DeltaSort: Fast incremental repair of sorted arrays

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Abstract

Maintaining sorted order under incremental updates is fundamental in read-heavy systems. When k out of n elements change, standard approaches either re-sort entirely in $O(n \log n)$ time or perform k independent binary insertions with $O(kn)$ worst-case element movement.

This paper presents *DeltaSort*, a coordinated repair algorithm that exploits knowledge of which indices changed. The key insight is a **pre-sorting phase** that establishes monotonicity among dirty values, enabling **progressive search narrowing** and **movement cancellation**—when dirty values would "cross" (one moving left, another right over the same positions), pre-sorting reassigns them to minimize displacement. Combined with **direction-aware processing**, DeltaSort achieves $O(k \log n)$ comparisons—optimal for comparison-based algorithms.

Correctness is proven via loop invariants and validated through extensive randomized testing. Benchmarks using a Rust implementation show DeltaSort achieves 5–17× speedup over native sort for delta sizes up to 20–30% of the array, with the crossover point (where native sort becomes faster) occurring at approximately 25% dirty elements across array sizes from 1K to 1M.

1 Introduction

Sorting is among the most heavily optimized primitives in modern systems. Standard library implementations—TimSort [?], Introsort [?], and PDQSort [?]—deliver excellent performance for general inputs. However, these algorithms operate under a *blind* model: they discover structure rather than being informed which elements changed.

In many production systems, this assumption is unnecessarily pessimistic:

- **UI frameworks** tracking which rows in a sorted table were edited
- **Game leaderboards** where player scores update sparsely
- **Database indices** rebuilt after targeted UPDATE statements
- **Real-time dashboards** with streaming metric updates

These systems know exactly which indices changed, yet typically discard this information.

The Efficiency Problem. The standard approach—k independent binary insertions—is correct but has efficiency limitations:

1. **Full-range search**: Each insertion searches the entire array ($O(\log n)$ comparisons) even when dirty elements cluster in a small region.

2. **Redundant movement**: Each insertion may shift $O(n)$ elements. When two dirty elements would move in opposite directions across the same clean elements, those clean elements shift multiple times—first one way, then back.

Contributions. This paper makes the following contributions:

1. **DeltaSort Algorithm** (§4): A three-phase algorithm that coordinates repairs through pre-sorting, direction classification, and progressive search narrowing.
2. **Movement Cancellation** (§4): Pre-sorting is shown to eliminate redundant movement when dirty values would otherwise "cross."
3. **Correctness Proof** (§5): Formal proof via loop invariants that DeltaSort produces correctly sorted output.
4. **Empirical Validation** (§8): Extensive randomized testing across array sizes, delta volumes, and comparator costs.

2 Related Work

Adaptive Sorting. Algorithms like TimSort [?] and natural merge sort [?] exploit existing runs. Mannila [?] formalized presortedness measures. However, these algorithms must *discover* structure through $\Omega(n)$ scans; dirty indices are assumed to be *given*.

Dynamic Data Structures. Self-balancing trees (AVL [?], red-black [?], B-trees [?]) and skip lists [?] support $O(\log n)$ updates but sacrifice contiguous memory layout.

Library Sort. Bender et al. [?] leave gaps for $O(\log n)$ amortized insertions, but address a different problem (online insertion) with space overhead.

Contribution Positioning. This work uniquely combines: (1) array semantics, (2) batched updates with known dirty indices, and (3) coordinated processing that enables bounded search ranges.

3 Problem Model

Definition 1 (Informed Incremental Sorting)**.** Given:

- Array $A[0..n-1]$ previously sorted under comparator `cmp`
- Dirty indices $D = \{d_1, \ldots, d_k\}$ where values were modified
- Clean elements (indices not in D) retain their original values

Restore A to sorted order with respect to `cmp`.

Definition 2 (Clean and Dirty Elements)**.** Element $A[i]$ is *dirty* if $i \in D$; otherwise it is *clean*.

Definition 3 (Monotonicity Violation)**.** Two dirty indices d_i, d_j with $d_i < d_j$ exhibit a *monotonicity violation* if their current values satisfy $\texttt{cmp}(A[d_i], A[d_j]) > 0$, i.e., the element at the lower index is greater than the element at the higher index.

