Optimal Scheme For 3-Node Network (Main-II) ($a_1 \neq a_2$ & $b_1 = b_2$)

Monday, 24 May 2021 12:42 PM

## Case (iii) $a_1 \neq a_2$ and $b_1 = b_2$

$$k = \max\{a_1, a_2\} \cdot \left( \frac{T - b + 1 - \max\{a_1, a_2\}}{b} \right)$$

w.l.o.g. assume $a_1 > a_2$

$$\Rightarrow k = a_1 \cdot \left( \frac{T - b + 1 - a_1}{b} \right)$$

$$m_1 = a_1 + a_1 \cdot \left( \frac{T - b + 1 - a_1}{b} \right) = a_1 \cdot \left( \frac{T + 1 - a_1}{b} \right)$$

$$m_2 = a_2 + a_1 \left( \frac{T - b + 1 - a_1}{b} \right) < m_1$$

### Dispersion-span Constraint :

Ineq-①

$$T - b - (m_1 - 1) - (b - a_1) \left[ \frac{m_1 - 1}{a_1} \right] \geq 0$$

$$T - b - a_1 \cdot \left( \frac{T + 1 - a_1}{b} \right) + 1 - (b - a_1) \left[ \frac{T + 1 - a_1}{b} - \frac{1}{a_1} \right]$$

$$\leq T - b - a_1 \cdot \left( \frac{T + 1 - a_1}{b} \right) + 1 - (b - a_1) \left( \frac{T + 1 - a_1}{b} - 1 \right)$$

$$= T - b + a - T - 1 + a_1 + b - a_1$$

$$= 0$$

$$\Rightarrow T - b_2 - (m_1 - 1) - (b_1 - a_1) \left[ \frac{m_1 - 1}{a} \right] \leq 0$$

Same Argument that was made in case(i)

Equality holds iff $b \mid T + 1 - a_1$

Therefore for Ineq ① to hold

$$b \mid T + 1 - a$$

$\therefore$ Dispersion Span constraint does not hold

Ineq-②

$$T - b - (m_2 - 1) - (b - a_2) \left[ \frac{m_2 - 1}{a_2} \right] \geq 0$$

$$T - b - a_2 - k + 1 - (b - a_2) \left[ \frac{m_2 - 1}{a_2} \right]$$

$$= T - b - a_2 - k + 1 - (b - a_2) \left[ \frac{m_2}{a_2} - \frac{1}{a_2} \right]$$

$$\leq T - b - a_2 - k + 1 - (b - a_2) \left( \frac{m_2}{a_2} - 1 \right)$$

$$= T - b - a_2 - k + 1 - (b - a_2) \left( \frac{k}{a_2} \right)$$

$$= T - b - a_2 + 1 - b \cdot \frac{k}{a_2}$$

$$= T - b - a_2 + 1 - \frac{a_1}{a_2} \cdot (T - b - a_1 + 1)$$

$$= \frac{(a_2 - a_1)(T - b + 1) - (a_2 - a_1)(a_1 + a_2)}{a_2}$$

$$= \frac{(a_2 - a_1)}{a_2} \underbrace{(T - b + 1 - a_1 - a_2)}_{+ve} < 0$$

(-ve)

$$\Rightarrow T - b_1 - (m_2 - 1) - (b - a_2) \left[ \frac{m_2 - 1}{a_2} \right] < 0$$

Therefore Ineq-② does not hold

Same Argument that was made in case(i)

$$\gamma < \frac{m_2}{a_2} < \gamma + \frac{1}{a_2}, \quad \gamma = \left[ \frac{m_2}{a_2} \right]$$

$$a_2 \gamma < m_2 < a_2 \gamma + 1$$

not possible

$\because m$ is an integer

If $T + 1 - a_1 = b$

then $k = 0$

but $k > 0$

$\therefore T + 1 - a_1 \geq 2b$

$\Rightarrow T + 1 - a_1 - b - a_2 \geq b - a_2 > 0$

Whenever $a_1 \neq a_2$ and $b_1 = b_2$ no valid SDF based construction is Rate-optimal