Math 7553 – Spring 2018

HW #1 (Hand In)

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Chapter 2

Problem 2 (a) We collect a set of data on the top 500 firms in the US. For each firm we record profit, number of employees, industry and the CEO salary. We are interested in understanding which factors affect CEO salary.

Ans: This problem is regression and inference problem because the quantitative output of the CEO salary here is based on CEO firm's feature.

Here, n is 500 because there are 500 firms in the US

The value of p is 3 because of profit, number of employees and industry.

Problem 2 (b) we are considering launching a new product and wish to know whether it will be a success or a failure. We collect data on 20 similar products that were previously launched. For each product we have recorded whether it was a success or failure, price charged for the product, marketing budget, competition price, and ten other variables.

Ans: This problem is classification and prediction problem because it will predict product's success or failure. Here, n is 20 – Because 20 similar products were previously launched.

The value of p is 13 because of price charged for the product, marketing budget, competition price, and ten other variables.

Problem 2 (c) We are interesting in predicting the % change in the US dollar in relation to the weekly changes in the world stock markets. Hence we collect weekly data for all of 2012. For each week we record the % change in the dollar, the % change in the US market, the % change in the British market, and the % change in the German market.

Ans: This problem is regression and prediction problem because quantitative output of % change.

Here, n is 52 – Because 52 weeks of 2012 weekly data is available.

The value of p is 3 because of % change in US market, % change in British market and % change in German market.

Problem 3. We now revisit the bias-variance decomposition.

Problem 3 (a) Provide a sketch of typical (squared) bias, variance, training error, test error, and Bayes (or irreducible) error curves, on a single plot, as we go from less flexible statistical learning methods Towards more flexible approaches. The x-axis should represent the amount of flexibility in the method, and the y-axis should represent the values for each curve. There should be five curves. Make sure to label each one.

Ans:

The graph is provided below in the below picture.

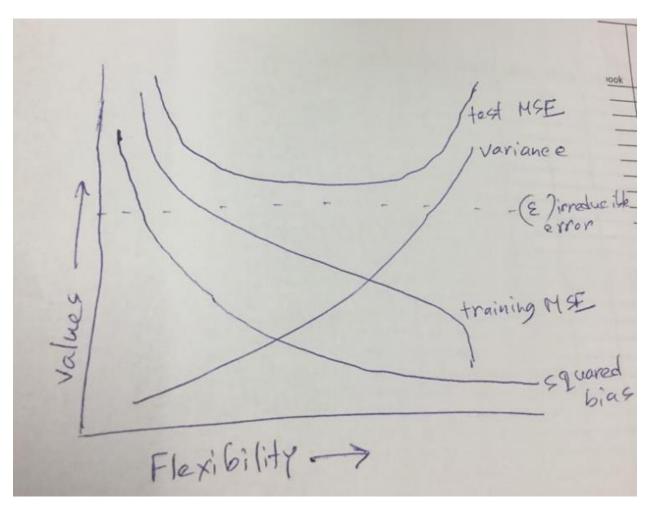


Fig 1: Sketch of typical (squared) bias, variance, training error, test error, and Bayes (or irreducible) error curves

Problem 3 (b) Explain why each of the five curves has the shape displayed in part

Ans: From the graph we see that all five lines are greater than zero. We see that the squared bias decreases monotonically because increases in flexibility yield a closer field. We also see that the variance increases monotonically because increases in flexibility yield over fit. A monotonic function is a function between ordered sets that preserves or reverses the given order. From the graph, training error decreases monotonically because increases in flexibility yield a closer fit. Test error is a concave up curve because increase in flexibility yields a fit before it overfits. Here we see that Bayes (irreducible error) – defines the lower limit. The test error is bounded below by the irreducible error due to variance in the error in the output values (0 \leq value). The Bayes error rate is defined for classification problems and is determined by the ratio of data points which lie at the wrong side of the decision boundary. (0 \leq value \leq 1)

Problem - 8

This exercise relates to the College data set, which can be found in the file College.csv. It contains a number of variables for 777 different universities and colleges in the US. The variables are

- · Private : Public/private indicator
- · Apps : Number of applications received
- · Accept : Number of applicants accepted
- · Enroll: Number of new students enrolled
- · Top10perc: New students from top 10% of high school class
- · Top25perc : New students from top 25% of high school class
- · F.Undergrad : Number of full-time undergraduates
- · P.Undergrad : Number of part-time undergraduates
- · Outstate : Out-of-state tuition
- · Room.Board: Room and board costs
- · Books: Estimated book costs
- · Personal : Estimated personal spending
- · PhD: Percent of faculty with Ph.D.'s
- · Terminal : Percent of faculty with terminal degree
- · S.F.Ratio : Student/faculty ratio
- · perc.alumni : Percent of alumni who donate
- · Expend : Instructional expenditure per student
- · Grad.Rate: Graduation rate

Before reading the data into R, it can be viewed in Excel or a text editor.