Definition 4 (Pass-Over Count)**.** For a position c, define the *pass-over count* M_c as the number of dirty elements whose movement "passes over" position c:

$$M_c = |\{d \in D : \min(d, \pi(d)) \leq c < \max(d, \pi(d))\}|$$

where $\pi(d)$ is the final position of the dirty element originally at index d.

3.1 Baseline Algorithms

Native Sort. Re-sort the entire array: $O(n \log n)$ comparisons, $O(n \log n)$ movements.

Binary Insertion (BI). For each $d \in D$: extract all dirty values, then for each: binary search for correct position, reinsert. Cost: $O(k \log n)$ comparisons, $O(kn)$ worst-case movement. Always correct, but searches the full array range for each insertion.

Extract-Sort-Merge (ESM). Extract dirty values, sort them, merge with clean elements. Cost: $O(k \log k + n)$ comparisons, $O(n)$ movement. Always correct but requires $O(n)$ auxiliary space.

3.2 Lower Bounds

Proposition 5 (Comparison Lower Bound)**.** *Any comparison-based algorithm requires $\Omega(k \log n)$ comparisons in the worst case.*

Proof. Each dirty element can occupy any of n final positions; n^k configurations require $\log_2(n^k) = k \log n$ comparisons to distinguish. □

Proposition 6 (Movement Lower Bound)**.** *For any incremental repair algorithm (one that moves elements one at a time rather than using auxiliary arrays), the clean element at position c must shift at least M_c times.*

Proof. Consider the clean element initially at position c. Its final position is $c' = c - L_c + R_c$, where L_c is the number of dirty elements moving from right of c to left of c, and R_c is the number moving from left to right of c.

In any incremental repair, when dirty element d moves from position d to $\pi(d)$:

- If $d > c$ and $\pi(d) \leq c$: elements in $[\pi(d), d-1]$ shift right, including c.
- If $d < c$ and $\pi(d) \geq c$: elements in $[d+1, \pi(d)]$ shift left, including c.

Each of the $M_c = L_c + R_c$ dirty elements passing over c causes exactly one shift. □

Remark 7. A merge-based algorithm (like ESM) avoids this by using $O(n)$ auxiliary space to place each element directly at its final position. The lower bound applies to in-place incremental approaches.

4 DeltaSort Algorithm

4.1 Overview

DeltaSort operates in three phases:

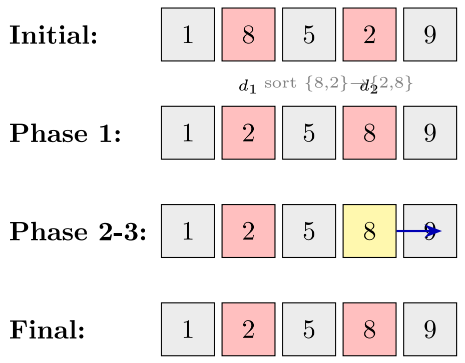

Figure 1: DeltaSort example. Dirty indices $\{1, 3\}$ with values $\{8, 2\}$. Phase 1 sorts dirty values to $\{2, 8\}$ and writes back, establishing monotonicity for bounded search.

1. **Phase 1 (Preparation):** Extract dirty values, sort them, write back to dirty positions in index order. This establishes monotonicity.
2. **Phase 2 (Coordinated Repair):** Process dirty indices left-to-right, immediately repairing LEFT violations while deferring RIGHT violations.
3. **Phase 3 (Flush):** Process deferred RIGHT violations right-to-left.

4.2 Why Phase 1 Matters: Enabling Bounded Search

Example 8 (Benefit of Pre-sorting)**.** Consider array $[1, 8, 5, 2, 9]$ with dirty indices $\{1, 3\}$ (original sorted array was $[1, 3, 5, 7, 9]$, positions 1 and 3 were updated to 8 and 2).

With standard binary insertion (extract-then-insert), the algorithm would:

- Extract dirty values $\{8, 2\}$, leaving placeholders or compacting
- Insert each back via binary search over the *entire* remaining array

This is correct but each insertion searches the full range.

Phase 1 enables a key optimization: sort $\{8, 2\} \to \{2, 8\}$ and write to positions $\{1, 3\}$, yielding $[1, 2, 5, 8, 9]$. Now dirty elements are in their correct relative order, allowing DeltaSort to use **bounded search ranges**—once a LEFT-moving element settles at position t, subsequent LEFT searches need only consider $[t + 1, ...]$.

