Case (i): $a_1 = a_2 = a$, $b_1 = b_2 = b$

$$k = a \cdot \frac{T+1-b-a}{b} \quad , \quad m_1 = m_2 = m = k+a$$

Dispersion-span Constraint:

$$T - b - (m-1) - (b-a)\left\lfloor \frac{m-1}{a} \right\rfloor \leq 0$$

L.H.S. $= T - b - \left(a\frac{T-a+1}{b} - 1\right) - (b-a)\left\lfloor \frac{T-a+1}{b} - \frac{1}{a} \right\rfloor$

$\leq T - b - a\cdot\frac{T-a+1}{b} + 1 - (b-a)\left(\frac{T-a+1}{b} - 1\right)$

$= \cancel{T} - \cancel{b} - \cancel{a\left(\frac{T-a+1}{b}\right)} + \cancel{1} - \cancel{T} + \cancel{a} + \cancel{a\left(\frac{T-a+1}{b}\right)} + \cancel{b} - \cancel{a}$

$= 0$

$\therefore$ L.H.S. ≤ 0

Therefore for dispersion-span constraint to hold

$$b \mid T-a+1$$

$n \in [g, g+1) \quad g = \lfloor n \rfloor$

✓ $\left\lfloor n - \frac{1}{a} \right\rfloor = n-1$ if $n = g$ D.S Cons. holds

? $\left\lfloor n - \frac{1}{a} \right\rfloor < n-1$ if $g \leq n < g + \frac{1}{a}$

✓ $\left\lfloor n - \frac{1}{a} \right\rfloor > n-1$ if $g + \frac{1}{a} < n < g+1$

NOPE

D.S. Cons. holds but does this case ever occur?

D.S. Cons. does not hold does this case ever occur (Yes)

$k = a\cdot\frac{(T-a-b+1)}{b}$ $\quad g < \frac{T-a+1}{b} < g + \frac{1}{a}$

$m = a\cdot\frac{(T-a+1)}{b}$ $\quad ag < a\cdot\frac{(T-a+1)}{b} < ag+1$

Integer Not true

$g + \frac{1}{a} < \frac{T-a+1}{b} < g+1$

$ag+1 < a\cdot\frac{T-a+1}{b} < ag+a$ True for some cases

$\Rightarrow \left\lfloor \frac{T-a+1}{b} - \frac{1}{a} \right\rfloor \geq \left(\frac{T-a+1}{b} - 1\right)$

$\Rightarrow -\left\lfloor \frac{T-a+1}{b} - \frac{1}{a} \right\rfloor \leq -\left(\frac{T-a+1}{b} - 1\right)$

Delay Constraint:

$2b + (b-a)\left(\left\lfloor \frac{k}{a} \right\rfloor + \left\lfloor \frac{k-1-k}{a} \right\rfloor\right) + (k-1)$

$= 2b + (b-a)\left(\left\lfloor \frac{k}{a} \right\rfloor + \frac{T-b-a+1}{b} + \left\lfloor -\frac{1+k}{a} \right\rfloor\right) + a\cdot\frac{(T-b-a+1)}{b} - 1$

$= 2b + (b-a)\cdot\underbrace{\left(\left\lfloor \frac{k}{a} \right\rfloor + \left\lfloor \frac{-1-k}{a} \right\rfloor + \frac{T-b-a+1}{b}\right)}_{c=-1} + a\cdot\frac{(T-b-a+1)}{b} - 1$

$= 2b + (b-a)\cdot\left(\frac{T-2b-a+1}{b}\right) + a\cdot\frac{(T-b-a+1)}{b} - 1$

$= 2b + T - 2b - a + \cancel{1} - a\cdot\frac{T-2b-a+1}{b} + \cancel{a}\cdot\frac{(T-b-a+1)}{b} - \cancel{1}$