Problem 8 (a) Use the read.csv() function to read the data into R. Call the loaded data "college". Make sure that you have the directory set to the correct location for the data.

Ans:

```
library(ISLR)
data(College)
college <- read.csv("C:\\Users\\shc422\\Desktop\\Dataset\\College.csv")
Reference: How to Read CSV in R - http://rprogramming.net/read-csv-in-r/</pre>
```

Problem 8 (b) Look at the data using the fix() function. You should notice that the first column is just the name of each university. We don't really want R to treat this as data. However, it may be handy to have these names for later.

Ans:

```
#R-Markdown has been used as a typesetting tool for this assignment.
library(ISLR)
data(College)
college <- read.csv("C:\\Users\\shc422\\Desktop\\Dataset\\College.csv")</pre>
head(college[, 1:5])
##
                                 X Private Apps Accept Enroll
## 1 Abilene Christian University
                                       Yes 1660
                                                   1232
                                                           721
## 2
               Adelphi University
                                       Yes 2186
                                                   1924
                                                           512
## 3
                   Adrian College
                                       Yes 1428
                                                   1097
                                                           336
## 4
              Agnes Scott College
                                       Yes 417
                                                    349
                                                           137
```

```
Alaska Pacific University
                                         Yes
                                              193
                                                      146
                                                               55
## 6
                 Albertson College
                                         Yes
                                              587
                                                      479
                                                              158
 rownames <- college[, 1]
 college <- college[, -1]</pre>
 head(college[, 1:5])
##
     Private Apps Accept Enroll Top10perc
## 1
         Yes 1660
                     1232
                              721
                                          23
## 2
         Yes 2186
                     1924
                                          16
                              512
## 3
         Yes 1428
                     1097
                              336
                                          22
## 4
         Yes 417
                      349
                              137
                                          60
## 5
         Yes 193
                      146
                               55
                                          16
## 6
         Yes 587
                      479
                              158
                                          38
```

#Reference: R head function - https://www.rdocumentation.org/packages/utils/v
ersions/3.4.3/topics/head

Problem 8 (c)

(i) Use the summary () function to produce a numerical summary of the variables in the data set.