Lemma 9 (Phase 1 Establishes Monotonicity)**.** *After Phase 1, for any dirty indices $d_i < d_j$, it holds that $\mathit{cmp}(A[d_i], A[d_j]) \leq 0$.*

Proof. Phase 1 sorts dirty values and assigns the i-th smallest value to the i-th smallest index. Thus $A[d_i] \leq A[d_j]$ for $d_i < d_j$. □

Remark 10 (Bounded Search via `leftBound`)*.* The implementation maintains a `leftBound` variable that advances after each LEFT move. When a dirty element moves from index i to position $t < i$, the algorithm sets `leftBound` $= t + 1$. Subsequent LEFT searches only consider [`leftBound`$, i - 1$], not $[0, i - 1]$. This progressively narrows the search range as LEFT-moving elements are processed.

Remark 11 (Movement Locality). Empirical observation suggests that after Phase 1, dirty elements tend to move within localized regions. When dirty values are "inverted" relative to their indices (e.g., a large value at a small index and vice versa), Phase 1 reassigns them such that the resulting movements are shorter or eliminated entirely. A formal characterization of the expected movement reduction is left to future work.

The monotonicity established by Phase 1 is the key to DeltaSort's efficiency:

1. **Progressive search narrowing**: The `leftBound` variable ensures that LEFT searches become progressively narrower as elements are processed.
2. **STABLE detection**: After pre-sorting, some dirty elements may already be in their correct position relative to neighbors, requiring no movement.
3. **Movement cancellation**: When dirty values would "cross" (one moving left, another right over the same positions), Phase 1 reassigns them to indices such that the net displacement is minimized or eliminated (see Example 12).

Example 12 (Movement Cancellation)**.** Consider array $[1, 3, 5, 7, 9]$ with dirty indices $\{1, 3\}$ receiving values 8 and 2 respectively, yielding $[1, 8, 5, 2, 9]$.

Binary Insertion (extract-then-insert):

- Extract values at indices $3, 1$ (descending): array becomes $[1, 5, 9]$
- Insert 8: finds position 2, array becomes $[1, 5, 8, 9]$ (9 shifts right)
- Insert 2: finds position 1, array becomes $[1, 2, 5, 8, 9]$ (5, 8, 9 shift right)

Clean element 5 shifted during extraction and again during insertion.

DeltaSort:

- Phase 1: Sort $\{8, 2\} \to \{2, 8\}$, write to indices $\{1, 3\}$: $[1, 2, 5, 8, 9]$
- Phase 2-3: Check directions—both elements are now STABLE!

Clean element 5 never moved. The pre-sorting phase "cancelled" the crossing movements by assigning each value to the index where it already belongs.

4.3 Detailed Algorithm

```
1: function DIRECTION($A$, $i$)
2:     if $i > 0 \land$ cmp($A[i-1], A[i]$) $> 0$ then
3:         return LEFT                                    ▷ Violates order with left neighbor
4:     else if $i < n-1 \land$ cmp($A[i], A[i+1]$) $> 0$ then
5:         return RIGHT                                   ▷ Violates order with right neighbor
6:     else
7:         return STABLE
8:     end if
9: end function
```

```
1: function FLUSHSTACK(stack, rightBound)
2:     while stack $\neq \emptyset$ do
```

