Optimal scheme for three-node network (Main I) (Introduction)

$(2,1)$ - achievable $(4,3,2,\{(3,1),(1,2)\})_{\mathbb{F}}$ streaming code

with rate $= \min\left\{ C_{T-N_1,M_1} \;,\; C_{T-N_2,M_2} \right\}$

$$= \frac{T-M_1-N_1+1}{T-\min(N_1,M_1)+1} \;,\; \frac{4-2+1}{4-2+2} = \frac{1}{2}$$

Code Construction :

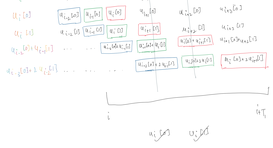
$(3,1)$ - channel : $(4,2,(3,2))_{\mathbb{F}}$ streaming code

$T_1 = T - N_3 = 3$

$N = N_1 = 2$

$D = (T_1, T_1+1, \ldots N) = (3,2)$

$k = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$



Streaming code over 3-Node Network paper

$$\frac{k}{\max\{N_1,M_1\}} = \frac{T-N_3-N_1+1}{T-\min\{N_1,M_1\}+1} \;\Big|\; \frac{k}{\max\{N_2,M_2\}} = \frac{T-N_3-N_2+1}{T-\min\{N_2,M_2\}+1}$$

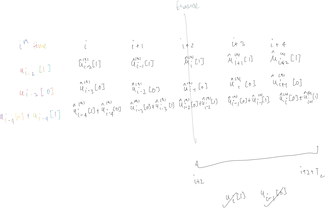
$$\frac{k}{k+\max\{N_1,M_1\}} = \frac{T-N_3-N_1+1}{T-N_1-N_3+1+\max\{N_1,M_1\}} \Rightarrow \frac{k}{\max\{N_1,M_1\}} = \frac{T-N_3-N_1+1}{\max\{N_1,M_1\}+1}$$

$$\Rightarrow k = T-N_3-N_1+1$$

$(3,1)$ channel $(3,2,(3,1))$

$T_2 = T - N_3 = 2$

$N = N_2 = 1$



Streaming codes that can correct burst and random erasures

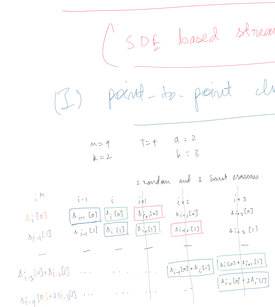
(SDF based streaming codes)

(I) point-to-point channel

$m = 4$ $\quad T = 4 \quad a = 2$

$k = 2 \quad b = 3$

2 random and 1 burst erasures



$w = 1+7$

Delay spectrum $= (4,3)$

Rate $= \frac{k}{m}$

$R_{opt} = \frac{T+1-a}{T+1-a+b} \;\&\; C_{T,(a,b)}$

For $k=2, m=4, b=1$

Rate $= \frac{1}{2}$, $R_{opt} = \frac{1}{2}$ $\therefore$ Rate $=$ Rate$_{opt}$

$\therefore$ $(b=3, a=2)$ - achievable $(4,2,(4,3))_{\mathbb{F}}$ streaming code with optimal rate

Note:

$b \mid T+1-a$

$\therefore$ streaming code based on SDF will be Rate optimal

Optimal Rate for 3-Node Relay Network

$$R_{opt\text{-}3\text{-}node} = \frac{k}{\max\{N_1,M_1\}} = \min\left\{ C_{T-b_1,a_1,b_1} \;,\; C_{T-b_2,a_2,b_2} \right\}$$

where $C_{T_i,a_i,b_i} = \frac{T_i - a_i + 1}{T_i - a_i + 1 + b_i}$

(II) Three-Node Relay channel

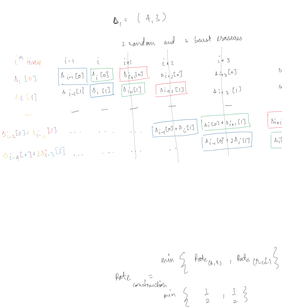
Example 1 : Some streaming codes for hop 1 & 2 $\quad T=7$

$((3,2),(3,2))$ - achievable $(4,4,2,((4,3),(3,4)))$ streaming code

$(3,2)$ - channel : $m = 4$, $k = 2$, $a_1 = m-k = 2$, $b_1 = 3$, $T_1 = T-b_2 = 4$

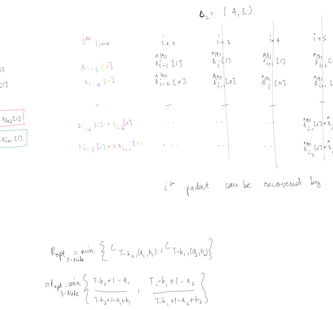
$D_1 = (4,2)$

2 random and 1 burst erasures



$(4,3)$ - channel : $m = 4$, $k = 2$, $a_2 = m-k = 2$, $b_2 = 3$, $T_2 = T-b_1 = 4$

