential.

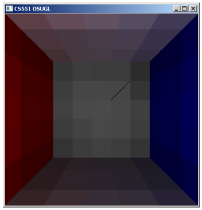
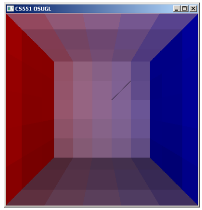
4 Results and Discussion

We have already seen *color bleeding* in Figure 1 which is one distinguishing aspect of radiosity. Next, for the same scene Figures 3 (a) and (b) show the difference between iteration 1 and iteration 4 of Gauss-Seidel algorithm. The scene is much brighter after iteration 4 which represents the aspect that the diffuse interactions managed to reflect and disperse more light as the iterations progressed. These images are not smoothed so that we can see the color contrasts clearly.

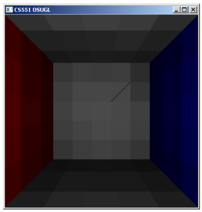
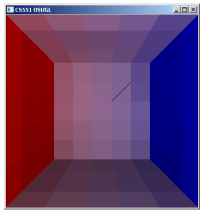
Figure 1(c) shows the final image after 10 iterations with smoothing. For smoothing, each vertex was assigned the average of adjacent patch colors and final rendering was done using color interpolation between vertices. Color bleeding is clearly apparent even after smoothing.

Figure 3 compares the iterations of Gauss-Seidel (GS) and Progressive Refinement (PR) algorithms for the Cornell Box scene. Both algorithms converged to similar results. Although, PR seems to require more iterations for convergence, its iterations are *much* less expensive than GS. [Cohen et al. 1988] adds an ambient intensity term ΔA as an estimate of the difference between the final total radiosity after convergence and the current total. We refer to this algorithm as APR (Ambient intensity with PR) in Figures 3 (e) and (f). This enables most objects to be 'illuminated' right from first iteration. As the algorithm converges towards the true solution, the estimated ΔA graduallly reduces to zero.

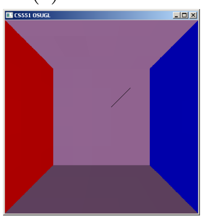
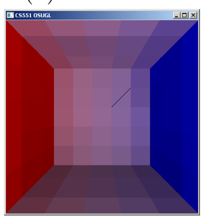
Figure 5, Figure 6, Figure 7, and Figure 8 demonstrate radiosity in slightly more complex scenes having occlusions. The default colors of walls in the rooms are shown in Figure 4. In Figure 5 and Figure 6 note the soft shadow on the right due to the pillar. This soft shadow is an automatic artifact of radiosity and looks very natural. This is one of the strengths of radiosity. For a good realization of the shadows in the smoothed image, the patches should be much smaller. Figure 7 and its smoothed rendering Figure 8 demonstrate the situation where the patches are not fine relative to all objects in the environment. Radiosity manages to compute the shadow effects (Figure 7) but the smoothing effectively loses them (Figure 8). In these scenes, most of the time was spent in the pair-wise patch

(a) GS 1 iter (b) GS 4 iters

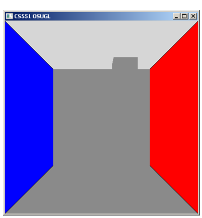
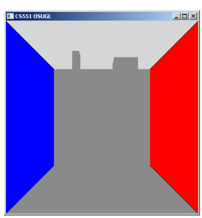



(c) PR 50 iters (d) PR 500 iters

(e) APR 1 iter (f) APR 500 iters

Figure 3: Cornell Box iterations with Gauss-Seidel (GS), Progressive Refinement (PR), and PR with Ambient intensity (APR). This scene has 175 patches. Note that one iteration of GS has $O(n^2)$ complexity whereas one iteration of PR and APR has only $O(n)$ complexity.