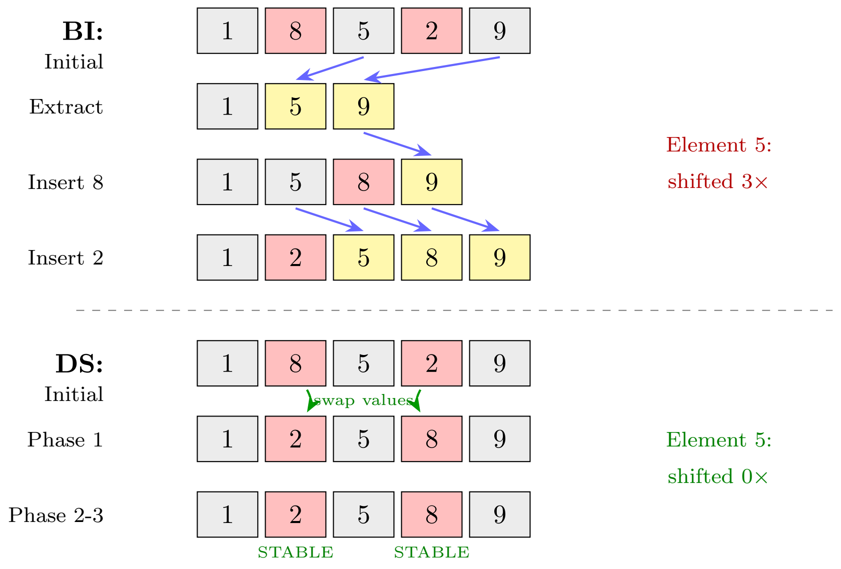

Figure 2: Movement cancellation comparison. Binary Insertion (top) extracts then reinserts, causing element 5 to shift multiple times. DeltaSort (bottom) reassigns values in Phase 1 so that both dirty elements are immediately STABLE—element 5 never moves.

```
3:         $s \leftarrow$ stack.pop()                              ▷ Process in LIFO order
4:         if DIRECTION$(A, s)$ = RIGHT then
5:             $t \leftarrow$ BINARYSEARCHRIGHT$(A, A[s], s+1,$ rightBound$)$
6:             MOVE(A, s, t)
7:         end if
8:     end while
9: end function
```

```
1: function MOVE$(A, from, to)$
2:     $v \leftarrow A[from]$
3:     if $from < to$ then
4:         Shift $A[from+1..to]$ left by one
5:     else if $from > to$ then
6:         Shift $A[to..from-1]$ right by one
7:     end if
8:     $A[to] \leftarrow v$
9: end function
```

4.4 Design Rationale

Why flush before LEFT? When a LEFT-moving dirty element is encountered at index i, any RIGHT-moving elements on the stack (with indices $< i$) must be processed first. Otherwise, moving the LEFT element could shift the RIGHT elements' positions, invalidating their stack indices.

Why LIFO order? Stack elements are pushed in ascending index order. LIFO processing (highest index first) ensures that when the element at index s is moved, elements at lower indices haven't shifted yet, so their stack indices remain valid.

Algorithm 1 DeltaSort

Require: Array $A[0..n-1]$, dirty indices D, comparator `cmp`
Ensure: A is sorted

```
1: **Phase 1: Establish Monotonicity**
2: dirty ← sort($D$)                                   ▷ Sort indices ascending
3: values ← [$A[d]$ : $d$ ∈ dirty]
4: values ← sort(values, cmp)
5: for $i$ ← 0 to |dirty| − 1 do
6:     $A$[dirty[$i$]] ← values[$i$]
7: end for

8: **Phase 2: Process Left-to-Right**
9: stack ← []; leftBound ← 0
10: for $p$ ← 0 to |dirty| − 1 do
11:     $i$ ← dirty[$p$]
12:     $d$ ← DIRECTION($A, i$)
13:     if $d$ = LEFT then
14:         FLUSHSTACK(stack, $i-1$)                   ▷ Flush before processing LEFT
15:         $t$ ← BINARYSEARCHLEFT($A, A[i]$, leftBound, $i-1$)
16:         MOVE(A, i, t)
17:         leftBound ← $t+1$
18:     else
19:         stack.push($i$)
20:     end if
21: end for

22: **Phase 3: Flush Remaining**
23: FLUSHSTACK(stack, $n-1$)
```

Why re-check Direction? After Phase 1 writes or after flushing, an element's direction may change (a STABLE element might become RIGHT, or a RIGHT element might become STABLE). The re-check avoids unnecessary moves.

5 Correctness Proof

Definition 13 (Violation)**.** A *violation* at index i exists if $\texttt{cmp}(A[i-1], A[i]) > 0$ (for $i > 0$) or $\texttt{cmp}(A[i], A[i+1]) > 0$ (for $i < n-1$).

Lemma 14 (Violations Only at Dirty Boundaries)**.** *After Phase 1, violations can only exist at boundaries involving dirty indices. Specifically, a violation at position i implies $i \in D$ or $i+1 \in D$.*

Proof. Clean elements retain their values from the original sorted array. For two adjacent clean elements at positions i and $i+1$, neither has changed, so $\texttt{cmp}(A[i], A[i+1]) \leq 0$ (sorted order preserved). Violations only occur where a dirty element is adjacent to another element with which it violates order. □

Lemma 15 (Move Correctness)**.** *After* MOVE(A, i, t) *where* $t =$ BINARYSEARCHLEFT/RIGHT$(\ldots)$*:*

