Chapter 8

```
library(tidyverse)
```

```
## Loading tidyverse: ggplot2
## Loading tidyverse: tibble
## Loading tidyverse: tidyr
## Loading tidyverse: readr
## Loading tidyverse: purrr
## Loading tidyverse: dplyr
```

```
## Conflicts with tidy packages ----------------------------------------------
```

```
## filter(): dplyr, stats
## lag():    dplyr, stats
```

```
library(AzureML)
ws <- workspace()

mlbook_red <- tbl_df(download.datasets(
  dataset = ws,
  name    = "mlbook2_r.csv"))
# what are the available variables
names(mlbook_red)
```

```
##  [1] "schoolnr"    "pupilNR_new" "langPOST"    "ses"         "IQ_verb"
##  [6] "sex"         "Minority"    "denomina"    "sch_ses"     "sch_iqv"
## [11] "sch_min"
```

```
# Attach library
library(nlme)
```

```
##
## Attaching package: 'nlme'
```

```
## The following object is masked from 'package:dplyr':
##
##     collapse
```

Table 8.1 Model 1

```
mod81 <- lme(langPOST ~ IQ_verb*ses + sex + sch_iqv*sch_ses,
               random=~IQ_verb|schoolnr, data = mlbook_red, method="ML")
summary(mod81)
```

```
## Linear mixed-effects model fit by maximum likelihood
##  Data: mlbook_red
##        AIC     BIC    logLik
##   24510.82 24585.6 -12243.41
##
## Random effects:
##  Formula: ~IQ_verb | schoolnr
##  Structure: General positive-definite, Log-Cholesky parametrization
##             StdDev    Corr
## (Intercept) 2.8846515 (Intr)
## IQ_verb     0.3821388 -0.815
## Residual    5.9995464
##
```

```
## Fixed effects: langPOST ~ IQ_verb * ses + sex + sch_iqv * sch_ses
##                    Value  Std.Error   DF   t-value p-value
## (Intercept)     40.42614 0.26534574 3543 152.35272  0.0000
## IQ_verb          2.24874 0.06166254 3543  36.46845  0.0000
## ses              0.17142 0.01145165 3543  14.96874  0.0000
## sex              2.40671 0.20164298 3543  11.93548  0.0000
## sch_iqv          0.76889 0.29375384  207   2.61746  0.0095
## sch_ses         -0.09280 0.04201643  207  -2.20861  0.0283
## IQ_verb:ses     -0.01957 0.00480006 3543  -4.07781  0.0000
## sch_iqv:sch_ses -0.10523 0.03299836  207  -3.18909  0.0016
##  Correlation:
##                 (Intr) IQ_vrb ses    sex    sch_qv sch_ss IQ_vr:
## IQ_verb         -0.292
## ses              0.017 -0.254
## sex             -0.374  0.026 -0.022
## sch_iqv         -0.087 -0.166  0.061  0.004
## sch_ses          0.051  0.053 -0.265 -0.009 -0.497
## IQ_verb:ses     -0.076  0.076 -0.122 -0.038 -0.015 -0.141
## sch_iqv:sch_ses -0.361 -0.009  0.023  0.036  0.183 -0.017 -0.128
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -4.14643392 -0.63402245  0.07982323  0.68121238  3.04626819
##
## Number of Observations: 3758
## Number of Groups: 211
```