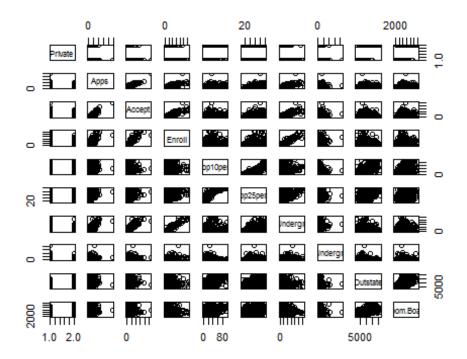
```
summary(college)
##
    Private
                                                     Enroll
                                                                   Top10perc
                    Apps
                                    Accept
                                           72
##
    No:212
              Min.
                          81
                                Min.
                                                 Min.
                                                        :
                                                           35
                                                                 Min.
                                                                        : 1.00
                                                                 1st Qu.:15.00
##
    Yes:565
               1st Qu.:
                         776
                                1st Qu.:
                                          604
                                                 1st Qu.: 242
##
              Median: 1558
                                Median: 1110
                                                 Median : 434
                                                                 Median :23.00
                      : 3002
##
              Mean
                                Mean
                                       : 2019
                                                 Mean
                                                        : 780
                                                                 Mean
                                                                         :27.56
##
               3rd Qu.: 3624
                                3rd Qu.: 2424
                                                 3rd Qu.: 902
                                                                 3rd Qu.:35.00
##
              Max.
                      :48094
                                Max.
                                       :26330
                                                 Max.
                                                         :6392
                                                                 Max.
                                                                         :96.00
##
      Top25perc
                      F. Undergrad
                                       P.Undergrad
                                                             Outstate
##
    Min.
           : 9.0
                     Min.
                                139
                                      Min.
                                                   1.0
                                                         Min.
                                                                 : 2340
##
                                992
    1st Qu.: 41.0
                     1st Qu.:
                                      1st Qu.:
                                                  95.0
                                                         1st Qu.: 7320
##
    Median: 54.0
                     Median: 1707
                                      Median :
                                                 353.0
                                                         Median: 9990
##
    Mean
           : 55.8
                     Mean
                            : 3700
                                      Mean
                                                 855.3
                                                         Mean
                                                                 :10441
##
    3rd Qu.: 69.0
                     3rd Qu.: 4005
                                      3rd Qu.:
                                                 967.0
                                                         3rd Qu.:12925
##
    Max.
           :100.0
                     Max.
                             :31643
                                      Max.
                                              :21836.0
                                                         Max.
                                                                 :21700
##
      Room.Board
                        Books
                                         Personal
                                                            PhD
##
   Min.
           :1780
                    Min.
                           :
                              96.0
                                      Min.
                                              : 250
                                                      Min.
                                                                 8.00
##
    1st Qu.:3597
                    1st Qu.: 470.0
                                      1st Qu.: 850
                                                      1st Qu.: 62.00
    Median:4200
                    Median : 500.0
                                      Median :1200
                                                      Median : 75.00
##
##
    Mean
           :4358
                    Mean
                           : 549.4
                                      Mean
                                              :1341
                                                      Mean
                                                              : 72.66
##
    3rd Qu.:5050
                    3rd Qu.: 600.0
                                      3rd Qu.:1700
                                                      3rd Qu.: 85.00
##
    Max.
           :8124
                    Max.
                           :2340.0
                                      Max.
                                              :6800
                                                      Max.
                                                              :103.00
##
       Terminal
                       S.F.Ratio
                                       perc.alumni
                                                            Expend
##
           : 24.0
                            : 2.50
    Min.
                     Min.
                                      Min.
                                              : 0.00
                                                       Min.
                                                               : 3186
##
    1st Qu.: 71.0
                     1st Ou.:11.50
                                      1st Ou.:13.00
                                                       1st Ou.: 6751
    Median: 82.0
                     Median :13.60
                                      Median :21.00
                                                       Median: 8377
##
```

```
##
   Mean : 79.7
                   Mean
                          :14.09
                                           :22.74
                                   Mean
                                                   Mean : 9660
    3rd Qu.: 92.0
                    3rd Qu.:16.50
##
                                   3rd Qu.:31.00
                                                   3rd Qu.:10830
           :100.0
##
   Max.
                    Max.
                          :39.80
                                   Max.
                                          :64.00
                                                   Max.
                                                          :56233
##
      Grad.Rate
##
   Min.
          : 10.00
    1st Qu.: 53.00
##
##
   Median : 65.00
           : 65.46
##
   Mean
    3rd Qu.: 78.00
##
##
   Max.
           :118.00
```

#Reference: R summary function - https://www.rdocumentation.org/packages/base/versions/3.4.3/topics/summary

ii) Use the pairs() function to produce a scatterplot matrix of the first ten columns or variables of the data.

pairs(college[, 1:10])

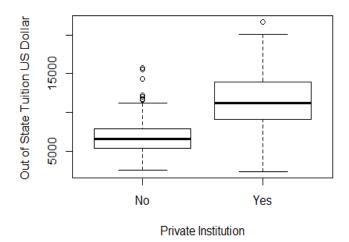


Reference: R pair function - https://www.rdocumentation.org/packages/pairwise/versions/0.4.3-2/topics/pair

iii) Use the plot() function to produce side-by-side boxplots of "Outstate" versus "Private".

```
plot(college$Private, college$Outstate, xlab = "Private Institution", ylab ="
Out of State Tuition US Dollar ", main = "Outstate Tuition Plot")
```

Outstate Tuition Plot



Reference: R plot function - https://www.rdocumentation.org/packages/graphics/versions/3.4.3/topics/plot

iv) Create a new qualitative variable, called "Elite", by binning the "Top10perc" variable. Use the summary() function to see how many elite universities there are. Now use the plot() function to produce side-by-side boxplots of "Outstate" versus "Elite".

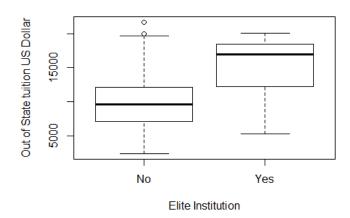
```
Elite <- rep("No", nrow(college))
Elite[college$Top10perc > 50] <- "Yes"
Elite <- as.factor(Elite)

college$Elite <- Elite
summary(college$Elite)

## No Yes
## 699 78

plot(college$Elite, college$Outstate, xlab = "Elite Institution", ylab ="Out of State tuition US Dollar", main = "Outstate Tuition Plot")</pre>
```

Outstate Tuition Plot



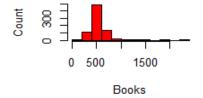
v) Use the hist() function to produce some histograms with numbers of bins for a few of the quantitative variables.