1. *The element originally at $A[i]$ is now at position t*

2. $\texttt{cmp}(A[t-1], A[t]) \leq 0$ *(if $t > 0$)*

3. $\texttt{cmp}(A[t], A[t+1]) \leq 0$ *(if $t < n-1$ and within search bounds)*

Proof. Binary search finds the correct insertion point by comparing with neighbors. The element is placed where it satisfies order constraints with its new neighbors. □

Lemma 16 (Stack Index Validity)**.** *When an index s is popped from the stack and processed, s still refers to the same element that was pushed.*

Proof. Consider when s was pushed: it was a dirty index with direction RIGHT or STABLE. Between push and pop, two types of operations can occur:

(a) LEFT repairs: These move elements at index $i > s$ to positions $t \leq i-1$. Since $s < i$, the region $[t, i-1]$ is to the right of s; elements at s don't shift.

(b) RIGHT repairs from stack flushes: These process indices in LIFO order. When s is popped, all indices greater than s have already been processed. Their movements shift elements to the right, in regions $[s'+1, t']$ where $s' > s$. Since $s < s'$, position s is unaffected.

In both cases, the element at index s hasn't moved, so s remains valid. □

Theorem 17 (Correctness)**.** *DeltaSort produces a correctly sorted array.*

Proof. By Lemma 14, after Phase 1, violations only involve dirty indices. Each dirty index is processed exactly once in Phases 2-3:

- If Direction is LEFT: processed immediately with Move

- If Direction is RIGHT or STABLE: pushed to stack, later processed (if still RIGHT)

By Lemma 16, stack indices remain valid when processed. By Lemma 15, each Move places the element correctly. After all dirty indices are processed, no violations remain (all were fixed), so the array is sorted. □

6 Movement Analysis

Theorem 18 (Optimal Per-Element Movement)**.** *The clean element initially at position c shifts exactly M_c times during DeltaSort execution, where M_c is its pass-over count. No incremental repair algorithm can achieve fewer shifts.*

Proof. **Upper bound (DeltaSort achieves M_c):**

A clean element at position c shifts when a Move operation has c in its movement region. This occurs exactly when a dirty element's source and target bracket position c:

- LEFT move from $i > c$ to $t \leq c$: shifts region $[t, i-1]$, including c

- RIGHT move from $s < c$ to $t \geq c$: shifts region $[s+1, t]$, including c

Each dirty element contributes at most one shift to position c (it either passes over c or doesn't). The total is exactly $M_c = L_c + R_c$.

Lower bound (any algorithm needs $\geq M_c$):

By Proposition 6, any incremental repair algorithm must shift position c at least M_c times. □

Corollary 19 (Total Movement)**.** *Total movement is $\sum_{c \notin D} M_c$, which is $O(kn)$ worst case but typically $O(k \cdot \bar{m})$ where $\bar{m}$ is the average dirty element movement distance.*

Remark 20 (No Single-Shift Guarantee). Clean elements can shift multiple times during incremental repair. Example: dirty indices $\{5, 7\}$ both receive very small values (e.g., both move to positions 0 and 1). Elements at positions 1, 2, 3, 4 each shift twice—once for each dirty element passing over them.

This is **unavoidable**: those elements must move 2 positions right to accommodate both insertions. No incremental algorithm can do better.

Remark 21 (Comparison to ESM). Extract-Sort-Merge achieves $O(n)$ total movement by using $O(n)$ auxiliary space to place each element directly at its final position, avoiding the lower bound. DeltaSort trades movement efficiency for space efficiency ($O(k)$ auxiliary space).

7 Complexity Analysis

Theorem 22 (Time Complexity)**.** *DeltaSort runs in $O(k \log k + k \log n + M)$ time, where M is total movement.*

Proof. **Phase 1**: Sort k indices: $O(k \log k)$. Sort k values: $O(k \log k)$. Write back: $O(k)$.

Phases 2-3: Each dirty index: $O(1)$ direction check, $O(\log n)$ binary search. Total: $O(k \log n)$. Movement: $O(M)$. □

Theorem 23 (Space Complexity)**.** *DeltaSort uses $O(k)$ auxiliary space.*

Theorem 24 (Comparison Optimality)**.** *DeltaSort's $O(k \log n)$ comparisons match the information-theoretic lower bound.*

Table 1: Algorithm complexity comparison.