Model 2

```
mod82 <- lme(langPOST ~ IQ_verb*ses + sex + sch_iqv*sch_ses,
               random=~IQ_verb|schoolnr, weights=varIdent(form=~1|sex),
               data = mlbook_red, method="ML")
summary(mod82)
```

```
## Linear mixed-effects model fit by maximum likelihood
##  Data: mlbook_red
##        AIC      BIC    logLik
##   24508.22 24589.23 -12241.11
##
## Random effects:
##  Formula: ~IQ_verb | schoolnr
##  Structure: General positive-definite, Log-Cholesky parametrization
##             StdDev    Corr
## (Intercept) 2.8746428 (Intr)
## IQ_verb     0.3826536 -0.824
## Residual    6.1523224
##
## Variance function:
##  Structure: Different standard deviations per stratum
##  Formula: ~1 | sex
##  Parameter estimates:
##         0         1
## 1.0000000 0.9488492
## Fixed effects: langPOST ~ IQ_verb * ses + sex + sch_iqv * sch_ses
##                    Value  Std.Error   DF   t-value p-value
```

```
## (Intercept)     40.43539 0.26654057 3543 151.70445  0.0000
## IQ_verb          2.24460 0.06166362 3543  36.40077  0.0000
## ses              0.17067 0.01143510 3543  14.92501  0.0000
## sex              2.40359 0.20140685 3543  11.93401  0.0000
## sch_iqv          0.74946 0.29262341  207   2.56117  0.0111
## sch_ses         -0.09067 0.04185758  207  -2.16623  0.0314
## IQ_verb:ses     -0.01948 0.00479239 3543  -4.06451  0.0000
## sch_iqv:sch_ses -0.10717 0.03295990  207  -3.25166  0.0013
##  Correlation:
##                 (Intr) IQ_vrb ses    sex    sch_qv sch_ss IQ_vr:
## IQ_verb         -0.293
## ses              0.017 -0.255
## sex             -0.391  0.026 -0.022
## sch_iqv         -0.085 -0.166  0.061  0.004
## sch_ses          0.051  0.053 -0.266 -0.009 -0.496
## IQ_verb:ses     -0.076  0.076 -0.122 -0.038 -0.015 -0.143
## sch_iqv:sch_ses -0.358 -0.009  0.024  0.037  0.179 -0.017 -0.128
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -4.04381796 -0.62822093  0.07918544  0.68185125  2.97744655
##
## Number of Observations: 3758
## Number of Groups: 211
```

nlme offers a number of variance functions that do not include the linear variance function used in Model 3. These are treated in Pinheiro & Bates (2004), Section 5.2. It turns out that the variance functions offered by nlme, when used in the default way, represent this data set slightly less well than a linear variance function.

Therefore we work towards the results in Table 8.2 by also using the result obtained by another program (MLwiN), namely, that the linear variance function for level 1 was estimated as

```
IQ_verb <- mlbook_red$IQ_verb
#36.382 - 2*1.689*IQ_verb
```

We define the transformed variable

```
IQ_trans <- 36.382 - 2*1.689*mlbook_red$IQ_verb
```

We can use this variance function (giving it a freely estimated multiplicative parameter, which indeed will be estimated very close to 1) by using varFixed:

```
mod83 <- lme(langPOST ~ IQ_verb*ses + sex + sch_iqv*sch_ses,
               random=~ IQ_verb|schoolnr, weights=varFixed(~IQ_trans),
               data = mlbook_red, method="ML")
summary(mod83)
```

```
## Linear mixed-effects model fit by maximum likelihood
##  Data: mlbook_red
##        AIC   BIC    logLik
##   24454.22 24529 -12215.11
##
## Random effects:
##  Formula: ~IQ_verb | schoolnr
##  Structure: General positive-definite, Log-Cholesky parametrization
##             StdDev    Corr
## (Intercept) 2.8649245 (Intr)
## IQ_verb     0.3288457 -0.778
```

```
## Residual    1.0000001
##
## Variance function:
##  Structure: fixed weights
##  Formula: ~IQ_trans
## Fixed effects: langPOST ~ IQ_verb * ses + sex + sch_iqv * sch_ses
##                    Value  Std.Error   DF   t-value p-value
## (Intercept)     40.51010 0.26314161 3543 153.94791  0.0000
## IQ_verb          2.20041 0.05820335 3543  37.80563  0.0000
## ses              0.17464 0.01145189 3543  15.24987  0.0000
## sex              2.31120 0.19832116 3543  11.65380  0.0000
## sch_iqv          0.68521 0.28960748  207   2.36601  0.0189
## sch_ses         -0.08687 0.04129195  207  -2.10385  0.0366
## IQ_verb:ses     -0.02232 0.00457096 3543  -4.88217  0.0000
## sch_iqv:sch_ses -0.10693 0.03258171  207  -3.28180  0.0012
##  Correlation:
##                 (Intr) IQ_vrb ses    sex    sch_qv sch_ss IQ_vr:
## IQ_verb         -0.308
## ses              0.025 -0.245
## sex             -0.371  0.037 -0.024
## sch_iqv         -0.078 -0.185  0.062  0.002
## sch_ses          0.052  0.062 -0.247 -0.011 -0.501
## IQ_verb:ses     -0.063  0.004 -0.242 -0.036 -0.014 -0.123
## sch_iqv:sch_ses -0.355 -0.013  0.044  0.034  0.172 -0.025 -0.152
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -4.42011752 -0.63248539  0.09102577  0.69345000  2.86020972
##
## Number of Observations: 3758
## Number of Groups: 211
```