(a) Scene 1 (b) Scene 2

Figure 4: Sample scenes rendered without any shading. These show the default diffuse (face color) values for each face. The gray columns inside the rooms have got merged with the far wall.

visibility test for form-factor determination. The Gauss-Seidel algorithm itself took very less time for 10 iterations.

5 Conclusion

Radiosity attempts to solve the rendering equation by energy balance. The simplifying assumptions make it practical but restrict its application to limited types of light interactions. However, they can be combined with other techniques to provide much richer forms of realistic visual effects. A major challenge is in computing the form factors. To build a scalable support for radiosity, optimized data structures must be used for storing the patches such as binary space partitioning (BSP) trees. Additionally, optimized algorithms like ray-casting or hemi-cubes should be used for complex environments. Moreover, the patches should be made much smaller compared to the environment so that finer aspects like soft shadows are not lost due to color smoothing. Alternatively, a better smoothing technique might be used (rather than simple averaging).

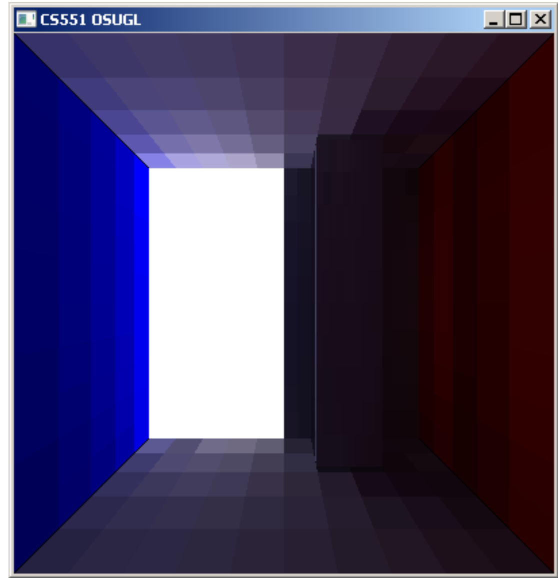


Figure 5: Scene 1 with occlusions. The left side of the far wall might be considered as open to daylight. Other walls do not emit any light. This scene has 1100 patches. Results shown are after 10 iterations of Gauss-Seidel algorithm.

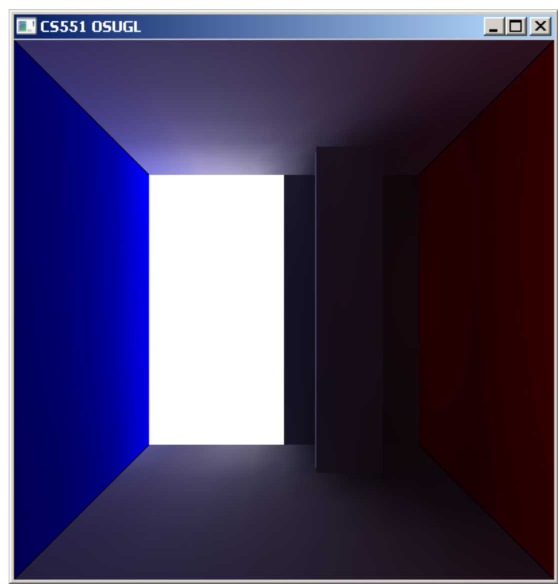


Figure 6: Scene 1 with smoothing and occlusions.

The radiosity demonstration program developed for this project can be run by following the instructions in the file **run-radiosity.txt**. This file is present in the submitted codebase.

Acknowledgments

I would like to thank Prof. Eugene Zhang for the guidance provided in the CS 551 course in Winter 2015.

