Assignment 6

Shubham Shrivastava

January 22, 2021

Question 1:

Solve the system of equations:
$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$
$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$
$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

Solution:

let, $\frac{1}{x}=p, \frac{1}{y}=q, \frac{1}{z}=r$
then the given system as follows,

$$2p+3q+10r=4 \tag{1}$$

$$4p-6q+5r=1 \tag{2}$$

$$6p+9q-20r=2 \tag{3}$$

This can be written in the form of $AX=B$, where
$A=\begin{pmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{pmatrix}$, $X=\begin{pmatrix}p\\ q\\ r\end{pmatrix}$ and $B=\begin{pmatrix}4\\ 1\\ 2\end{pmatrix}$

$$\begin{aligned}|A| &= 2(120-45)-3(-80-30)+10(36+360)\\ &= 150+330+720\\ &= 1200\end{aligned} \tag{4}$$

Now,

Thus, A is non singular. Therefore, its inverse exists.
Now,
A_{11}=75,A_{12}=110,A_{13}=72
A_{21}=150,A_{22}=-100,A_{23}=0
A_{31}=75,A_{32}=32,A_{33}=-24
WKT;

$A^{-1} = \frac{1}{|A|}$adjA

$$=\frac{1}{1200}\begin{pmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{pmatrix}$$

Now,

$$X=A^{(}-1)B$$

$$=>\begin{pmatrix} p \\ q \\ r \end{pmatrix}=\frac{1}{1200}\begin{pmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$$

$$=\frac{1}{1200}\begin{pmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{pmatrix} =1200\begin{pmatrix} 600 \\ 400 \\ 240 \end{pmatrix}= \begin{pmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{pmatrix}$$

therefore

$$p=\frac{1}{2}, q=\frac{1}{3} and r=\frac{1}{5} Hence, x=2, y=3 and z=5$$

Question 2

If a,b,c are in A.P, then the determinant
$\begin{pmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{pmatrix}$ is

Solution

Let, A = $\begin{pmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{pmatrix}$

$$R_2 => 2R_2 - R_1 - R_3$$

$=\frac{1}{2}$ $\begin{pmatrix} x+2 & x+3 & x+2a \\ 0 & 0 & 2(2b-a-c) \\ x+4 & x+5 & x+2c \end{pmatrix}$

Since a, b and c are in AP using 2b = a + c, we get

$A=$ $\begin{pmatrix} x+2 & x+3 & x+2a \\ 0 & 0 & 0 \\ x+4 & x+5 & x+2c \end{pmatrix}$

Since, all elements of R_2 are zero

Question 3

Two farmers Ramkishan and Gurcharan Singh cultivates only three varieties of rice namely Basmati, Permal and Naura. The sale (in Rupees) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B. September Sales(in Rupees)

$$A = \begin{array}{c} \begin{matrix} Basmati & Permal & Naura \end{matrix} \\ \begin{pmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{pmatrix} \end{array} \begin{array}{c} \\ Ramakishan \\ GurucharanSingh \end{array}$$

October sales (in Rupees)

$$B = \begin{array}{c} \begin{matrix} Basmati & Permal & Naura \end{matrix} \\ \begin{pmatrix} 5,000 & 10,000 & 6,000 \\ 50,000 & 10,000 & 10,000 \end{pmatrix} \end{array} \begin{array}{c} \\ Ramakishan \\ GurucharanSingh \end{array}$$

(i) Find the combined sales in September and October for each farmer in each variety.
(ii) Find the decrease in sales from September to October.
(iii) If both farmers receive 2sales, compute the profit for each farmer and for each variety sold in October.

Solution

(i) The combined sales from Setember to October will be

$$\begin{aligned} A + B &= \begin{pmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{pmatrix} + \begin{pmatrix} 5,000 & 10,000 & 6,000 \\ 50,000 & 10,000 & 10,000 \end{pmatrix} \\ &= \begin{pmatrix} 10,000 + 5,000 & 20,000 + 10,000 & 30,000 + 6,000 \\ 50,000 + 20,000 & 30,000 + 10,000 & 10,000 + 10,000 \end{pmatrix} \\ &= \begin{pmatrix} 15,000 & 30,000 & 36,000 \\ 70,000 & 40,000 & 20,000 \end{pmatrix} \end{aligned}$$

thefore the combined sales would be as:

$$A + B = \begin{array}{c} \begin{matrix} Basmati & Permal & Naura \end{matrix} \\ \begin{pmatrix} 15,000 & 30,000 & 36,000 \\ 70,000 & 40,000 & 20,000 \end{pmatrix} \end{array} \begin{array}{c} \\ Ramakishan \\ GurucharanSingh \end{array}$$

(ii) Decrease in sales from september to october will be $A - B$

$$\begin{aligned} A - B &= \begin{pmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} - \begin{pmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{pmatrix} \\ &= \begin{pmatrix} 10000 - 5000 & 20000 - 10000 & 30000 - 6000 \\ 50000 - 20000 & 30000 - 10000 & 10000 - 10000 \end{pmatrix} \\ &= \begin{pmatrix} 5000 & 10000 & 24000 \\ 30000 & 20000 & 0 \end{pmatrix} \end{aligned}$$

Hence decrease in sales from september to october

$$\begin{matrix} Basmati & Permal & Naura & \\ \begin{pmatrix} 5,000 & 10,000 & 24,000 \\ 30,000 & 20,000 & 0 \end{pmatrix} & & & \begin{matrix} Ramakishan \\ GurucharanSingh \end{matrix} \end{matrix}$$

(iii)

$$\begin{aligned} Profit &= 2\% * (salesinoctober) \\ &= \frac{2}{100} * B \\ &= 0.02 * B \\ &= 0.02 \begin{pmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{pmatrix} \\ &= \begin{pmatrix} 0.02 * 5000 & 0.02 * 10000 & 0.02 * 6000 \\ 0.02 * 20000 & 0.02 * 10000 & 0.02 * 10000 \end{pmatrix} \\ &= \begin{pmatrix} 100 & 200 & 120 \\ 400 & 200 & \end{pmatrix} \end{aligned}$$

Hence,

$$profit = \begin{matrix} Basmati & Permal & Naura & \\ \begin{pmatrix} 100 & 200 & 120 \\ 400 & 200 & 200 \end{pmatrix} & & & \begin{matrix} Ramakishan \\ GurucharanSingh \end{matrix} \end{matrix}$$

Hence,
Ramkrishan recieves

RS 100 profit on sale of Basmati Rice,

RS 200 profit on sale of Permal

RS 120 profit in the sale of Naura in the month of October

Hence,
Gurucharan Singh recieves

Rs.400 profit on sales of basmati rice

Rs 200 profit on sales of permal

Rs.200 profit on sales of nora in the month of october