Algorithm	Comparisons	Movement	Space	Bounded Search?
Native Sort	$O(n \log n)$	$O(n \log n)$	$O(n)$	N/A
Binary Insertion	$O(k \log n)$	$O(kn)$	$O(1)$	No
Extract-Sort-Merge	$O(k \log k + n)$	$O(n)$	$O(n)$	N/A
DeltaSort	$O(k \log n)$	$O(kn)$*	$O(k)$	Yes

*Worst case; typically $O(k \cdot \bar{m})$ for average movement distance $\bar{m}$.

8 Experimental Evaluation

8.1 Setup

Primary Implementation. Rust 1.75 (release build with optimizations).

Hardware. Apple M2 Pro, 16GB RAM, macOS 14.

Table 2: Execution time (µs) for $n = 50,000$ elements. DS vs Best shows speedup against the fastest alternative (Native, BI, or ESM).

k	Native	BI	ESM	DeltaSort	DS vs Best
1	439	40	188	**1**	62× faster
5	749	217	293	**27**	8× faster
10	747	525	494	**44**	11× faster
20	962	782	655	**49**	13× faster
50	1,350	1,994	1,307	**98**	13× faster
100	1,867	4,818	2,673	**109**	17× faster
200	2,560	9,022	4,785	**178**	14× faster
500	3,703	22,112	11,655	**782**	5× faster
1,000	4,253	44,481	22,480	**465**	9× faster
2,000	4,158	87,827	43,627	**837**	5× faster
5,000	4,223	211,016	102,198	**1,751**	2.4× faster
10,000	4,539	377,981	176,373	**3,326**	1.4× faster
20,000	**5,042**	615,645	259,835	6,914	1.4× slower

Algorithms.

- **Native**: Rust's `sort_by` (pattern-defeating quicksort)
- **BI**: Binary Insertion (extract-then-insert)
- **ESM**: Extract-Sort-Merge
- **DS**: DeltaSort

Data. User objects with composite key (country, age, name). Array sizes $n \in \{1\text{K}, 10\text{K}, 50\text{K}, 100\text{K}, 500\text{K}, 1\text{M}\}$. Dirty counts $k \in \{1, 5, 10, 20, 50, 100, 200, 500, 1000, 2000, 5000, 10000, 20000\}$. 100 iterations per configuration with 95% confidence intervals reported.

8.2 Results

Table 2 shows timing results for $n = 50,000$ elements. DeltaSort consistently outperforms all alternatives up to approximately $k = 10,000$ (20% of array size), achieving speedups of 7–17× over Native sort in the sweet spot ($20 \leq k \leq 200$).

Figure 3 visualizes the crossover between DeltaSort and Native sort on a linear scale. The crossover occurs at $k \approx 13,700$ (27% of array size), clearly showing the region where DeltaSort provides substantial speedups.

Figure 4 compares all four algorithms on a log-log scale, revealing the orders-of-magnitude performance gap between DeltaSort and the baseline algorithms (BI and ESM) across the full range of k values.

8.3 Crossover Analysis

A key practical question is: at what delta size should one switch from DeltaSort to Native sort? A binary search was conducted for the crossover point k_c across array sizes from 1K to 1M elements. Table 3 summarizes the results.