$= T - a + a\cdot\frac{b}{b}$

$= T$ $\quad \therefore \geq T$

$k = a\cdot\frac{(T-b-a+1)}{b}$

$m = a\left(\frac{T-a+1}{b}\right)$

$b \mid T-a+1 \Rightarrow b \mid T-b-a+1$

$\left\lfloor \frac{k}{a} \right\rfloor = \frac{k - (k)_{1,a}}{a}$

$\left\lfloor \frac{-1-k}{a} \right\rfloor = \frac{-1-k-((-1-(k))+1, a)}{a}$

$\Rightarrow \left\lfloor \frac{k}{a} \right\rfloor + \left\lfloor \frac{-1-k}{a} \right\rfloor = \frac{-1-[(-1-(k))+a]}{a} = -1$

Constraints on parameters:

- $a_1 = a_2 \ (= a)$
- $b_1 = b_2 \ (= b)$
- $b \mid T-a+1$
- $N_1 = N_2 = T-b+1$
- $k = a\cdot\frac{T-b-a+1}{b}$
- $m_1 = m_2 = k+a$

$$R_1 = R_2 = R_{opt} = \frac{T-b-a+1}{T-a+1} = R_{opt,2\text{-Node}}$$

An optimal SDE based construction exists with above constraints

Example: $T = 7 \quad a_1 = a_2 = 2 \quad b_1 = b_2 = 3$

$k = 2 \quad m_1 = m_2 = 4 \quad N_1 = N_2 = 5$

$\delta_1 = (\underset{0^{th}}{4}, \underset{1^{st}}{3})$

At Node 1:

j^{th} time	$i-1$	i	$i+1$	$i+2$	$i+3$	$i+4$
$\delta_j[0]$	$\delta_{i-1}[0]$	$\delta_i[0]$	$\delta_{i+1}[0]$	$\delta_{i+2}[0]$	$\delta_{i+3}[0]$	$\delta_{i+4}[0]$
$\delta_j[1]$	$\delta_{i-1}[1]$	$\delta_i[1]$	$\delta_{i+1}[1]$	$\delta_{i+2}[1]$	$\delta_{i+3}[1]$	$\delta_{i+4}[1]$
—	—	—	—	—	—	—
$\delta_{j-2}[0] + \delta_{j-2}[1]$	...	...	...	$\delta_{i-1}[0] + \delta_i[1]$	$\delta_i[0] + \delta_{i+1}[1]$	$\delta_{i+1}[0] + \delta_{i+2}[1]$
$\delta_{j-4}[0] + 2\delta_{j-3}[1]$	...	...	...	...	$\delta_{i-1}[0] + 2\delta_i[1]$	$\delta_i[0] + 2\delta_{i+1}[1]$

At Node 2: $\delta_2 = (\underset{0^{th}}{3}, \underset{1^{st}}{4})$

j^{th} time	$i+2$	$i+3$	$i+4$	$i+5$	$i+6$	$i+7$
$\delta_{j-3}[1]$	$\delta_{i-1}[1]$	$\delta_i[1]$	$\delta_{i+1}[1]$	$\delta_{i+2}[1]$		
$\delta_{j-4}[0]$	$\delta_{i-1}[0]$	$\delta_i[0]$	$\delta_{i+1}[0]$			
—	—	—	—	—	—	—
$\delta_{j-6}[1] + \delta_{j-6}[0]$	...	...	...	$\delta_{i-1}[1] + \delta_{i-1}[0]$	$\delta_i[1] + \delta_i[0]$	$\delta_{i+1}[1] + \delta_{i+1}[0]$
$\delta_{j-7}[1] + 2\delta_{j-7}[0]$	...	...	...	...	$\delta_{i-1}[1] + 2\delta_{i-1}[0]$	$\delta_i[1] + 2\delta_i[0]$

At Node 3:

j^{th} time	$i+5$	$i+6$	$i+7$	$i+8$	$i+9$	$i+10$
$\delta_{j-7}[0]$	$\delta_{i-2}[0]$	$\delta_{i-1}[0]$	$\delta_i[0]$	$\delta_{i+1}[0]$	$\delta_{i+2}[0]$	$\delta_{i+3}[0]$
$\delta_{j-7}[1]$	$\delta_{i-2}[1]$	$\delta_{i-1}[1]$	$\delta_i[1]$	$\delta_{i+1}[1]$	$\delta_{i+2}[1]$	$\delta_{i+3}[1]$