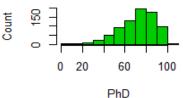
```
par(mfrow = c(2,2))
hist(college$Books, col = 2, xlab = "Books", ylab = "Count")
hist(college$PhD, col = 3, xlab = "PhD", ylab = "Count")
hist(college$Grad.Rate, col = 4, xlab = "Grad Rate", ylab = "Count")
hist(college$perc.alumni, col = 6, xlab = "% alumni", ylab = "Count")
```

Reference: R hist function - https://www.rdocumentation.org/packages/graphics/versions/3.4.3/topics/hist

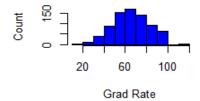
Histogram of college\$Books

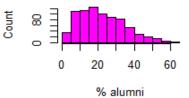
Histogram of college\$PhD





Histogram of college\$Grad.Ra Histogram of college\$perc.alur





vi) Continue exploring the data, and provide a brief summary of what you discover.

```
summary(college$PhD)
```

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 8.00 62.00 75.00 72.66 85.00 103.00

weird.phd <- college[college$PhD == 103, ]
    nrow(weird.phd)

## [1] 1

rownames[as.numeric(rownames(weird.phd))]

## [1] Texas A&M University at Galveston
## 777 Levels: Abilene Christian University ... York College of Pennsylvania</pre>
```

#Reference: R row.names function - https://www.rdocumentation.org/packages/base/versi
ons/3.4.3/topics/row.names

Question – 9. This exercise involves the "Auto" data set studied in the lab. Make sure the missing values have been removed from the data.

```
a)
auto <- read.csv("C:\\Users\\shc422\\Desktop\\Dataset\\Auto(1).csv", na.strings = "?")</pre>
auto <- na.omit(auto)</pre>
str(auto)
## 'data.frame':
                   392 obs. of 9 variables:
                 : num 18 15 18 16 17 15 14 14 14 15 ...
## $ mpg
## $ cylinders : int 8 8 8 8 8 8 8 8 8 ...
## $ displacement: num 307 350 318 304 302 429 454 440 455 390 ...
## $ horsepower : int 130 165 150 150 140 198 220 215 225 190 ...
## $ weight
                 : int
                        3504 3693 3436 3433 3449 4341 4354 4312 4425 3850 ...
## $ acceleration: num 12 11.5 11 12 10.5 10 9 8.5 10 8.5 ...
                 : int 70 70 70 70 70 70 70 70 70 70 ...
##
   $ year
                : int 111111111...
   $ origin
                 : Factor w/ 304 levels "amc ambassador brougham",..: 49 36 231 14 161
## $ name
 141 54 223 241 2 ...
## - attr(*, "na.action")=Class 'omit' Named int [1:5] 33 127 331 337 355
## ....- attr(*, "names")= chr [1:5] "33" "127" "331" "337" ...
```

Here all the variables rather than "horsepower" and "name" are quantitative.

b. What is the range of each quantitative predictor?

```
summary(auto[, -c(4, 9)])
                                 displacement
                                                   weight
##
                   cylinders
        mpg
## Min. : 9.00
                                      : 68.0 Min.
                  Min.
                        :3.000
                                Min.
                                                     :1613
## 1st Qu.:17.00
                  1st Qu.:4.000
                                1st Qu.:105.0
                                               1st Qu.:2225
## Median :22.75
                                Median :151.0
                  Median :4.000
                                               Median:2804
                                      :194.4
## Mean
         :23.45
                  Mean
                        :5.472
                                Mean
                                               Mean
                                                    :2978
## 3rd Qu.:29.00 3rd Qu.:8.000 3rd Qu.:275.8 3rd Qu.:3615
```

```
##
    Max.
           :46.60
                    Max.
                            :8.000
                                     Max.
                                             :455.0
                                                      Max.
                                                             :5140
##
     acceleration
                         year
                                         origin
           : 8.00
##
  Min.
                    Min.
                            :70.00
                                     Min.
                                             :1.000
    1st Qu.:13.78
                    1st Qu.:73.00
                                     1st Qu.:1.000
##
                    Median :76.00
##
   Median :15.50
                                     Median :1.000
           :15.54
                            :75.98
##
    Mean
                    Mean
                                     Mean
                                             :1.577
    3rd Qu.:17.02
                    3rd Qu.:79.00
                                     3rd Ou.:2.000
##
  Max.
##
         :24.80
                    Max.
                           :82.00
                                     Max. :3.000
```

c. What is the mean and standard deviation of each quantitative predictor?