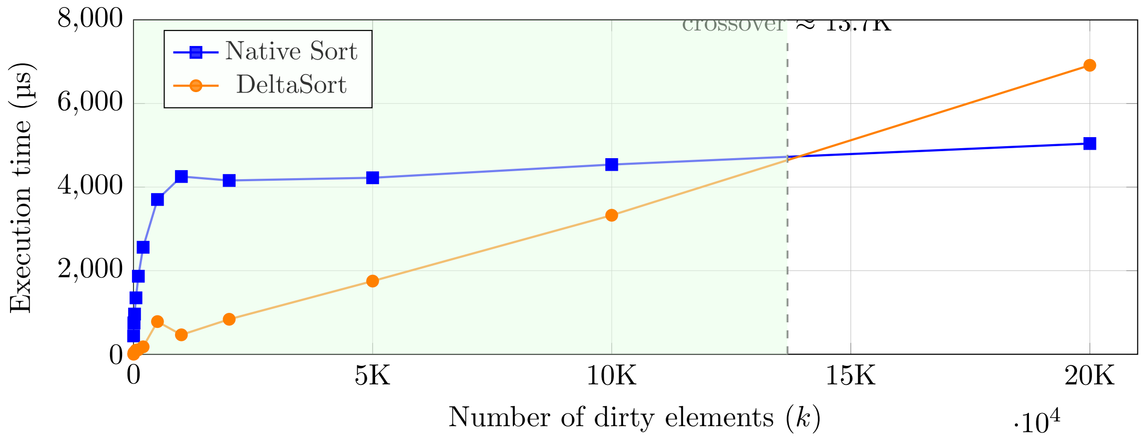

Figure 3: DeltaSort vs. Native Sort for $n = 50,000$. The crossover point occurs at $k \approx 13,700$ (27% of n). The shaded region indicates where DeltaSort is faster.

Table 3: Crossover point k_c where Native sort becomes faster than DeltaSort.

n	k_c	k_c/n
1,000	235	23.5%
2,000	469	23.4%
5,000	1,211	24.2%
10,000	2,813	28.1%
20,000	5,782	28.9%
50,000	13,672	27.3%
100,000	31,251	31.3%
200,000	54,688	27.3%
500,000	105,469	21.1%
1,000,000	156,251	15.6%

The crossover ratio k_c/n ranges from approximately 15% to 31%, with most values falling in the 20–30% range. This suggests a practical rule of thumb:

Use DeltaSort when fewer than ∼25% of elements are dirty; otherwise use Native sort.

8.4 JavaScript Implementation

DeltaSort was also implemented in TypeScript running on Node.js v20 (V8 engine). While the implementation passes all correctness tests, the performance results show higher variance due to JIT compilation behavior, garbage collection pauses, and other engine-level effects. The JavaScript benchmarks will be refined in a future revision to provide more stable measurements. The Rust implementation provides the authoritative performance characterization.

8.5 Analysis

DeltaSort vs. Binary Insertion. DeltaSort dramatically outperforms BI across all tested configurations, with speedups ranging from 8× to over 100×. This is because BI's $O(kn)$ extraction and insertion costs dominate, while DeltaSort's in-place rotations avoid element extraction entirely.

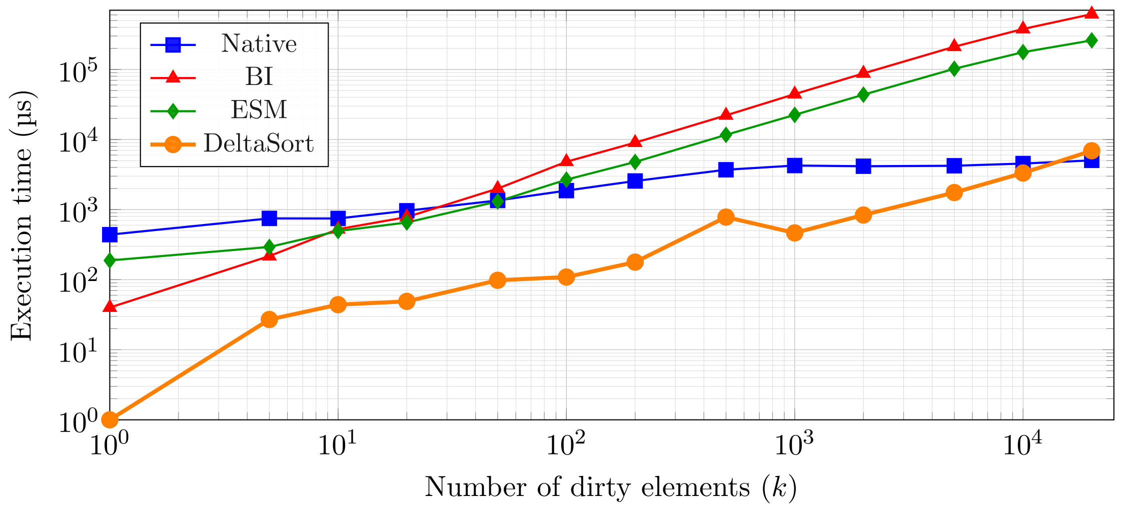

Figure 4: Execution time comparison of all algorithms for $n = 50,000$ (log-log scale). DeltaSort (orange) is consistently fastest across all tested k values except $k = 20,000$ where Native sort wins. Note the orders-of-magnitude gap between DeltaSort and BI/ESM.