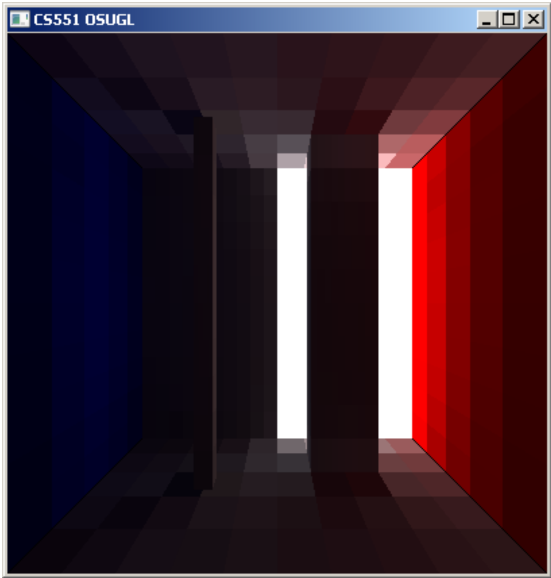


Figure 7: Scene 2 with occlusions. The right side of the far wall might be considered as open to daylight. Other walls do not emit any light. This scene has 1500 patches. Results shown are after 10 iterations of Gauss-Seidel algorithm.

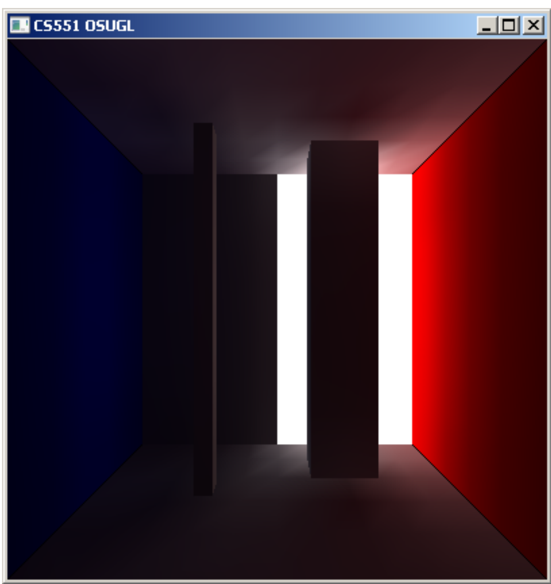


Figure 8: Scene 2 with smoothing and occlusions.

References

COHEN, M. F., AND GREENBERG, D. P. 1985. The hemi-cube: a radiosity solution for complex environments. In *SIGGRAPH '85: Proceedings of the 12th annual conference on Computer graphics and interactive techniques*, ACM Press, New York, NY, USA, 31–40.

COHEN, M. F., CHEN, S. E., WALLACE, J. R., AND GREENBERG, D. P. 1988. A progressive refinement approach to fast radiosity image generation. In *SIGGRAPH '88: Proceedings of the 15th annual conference on Computer graphics and interactive techniques*, ACM Press, New York, NY, USA, 75–84.

GORAL, C. M., TORRANCE, K. E., GREENBERG, D. P., AND BATTAILE, B. 1984. Modeling the interaction of light between diffuse surfaces. In *SIGGRAPH '84: Proceedings of the 11th annual conference on Computer graphics and interactive techniques*, ACM Press, New York, NY, USA, vol. 18, 213–222.

IMMEL, D. S., COHEN, M. F., AND GREENBERG, D. P. 1986. A radiosity method for non-diffuse environments. *SIGGRAPH Comput. Graph. 20*, 4 (Aug.), 133–142.

KAJIYA, J. T. 1986. The Rendering Equation. In *Proceedings of the 13th Annual Conference on Computer Graphics and Interactive Techniques*, ACM, New York, NY, USA, vol. 20 of *SIGGRAPH '86*, 143–150.

WALLACE, J. R., COHEN, M. F., AND GREENBERG, D. P. 1987. A two-pass solution to the rendering equation: A synthesis of ray tracing and radiosity methods. In *SIGGRAPH '87: Proceedings of the 14th annual conference on Computer graphics and interactive techniques*, ACM Press, New York, NY, USA, 311–320.

WATT, A. 2000. *3D Computer Graphics*. Addison-Wesley.