```
sapply(auto[, -c(4, 9)], mean)
                                                  weight acceleration
##
            mpg
                    cylinders displacement
##
                                194.411990
                                             2977.584184
                                                             15.541327
      23.445918
                     5.471939
##
                       origin
           year
##
      75.979592
                     1.576531
sapply(auto[, -c(4, 9)], sd)
##
                    cylinders displacement
                                                  weight acceleration
            mpg
##
      7.8050075
                    1.7057832
                               104.6440039 849.4025600
                                                             2.7588641
##
                       origin
           year
##
                    0.8055182
      3.6837365
```

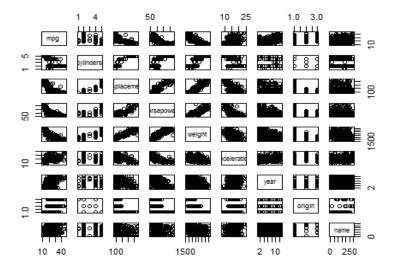
d)Now remove the 10th through 85th observations. What is the range, mean, and standard deviation of each predicto r in the subset of the data that remains?

```
subset <- auto[-c(10:85), -c(4,9)]
sapply(subset, range)
##
         mpg cylinders displacement weight acceleration year origin
                                                       8.5
                                                             70
## [1,] 11.0
                      3
                                  68
                                        1649
## [2,] 46.6
                      8
                                 455
                                        4997
                                                      24.8
                                                             82
                                                                     3
sapply(subset, mean)
##
                    cylinders displacement
                                                  weight acceleration
            mpg
##
      24.404430
                     5.373418
                                187.240506 2935.971519
                                                             15.726899
##
           year
                       origin
##
      77.145570
                     1.601266
sapply(subset, sd)
##
                    cylinders displacement
                                                  weight acceleration
            mpg
##
                     1.654179
                                 99.678367
                                              811.300208
                                                              2.693721
       7.867283
##
                       origin
           year
##
       3.106217
                     0.819910
```

#Reference: R sapply function - https://www.datacamp.com/community/tutorials/r-tutori al-apply-family

e) Using the full data set, investigate the predictors graphically, using scatterplots or other tools of your choice. Create some plots highlighting the relationships among the predictors. Comment on your findings.

```
auto$cylinders <- as.factor(auto$cylinders)
auto$year <- as.factor(auto$year)
auto$origin <- as.factor(auto$origin)
pairs(auto)</pre>
```



From the graph we see that we will to get more mileage per gallon on a 4 cylinders vehicle than the others. Weight, displacement and horsepower have an inverse effect with mpg. There is an overall increase in mpg over the years. It almost doubled in one decade. We also notice that Japanese cars have higher mpg than US or European cars.

f) Suppose that we wish to predict gas mileage ("mpg") on the basis of other variables. Do your plots suggest that any of the other variables might be useful in predicting "mpg"?

```
auto$horsepower <- as.numeric(auto$horsepower)
cor(auto$weight, auto$horsepower)

## [1] 0.8645377

cor(auto$weight, auto$displacement)

## [1] 0.9329944

cor(auto$displacement, auto$horsepower)

## [1] 0.897257</pre>
```

```
#Reference: Correlation function - https://www.rdocumentation.org/packages/stats/versions/3.4.3/topics/cor
```

Chapter -3

Problem - 8

This question involves the use of simple linear regression on the Auto data set.

lm(formula = mpg ~ horsepower)

##

Problem 8 (a) Use the lm() function to perform a simple linear regression with mpg as the response and horsepower as the predictor. Use the summary() function to print the results. Comment on the output.

```
auto <- read.csv("C:\\Users\\shc422\\Desktop\\Dataset\\Auto(1).csv", header=T</pre>
, na.strings="?")
 auto <- na.omit(auto)</pre>
 summary(auto)
##
                       cylinders
                                      displacement
                                                        horsepower
         mpg
##
   Min.
          : 9.00
                    Min.
                            :3.000
                                     Min.
                                             : 68.0
                                                      Min.
                                                              : 46.0
    1st Qu.:17.00
                                                      1st Qu.: 75.0
                     1st Qu.:4.000
                                     1st Qu.:105.0
##
## Median :22.75
                     Median :4.000
                                     Median :151.0
                                                      Median: 93.5
##
   Mean
           :23.45
                     Mean
                            :5.472
                                     Mean
                                             :194.4
                                                      Mean
                                                              :104.5
                                     3rd Qu.:275.8
                                                      3rd Qu.:126.0
##
    3rd Qu.:29.00
                     3rd Qu.:8.000
##
   Max.
           :46.60
                            :8.000
                                             :455.0
                                                              :230.0
                    Max.
                                     Max.
                                                      Max.
##
##
                    acceleration
        weight
                                          year
                                                         origin
##
   Min.
           :1613
                          : 8.00
                                    Min.
                                            :70.00
                                                     Min.
                                                             :1.000
                   Min.
    1st Qu.:2225
                   1st Qu.:13.78
                                                     1st Qu.:1.000
##
                                    1st Qu.:73.00
##
    Median :2804
                   Median :15.50
                                    Median :76.00
                                                     Median :1.000
           :2978
##
   Mean
                   Mean
                           :15.54
                                    Mean
                                            :75.98
                                                     Mean
                                                             :1.577
##
    3rd Qu.:3615
                    3rd Qu.:17.02
                                    3rd Qu.:79.00
                                                     3rd Qu.:2.000
           :5140
                           :24.80
##
   Max.
                   Max.
                                    Max.
                                            :82.00
                                                     Max.
                                                             :3.000
##
##
                     name
##
    amc matador
                       :
                          5
##
   ford pinto
                          5
                          5
##
   toyota corolla
##
    amc gremlin
                          4
    amc hornet
##
                          4
   chevrolet chevette:
##
                          4
##
    (Other)
                       :365
attach(auto)
lm.fit = lm(mpg \sim horsepower)
summary(lm.fit)
##
## Call:
```

