Assignment2

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Question 1:

Let $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}$. Find a matrix D such that $CD - AB = 0$.

Solution:

Let $D = \begin{pmatrix} x & y \\ z & p \end{pmatrix}$
Now, we have to calculate CD and AB,
$AB = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 43 & 22 \end{pmatrix}$
Now

$$CD = AB \tag{1}$$

$C^{-1}CD = C^{-1}AB$
$D = C^{-1}(AB)$

C^{-1} :

$$\begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} C$$

$$R_2 \to R_2 - \frac{3}{2} R_1$$

$$\begin{pmatrix} 2 & 5 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-3}{2} & 1 \end{pmatrix} C$$

$$R_1 \to \frac{R_1}{2}$$

$$\begin{pmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{-3}{2} & 1 \end{pmatrix} C$$

$$R_1 \to R_1 - 5R_2$$

$$\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{16}{2} & -5 \\ \frac{-3}{2} & 1 \end{pmatrix} C$$

$$R_2 \to 2R_2$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 8 & -5 \\ -3 & 2 \end{pmatrix} C$$

$$C^{-1} = \begin{pmatrix} 8 & -5 \\ -3 & 2 \end{pmatrix}$$

$$C^{-1}(AB) = \begin{pmatrix} 8 & -5 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 43 & 22 \end{pmatrix}$$

$$C^{-1}(AB) = \begin{pmatrix} 24 - 215 & 0 - 110 \\ -9 + 86 & 0 + 44 \end{pmatrix}$$

$$D = C^{-1}(AB) = \begin{pmatrix} -191 & -110 \\ 77 & 44 \end{pmatrix}$$

Question 2:

Find the values of a,b,c and d from the equations:

$$\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 0 & 13 \end{pmatrix}$$

Solution:

On equating we get

$$a - b = -1 \tag{2}$$

$$2a + c = 5 \tag{3}$$

$$2a - b = 0 \tag{4}$$

$$3c + d = 13 \tag{5}$$

solving these equations
from(4)
$2a - b = 0$
$b = 2a$
solving (2)
$a - b = -1$
putting $b = 2a$
$a - 2a = -1$
=> $a = 1$
now, we already know
$b = 2a$
=> $b = 2$

solving (3)
$2a + c = 5$
$2 + c = 5$
=> $c = 3$

solving (4)
$3c + d = 13$
$3 * 3 + d = 13$
$d = 13 - 9$
=> $d = 4$
Hence, $a = 1$, $b = 2$, $c = 3$, $d = 4$