Another way is by using varPower

```
mod83a <- lme(langPOST ~ IQ_verb*ses + sex + sch_iqv*sch_ses,
              random=~ IQ_verb|schoolnr, weights=varPower(form=~IQ_trans),
              data = mlbook_red, method="ML")
summary(mod83a)
```

```
## Linear mixed-effects model fit by maximum likelihood
##  Data: mlbook_red
##       AIC      BIC   logLik
##   24456.2 24537.21 -12215.1
##
## Random effects:
##  Formula: ~IQ_verb | schoolnr
##  Structure: General positive-definite, Log-Cholesky parametrization
##             StdDev    Corr
## (Intercept) 2.8654184 (Intr)
## IQ_verb     0.3287286 -0.781
## Residual    1.0340154
##
## Variance function:
##  Structure: Power of variance covariate
##  Formula: ~IQ_trans
```

```
##  Parameter estimates:
##     power
## 0.4906359
## Fixed effects: langPOST ~ IQ_verb * ses + sex + sch_iqv * sch_ses
##                    Value  Std.Error   DF   t-value p-value
## (Intercept)     40.50824 0.26317705 3543 153.92011  0.0000
## IQ_verb          2.20136 0.05822917 3543  37.80505  0.0000
## ses              0.17456 0.01145027 3543  15.24527  0.0000
## sex              2.31319 0.19838936 3543  11.65987  0.0000
## sch_iqv          0.68610 0.28964222  207   2.36879  0.0188
## sch_ses         -0.08693 0.04130328  207  -2.10478  0.0365
## IQ_verb:ses     -0.02227 0.00457406 3543  -4.86948  0.0000
## sch_iqv:sch_ses -0.10689 0.03258255  207  -3.28074  0.0012
##  Correlation:
##                 (Intr) IQ_vrb ses    sex    sch_qv sch_ss IQ_vr:
## IQ_verb         -0.308
## ses              0.024 -0.245
## sex             -0.371  0.036 -0.024
## sch_iqv         -0.078 -0.185  0.062  0.002
## sch_ses          0.052  0.062 -0.247 -0.011 -0.501
## IQ_verb:ses     -0.063  0.006 -0.240 -0.036 -0.014 -0.123
## sch_iqv:sch_ses -0.355 -0.013  0.044  0.034  0.172 -0.025 -0.152
##
## Standardized Within-Group Residuals:
##        Min         Q1        Med         Q3        Max
## -4.4049968 -0.6328550  0.0914132  0.6934014  2.8579850
##
## Number of Observations: 3758
## Number of Groups: 211
```