```
## Residuals:
                       Median
##
        Min
                  1Q
                                    3Q
                                            Max
## -13.5710 -3.2592 -0.3435
                                2.7630
                                        16.9240
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 39.935861
                           0.717499
                                      55.66
                                               <2e-16 ***
                                     -24.49
## horsepower -0.157845
                           0.006446
                                               <2e-16 ***
## ---
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
## Residual standard error: 4.906 on 390 degrees of freedom
## Multiple R-squared: 0.6059, Adjusted R-squared: 0.6049
## F-statistic: 599.7 on 1 and 390 DF, p-value: < 2.2e-16
```

#R Tutorial Series: Simple Linear Regression- https://www.r-bloggers.com/r-tutorial-series-simple-linear-regression/

Problem 8 (a)

i. Is there a relationship between the predictor and the response?

Ans: There is a relationship between horsepower and mpg. When we determine that testing the null hypothesis of all the regression coefficient equal to zero. We see that the F- statistic is far larger than 1 and the p value of the F- statistic is close to zero we can reject the null hypothesis and conclude there is a statistically significant relationship between horsepower and mpg.

ii. How strong is the relationship between the predictor and the response?

Ans: If we want to calculate the residual error relative to the response we use the mean of the response and the RSE. The mpg's mean is 23.445. The RSE of the lm.fit is 4.90. It indicates a percentage error of 20.92. The R^2 of the lm.fit 0.6059 mean 60.59% of the variance in mpg is explained by horsepower.

iii) Is the relationship between the predictor and the response positive or negative?

Ans: The relationship between mpg and horsepower is negative. The more horsepower an automobile has the linear regression indicates the less mpg fuel efficiency the automobile have.

iv) What is the predicted mpg associated with a horsepower of 98? What are the associated 95% confidence and prediction intervals?

Ans:

```
predict(lm.fit, data.frame(horsepower=c(98)), interval="confidence")

## fit lwr upr

## 1 24.46708 23.97308 24.96108

predict(lm.fit, data.frame(horsepower=c(98)), interval="prediction")

## fit lwr upr

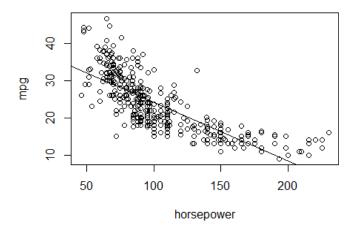
## 1 24.46708 14.8094 34.12476
```

Problem 8 (b)

Plot the response and the predictor. Use the abline() function to display the least squares regression line.

Ans:

```
plot(horsepower, mpg)
abline(lm.fit)
```

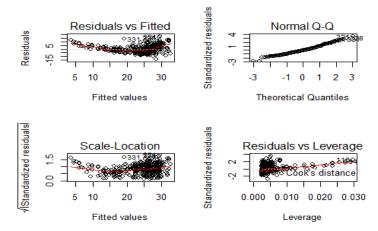


 $Reference: abline \ R \ function-http://www.sthda.com/english/wiki/abline-r-function-an-easy-way-to-add-straight-lines-to-a-plot-using-r-software$

Problem 8 (c)

Use the plot() function to produce diagnostic plots of the least squares regression fit. Comment on any problems you see with the fit.

```
par(mfrow=c(2,2))
plot(lm.fit)
```



There is some evidence of non-linearity based on the residuals plots.

Problem 13

In this exercise you will create some simulated data and will fit simple linear regression models to it. Make sure to use set.seed(1) prior to starting part (a) to ensure consistent results.

Problem 13 (a)

Using the rnorm() function, create a vector, x, containing 100 observations drawn from a N(0, 1) distribution. This represents a feature, X.

