) ① Let $F_Y(y)$ be the cdf of $Y = F(X)$, then for any $y \in [0,1]$,
we have:

$F_Y(y) = Pr[Y \le y] = Pr[F(X) \le y]$

$= Pr[X \le F^{-1}(y)] = F(F^{-1}(y)) = y$

② Let $F_X(x)$ be the cdf of $X = F^{-1}(U)$, then for any $x \in [0,1]$.

$F_X(x) = Pr(X \le x)$

$= Pr(F^{-1}(U) \le x)$

$= Pr(U \le F(x))$

$= F(x)$ since $U \sim Uniform(0,1)$

thus, $X \sim F$.

$P(Z \le z) = P(X - Y \le z)$ $P(X \le Y + z)$

If $z \ge 1-Y$, $P(X \le Y+z) = 0$

If $z < -Y$, $P(X \le Y+z) = 0$

$P(Z \le z) = Y + z$ for $-Y \le z \le 1-Y$

$Z = X - Y$

$f_Z(z) = \int_0^1 \int_0^1 f_X(x) f_Y(y) dx dy$

$= \int_0^1 \int_0^1 f_X(y+z) f_Y(y) dx dy$

$= \int_0^1 f_X(y+z) dx = \int_{t-z}^{z+1} 1_{0 \le t \le 1} dt$

$= [0,1] \cap [z, z+1] = \begin{cases} 1-z & z \ge 0 \\ 1+z & z \le 0 \end{cases}$

$Z = \min\{X, Y\}$. $\Rightarrow$ $P(Z \ge z) = P(X \ge z) P(Y \ge z)$

$= (1-z)^2 = 1 - 2z + z^2$

$P(Z < z) = 1 - (1-z)^2 = 2z - z^2$

$f(z) = \frac{\partial F}{\partial z} = 2 - 2z$ for $z \in [0,1]$.

$E(Y) = \int_{-\infty}^{\infty} e^x \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2} dx$

$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{x - x^2/2} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{x - x^2/2} dx$.

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x-1)^2} dx \cdot e^{\frac{1}{2}}$

$= \sqrt{\frac{e}{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x-1)^2} dx$

$= \sqrt{\frac{e}{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x-1)^2} d(x-1) = \sqrt{\frac{e}{2\pi}}$

$Var(Y)$

$= \frac{e^2}{\sqrt{2\pi}} - \frac{e}{2\pi}$

$Var(Y) = E(Y^2) - [E(Y)]^2$, $E(Y^2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{2x - x^2/2} dx = \frac{1}{\sqrt{2\pi}} e^2 \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x-2)^2} d(x-2) = \frac{e^2}{\sqrt{2\pi}}$