$D_2 = (4,3)$



i^{th} packet can be recovered by $i+T_2 = 7$

$$\text{Rate}_{\text{construction}} = \min\left\{ \text{Rate}_{(3,2)} \;,\; \text{Rate}_{(4,3)} \right\} = \min\left\{ \frac{1}{2} \;,\; \frac{3}{4} \right\} = \frac{1}{2}$$

$$R_{opt\text{-}3\text{-}node} = \min\left\{ C_{T-b_1,a_1,b_1} \;,\; C_{T-b_2,a_2,b_2} \right\}$$

$$\Rightarrow R_{opt\text{-}3\text{-}node} = \min\left\{ \frac{T-b_1+1-a_1}{T-b_1+1-a_1+b_1} \;,\; \frac{T-b_2+1-a_2}{T-b_2+1-a_2+b_2} \right\}$$

$$\Rightarrow R^{opt}_{3\text{-}node} = \frac{7-3+1-2}{7-3+1-2+3} = \frac{1}{2}$$

$\therefore$ Rate$_{\text{construction}}$ $=$ R$_{opt\text{-}3\text{-}node}$

Test cases for 3-Node Relay Network

Code 1 : $b_1 = 2$, $a_1 = 2$, $M_1 = 6$, $k_1 = 4$, $T_1 = 7$ $\quad R = R_{opt}$

$D_1 = (7,6,4,3)$

Code 2 : $b_2 = 5$, $a_2 = 4$, $M_2 = 8$, $k_2 = 4$, $T_2 = 8$ $\quad R = R_{opt}$

$D_2 = (8,7,6,5)$

Code 3 : $b_3 = 6$, $a_3 = 4$, $M_3 = 8$, $k_3 = 4$, $T_3 = 9$ $\quad R = R_{opt}$

$D_3 = (9,8,7,6)$ $\quad T = 14$

Code 4 :

$b_4 = 4$, $a_4 = 2$, $M_4 = 6$, $k_4 = 4$, $T_4 = 9$, $R = R_{opt} = \frac{8}{4+4} = \frac{2}{3}$

$D_4 = (9,8,5,4)$

$4,5,6,9$

1-4 $\quad T=12$

$$R^{opt}_{3\text{-}node} = \min\left(\frac{T-b_4-a_1+1}{T-b_4-a_1+1+b_1} \;,\; \frac{T-b_1-a_2+1}{T-b_1-a_2+1+b_2} \right)$$

$T - b_4 = 8 \neq T_1$

Not - optimal

Though there exists a Rate optimal streaming code with different values of M_1 & T for $a_1 = a_2 = 2$ & $b_1 = 3$ & $b_2 = 4$

1-2 $\quad T=12$

$$R_{opt\text{-}3\text{-}node} = \min\left(\frac{T-b_1-a_1+1}{T-b_1-a_1+1+b_1} \;,\; \frac{T-b_2-a_2+1}{T-b_2-a_2+1+b_2} \right) = \min\left(\frac{6}{6+3} \;,\; \frac{6}{6+5} \right)$$

$$= \frac{6}{11} > \frac{1}{2}$$

$\therefore$ 1 - 2 $\quad$ Not optimal

1-1 $\quad T=10$

$$R^{opt}_{3\text{-}node} = \frac{10-3-2+1}{10-3-2+1+3} = \frac{6}{9} = \frac{2}{3}$$

$\therefore$ 1 - 1 $\quad$ optimal

1-3 $\quad T=13$

$$R_{opt\text{-}3\text{-}node} = \min\left(\frac{T-b_1-a_1+1}{T-b_1-a_1+1+b_1} \;,\; \frac{T-b_3-a_2+1}{T-b_3-a_2+1+b_3} \right) = \min\left(\frac{6}{6+3} \;,\; \frac{7}{7+6} \right)$$

$$= \frac{7}{13} > \frac{1}{2}$$

$\therefore$ 1 - 3 $\quad$ Not optimal

2-2 $\quad T=13$

$$R_{opt\text{-}3\text{-}node} = \frac{13-5-4+1}{13-5-4+1+5} = \frac{1}{2}$$

$\therefore$ 2 - 2 $\quad$ optimal

2-3 $\quad T=14$

$$R_{opt\text{-}3\text{-}node} = \min\left(\frac{T-b_1-a_1+1}{T-b_1-a_1+1+b_1} \;,\; \frac{T-b_3-a_2+1}{T-b_3-a_2+1+b_3} \right) = \min\left(\frac{6}{6+6} \;,\; \frac{5}{5+5} \right)$$

$$= \frac{1}{2}$$

$\therefore$ 2 - 3 $\quad$ optimal

3-3 $\quad T=15$

$$R_{opt\text{-}3\text{-}node} = \frac{15-6-4+1}{15-6-4+1+6} = \frac{1}{2}$$

$\therefore$ 3 - 3 $\quad$ optimal