```
set.seed(1)
x = rnorm(100)
```

Problem 13 (b)

Using the rnorm() function, create a vector, eps, containing 100 observations drawn from a N(0, 0.25) distribution i.e. a normal distribution with mean zero and variance 0.25.

Problem 13 (c)

Using x and eps, generate a vector y according to the model Y = -1 + 0.5X + E. (1)

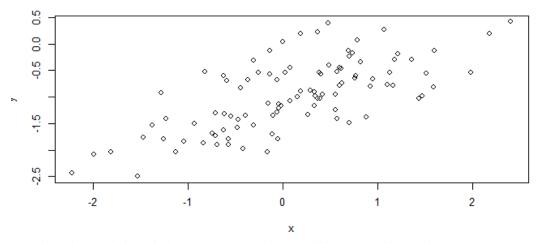
What is the length of the vector y? What are the values of $\beta 0$ and $\beta 1$ in this linear model?

$$y = -1 + 0.5*x + eps$$

y is of length 100. β 0 is -1, β 1 is 0.5.

Problem 13 (d)

Create a scatterplot displaying the relationship between x and y. Comment on what you observe. plot(x,y)



There is a linear relationship between x and y with a positive slope, with a variance as to be expected.

Problem 13 (e)

Fit a least squares linear model to predict y using x. Comment on the model obtained. How do $\hat{\beta}$ 0 and $\hat{\beta}$ 1 compare to $\hat{\beta}$ 0 and $\hat{\beta}$ 1?

```
lm.fit = lm(y\sim x)
summary(lm.fit)
##
## Call:
## lm(formula = y \sim x)
##
## Residuals:
##
        Min
                  1Q
                        Median
                                     3Q
                                              Max
  -0.93842 -0.30688 -0.06975
                                0.26970
                                         1.17309
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
   (Intercept) -1.01885
                            0.04849 -21.010 < 2e-16 ***
## x
                0.49947
                            0.05386
                                      9.273 4.58e-15 ***
## ---
                            0.001 '**' 0.01 '*' 0.05
## Signif. codes:
## Residual standard error: 0.4814 on 98 degrees of freedom
## Multiple R-squared: 0.4674, Adjusted R-squared:
## F-statistic: 85.99 on 1 and 98 DF, p-value: 4.583e-15
```

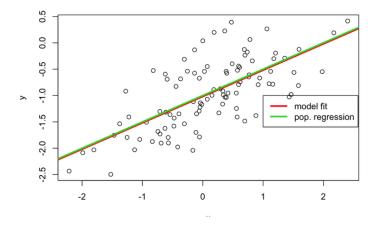
The linear regression fits a model close to the true value of the coefficient as it is constructed. This model has a large F- statistic with a near zero p-value. So we can reject the null hypothesis.

Problem 13 (f)

Display the least squares line on the scatterplot obtained in (d). Draw the population regression line on the plot, in a different color. Use the legend () command to create an appropriate legend.

Ans:

```
plot(x, y)
abline(lm.fit, lwd=3, col=2)
abline(-1, 0.5, lwd=3, col=3)
legend(-1, legend = c("model fit", "pop. regression"), col=2:3, lwd=3)
```



Problem 13 (g)

```
lm.fit_sq = lm(y\sim x+I(x^2))
summary(lm.fit_sq)
##
## Call:
## lm(formula = y \sim x + I(x^2))
##
## Residuals:
##
        Min
                   1Q
                        Median
                                      3Q
                                              Max
## -0.98252 -0.31270 -0.06441 0.29014
                                          1.13500
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
                            0.05883 -16.517
                                             < 2e-16 ***
## (Intercept) -0.97164
                                       9.420
                                              2.4e-15 ***
## X
                 0.50858
                            0.05399
## I(x^2)
                -0.05946
                            0.04238
                                     -1.403
                                                0.164
## ---
## Signif. codes:
                    0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

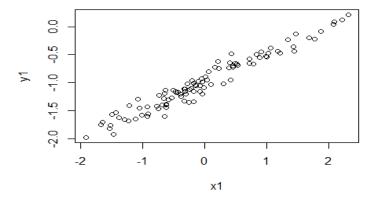
```
## Residual standard error: 0.479 on 97 degrees of freedom
## Multiple R-squared: 0.4779, Adjusted R-squared: 0.4672
## F-statistic: 44.4 on 2 and 97 DF, p-value: 2.038e-14
```

There is evidence that model fit has increased over the training data given the slight increase in R2 and RSE. Although, the p-value of the t-statistic suggests that there isn't a relationship between y and x2.