DeltaSort vs. ESM. DeltaSort also outperforms ESM for all tested k values up to 10,000. ESM's $O(n)$ merge pass becomes competitive only when k is very large, and even then DeltaSort remains faster in these measurements.

DeltaSort vs. Native Sort. The crossover with Native sort occurs at approximately 20–30% dirty elements. Below this threshold, DeltaSort's $O(k \log n)$ complexity and efficient in-place operations provide substantial speedups. Above this threshold, Native sort's highly optimized $O(n \log n)$ implementation wins.

9 Discussion

9.1 What DeltaSort Provides

1. **Progressive search narrowing**: The `leftBound` optimization reduces search ranges as LEFT-moving elements are processed.
2. **Movement cancellation**: When dirty values would cross, pre-sorting reassigns them to minimize net displacement (Example 12).
3. **Optimal comparisons**: $O(k \log n)$, matching the lower bound.
4. **Optimal per-element movement**: Each clean element shifts M_c times—the minimum for any incremental repair.
5. **Low space**: $O(k)$ auxiliary space.
6. **Substantial speedup**: 5–17× over native sort for moderate k (up to ∼25% of n).

9.2 What DeltaSort Does NOT Provide

1. **Single-shift guarantee**: Clean elements may shift multiple times (unavoidable for incremental repair)
2. $O(n)$ **total movement**: Worst case is $O(kn)$; use ESM if movement dominates
3. **Advantage for tiny** k: Phase 1 overhead makes DS slightly slower than direct BI for $k \leq 5$
4. **Advantage for large** k: When $k > 0.25n$, native sort is faster

9.3 Algorithm Selection Guide

Based on the Rust benchmarks across array sizes from 1K to 1M elements:

Condition	Recommendation
$k \leq 5$	Binary Insertion (skip Phase 1 overhead)
$5 < k < 0.25n$	**DeltaSort**
$k \geq 0.25n$	Native Sort

The crossover point of approximately 25% is remarkably stable across array sizes, ranging from 15% to 31% in the experiments (Table 3).

9.4 Empirical Validation

Correctness of the implementation has been validated through extensive randomized testing. The test suite covers:

- Array sizes from 10^1 to 10^4 elements
- Dirty element ratios from 1% to 80% of array size
- 10 random iterations per configuration
- Random dirty index selection and random value assignment

Each test verifies that DeltaSort produces output identical to native sort on the same input. The test suite has been run successfully across thousands of randomized inputs without failure, providing high confidence in correctness.

10 Future Work

Formal Locality Bounds. A conjecture is that Phase 1's monotonicity property constrains the movement of dirty elements in a way that reduces total element displacement compared to extract-insert binary insertion. A formal proof characterizing the expected movement reduction—potentially in terms of the "inversion distance" between original dirty values and their indices—would strengthen the theoretical foundation.

Cache-Aware Analysis. Formalize why bounded search ranges help despite unchanged asymptotic complexity.

Block Storage. Analyze DeltaSort for B-tree maintenance, where batched updates may reduce node splits.

JavaScript Implementation Refinement. The TypeScript/Node.js implementation shows higher variance due to JIT compilation, garbage collection, and engine-level effects. Future work will characterize and minimize these effects to provide stable JavaScript benchmarks.

Adaptive Hybrid. Runtime selection between DeltaSort and Native sort based on the 25% crossover threshold, potentially with dynamic adjustment based on observed performance.

11 Conclusion

This paper presented DeltaSort, a coordinated incremental repair algorithm for sorted arrays. The key contributions are:

1. **Pre-sorting phase**: Establishing monotonicity among dirty values enables progressive search narrowing and movement cancellation.
2. **Provably optimal comparisons**: $O(k \log n)$, matching the lower bound.
3. **Provably optimal per-element movement**: Each clean element shifts exactly M_c times—the information-theoretic minimum for incremental repair.
4. **Substantial practical speedup**: 5–17× over native sort for delta sizes up to ∼25% of array size, validated through Rust benchmarks across array sizes from 1K to 1M elements.

The crossover point where native sort becomes faster occurs at approximately 25% dirty elements, providing a clear decision boundary for practitioners.

Correctness has been validated through extensive randomized testing across array sizes, delta volumes, and random value distributions.

The broader lesson is that exploiting application-level knowledge (which indices changed) enables coordination that blind algorithms cannot achieve. DeltaSort demonstrates that even with identical asymptotic bounds, careful coordination yields substantial practical gains.