

Boolean Algebra and Basic Operators

Due to historical reasons, digital circuits are called switching circuits, digital circuit functions are called switching functions and the algebra is called switching algebra. The algebraic system known as Boolean algebra named after the mathematician George Boole. George Boole Invented multi-valued discrete algebra (1854) and E. V. Huntington developed its postulates and theorems (1904). Historically, the theory of switching networks (or systems) is credited to Claude Shannon, who applied mathematical logic to describe relay circuits (1938). Relays are controlled electromechanical switches and they have been replaced by electronic controlled switches called logic gates. A special case of Boolean Algebra known as Switching Algebra is a useful mathematical model for describing the combinational circuits. In this section we will briefly discuss how the Boolean algebra is applied to the design of digital systems.

Examples of Huntington 's postulates are given below:

Closure

If X and Y are in set (0, 1) then operations $X+Y$ and $X \cdot Y$ are also in set (0, 1)

Identity

$$X + 0 = X \qquad X \cdot 1 = X$$

Distributive

$$X \cdot (Y + Z) = (X \cdot Y) + (X \cdot Z)$$

$$X + (Y \cdot Z) = (X + Y) \cdot (X + Z)$$

Complement

$$X + \overline{X} = 1$$

$$X \cdot \overline{X} = 0$$

Note that for each property, one form is the dual of the other; (zeros to ones, ones to zeros, '.' operations to '+' operations, '+' operations to '.' operations).

From the above postulates the following theorems could be derived.

Associative

$$X + (Y + Z) = (X + Y) + Z$$

$$X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$$

Idempotence

$$X \cdot X = X$$

$$X + X = X$$

Absorption

$$X + (X \cdot Y) = X$$

$$X \cdot (X + Y) = X$$

Simplification

$$X + (\bar{X} \cdot Y) = X + Y$$

$$X \cdot (\bar{X} + Y) = X \cdot Y$$

Consensus

$$X \cdot Y + \bar{X} \cdot Z + Y \cdot Z = X \cdot Y + \bar{X} \cdot Z$$

$$(X + Y) \cdot (\bar{X} + Z) \cdot (Y + Z) = (X + Y) \cdot (\bar{X} + Z)$$

Adjacency

$$X \cdot Y + X \cdot \bar{Y} = X$$

$$(X + Y) \cdot (X + \bar{Y}) = X$$

Demorgans

$$\overline{X + Y} = \bar{X} \cdot \bar{Y}$$

$$\overline{X \cdot Y} = \bar{X} + \bar{Y}$$

In a general form

$$\overline{F(+, \cdot, X_1, \dots, X_n)} = G(+, \cdot, \bar{X}_1, \dots, \bar{X}_n)$$

Very useful for complementing function expressions; for example

$$F = X + Y \cdot Z; \quad \bar{F} = \overline{X + Y \cdot Z}$$

$$\bar{F} = \bar{X} \cdot \overline{Y \cdot Z} \quad F = \bar{X} \cdot (\bar{Y} + \bar{Z})$$

$$\bar{F} = \bar{X} \cdot \bar{Y} + \bar{X} \cdot \bar{Z}$$

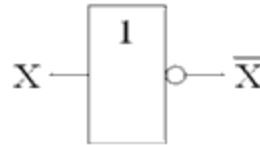
Switching Algebra Operations

A set is a collection of objects (or elements) and for example, a set $Z \{0, 1\}$ means that Z is a set containing two elements distinguished by the symbols 0 and 1. There are three primary operations AND, OR and NOT.

NOT

It is an inverse complement or inversion operation. Usually shown as overbar (\bar{X}), other forms are X' and $\sim X$

X	\bar{X}
0	1
1	0

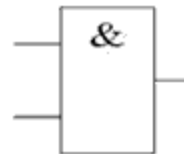


AND

Also known as the conjunction operation; output is true (1) only if all inputs are true.

Algebraic operators are '·', '&', ' \wedge '

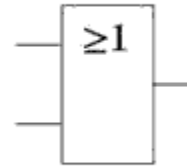
X	Y	$X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1



OR

Also known as the disjunction operation; output is true (1) if any input is true. Algebraic operators are '+', '|', '∨'

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1



AND and OR are called binary operations because they are defined on two operands X and Y. Not is called a unary operation because it is defined on a single operand X. All of these operations are closed. That means if one applies the operation to two elements in a set Z {0, 1}, the result will always be an element in the set B and not something else.

Like standard algebra, switching algebra operators has a precedence of evaluation. The following rules are useful in this regard.

1. NOT operations have the highest precedence
2. AND operations are next
3. OR operations are lowest
4. Parentheses explicitly define the order of operator evaluation and it is a good practice to use parentheses, especially for situations which can cases doubt.

Note that in Boolean algebra the operators AND and OR are not linear group operations; so one cannot solve equations by "adding to" or "multiplying" on both sides of the equal sign as is done with real, complex numbers in standard algebra.