Problem 13 (h)

Repeat (a)—(f) after modifying the data generation process in such a way that there is less noise in the data. The model (3.39) should remain the same. You can do this by decreasing the variance of the normal distribution used to generate the error term in (b). Describe your results.

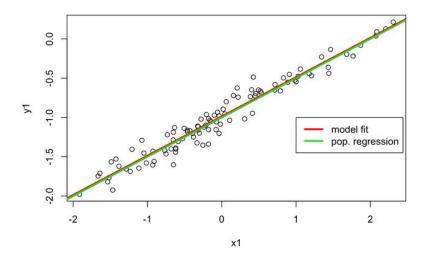
```
set.seed(1)
eps1 = rnorm(100, 0, 0.125)
x1 = rnorm(100)> y1 = -1 + 0.5*x1 + eps1
y1 = -1 + 0.5*x1 + eps1
plot(x1, y1)
```



```
lm.fit1 = lm(y1\sim x1)
 summary(lm.fit1)
##
## Call:
## lm(formula = y1 \sim x1)
##
## Residuals:
##
        Min
                  10
                        Median
                                     30
                                              Max
## -0.29052 -0.07545 0.00067 0.07288 0.28664
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.98639
                            0.01129
                                     -87.34
                                               <2e-16 ***
## x1
                0.49988
                            0.01184
                                      42.22
                                               <2e-16 ***
## ---
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
## Residual standard error: 0.1128 on 98 degrees of freedom
```

```
## Multiple R-squared: 0.9479, Adjusted R-squared: 0.9474
## F-statistic: 1782 on 1 and 98 DF, p-value: < 2.2e-16

plot(x, y)
abline(lm.fit, lwd=3, col=2)
abline(-1, 0.5, lwd=3, col=3)
legend(-1, legend = c("model fit", "pop. regression"), col=2:3, lwd=3)</pre>
```



Here we can see that the error observed in R² and RSE decrease considerably.

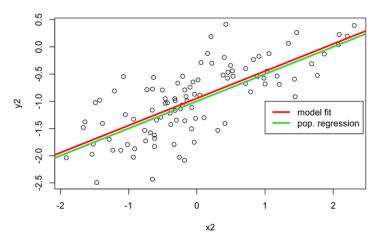
Problem 13 (i)

Repeat (a)–(f) after modifying the data generation process in such a way that there is more noise in the data. The model (3.39) should remain the same. You can do this by increasing the variance of the normal distribution used to generate the error term _ in (b). Describe your results.

```
set.seed(1)
eps2 = rnorm(100, 0, 0.5)
x2 = rnorm(100)
y2 = -1 + 0.5*x2 + eps2
plot(x2, y2)
```

```
lm.fit2 = lm(y2\sim x2)
 summary(lm.fit2)
##
## Call:
## lm(formula = y2 \sim x2)
##
## Residuals:
##
        Min
                   1Q
                        Median
                                      3Q
                                               Max
## -1.16208 -0.30181
                       0.00268 0.29152
                                           1.14658
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
##
```

```
## (Intercept) -0.94557
                           0.04517
                                     -20.93
                                              <2e-16 ***
## x2
                0.49953
                           0.04736
                                      10.55
                                              <2e-16 ***
## ---
                           0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
                   0
##
## Residual standard error: 0.4514 on 98 degrees of freedom
## Multiple R-squared: 0.5317, Adjusted R-squared: 0.5269
## F-statistic: 111.2 on 1 and 98 DF, p-value: < 2.2e-16
```



RSE increase considerably here.

Problem 13 (j)

What are the confidence intervals for $\beta 0$ and $\beta 1$ based on the original data set, the noisier data set, and the less noisy data set? Comment on your results.

```
confint(lm.fit)
##
                    2.5 %
                              97.5 %
## (Intercept) -1.1150804 -0.9226122
## x
                0.3925794 0.6063602
confint(lm.fit1)
                   2.5 %
                             97.5 %
## (Intercept) -1.008805 -0.9639819
## x1
                0.476387 0.5233799
confint(lm.fit2)
##
                    2.5 %
                              97.5 %
## (Intercept) -1.0352203 -0.8559276
## x2
                0.4055479 0.5935197
```

Here All intervals here seem to be centered on approximately 0.5, with the second fit's interval being narrower than the first fit's interval and the last fit's interval is wider than the first fit's interval.

#Reference: [1] set.seed function - http://rfunction.com/archives/62
 [2] rnorm function - https://www.rdocumentation.org/packages/compositions
/versions/1.40-1/topics/rnorm