and a further additional parameter is allowed by varConstPower

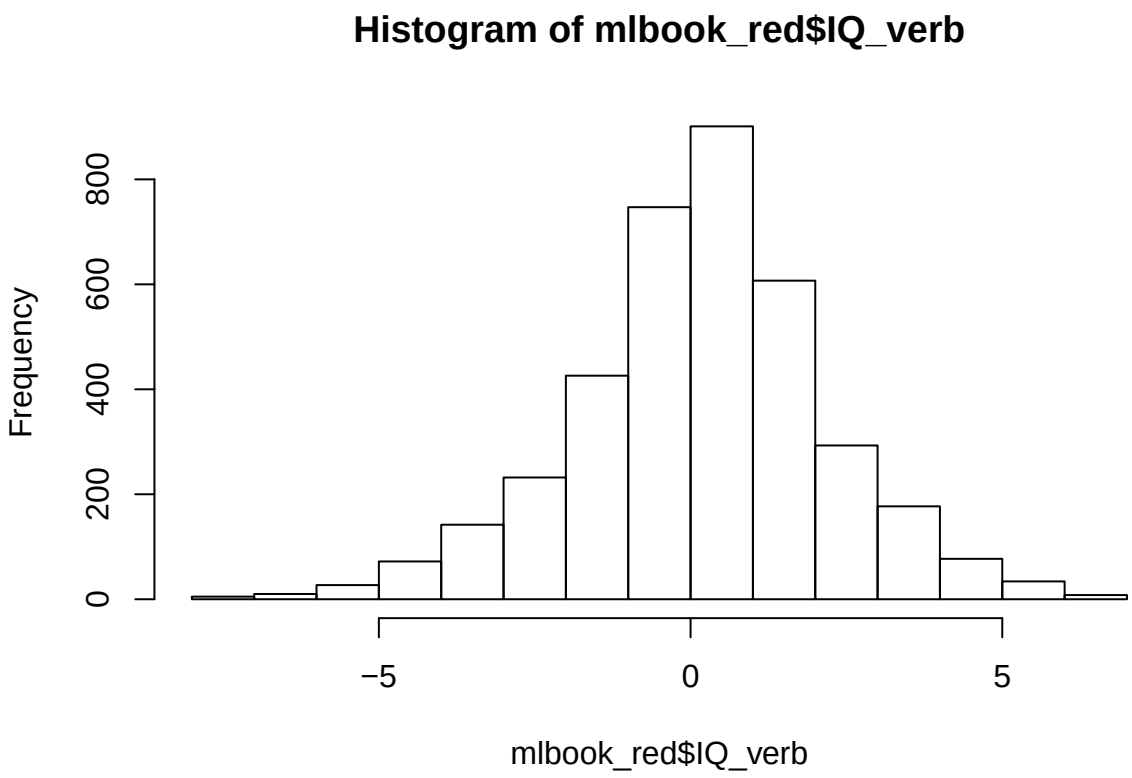
```
mod83b <- lme(langPOST ~ IQ_verb*ses + sex + sch_iqv*sch_ses,
              random=~ IQ_verb|schoolnr,
              weights=varConstPower(form=~IQ_trans),
              data = mlbook_red, method="ML")
summary(mod83b)
```

```
## Linear mixed-effects model fit by maximum likelihood
##  Data: mlbook_red
##       AIC      BIC   logLik
##   24458.2 24545.44 -12215.1
##
## Random effects:
##  Formula: ~IQ_verb | schoolnr
##  Structure: General positive-definite, Log-Cholesky parametrization
##             StdDev    Corr
## (Intercept) 2.8654106 (Intr)
## IQ_verb     0.3287414 -0.781
## Residual    1.0332895
##
## Variance function:
##  Structure: Constant plus power of variance covariate
##  Formula: ~IQ_trans
##  Parameter estimates:
```

```
##       const       power
## 0.001440342 0.490762235
## Fixed effects: langPOST ~ IQ_verb * ses + sex + sch_iqv * sch_ses
##                    Value  Std.Error   DF   t-value p-value
## (Intercept)     40.50824 0.26317664 3543 153.92035  0.0000
## IQ_verb          2.20136 0.05822974 3543  37.80475  0.0000
## ses              0.17456 0.01145027 3543  15.24526  0.0000
## sex              2.31319 0.19838940 3543  11.65986  0.0000
## sch_iqv          0.68611 0.28964212  207   2.36882  0.0188
## sch_ses         -0.08693 0.04130322  207  -2.10479  0.0365
## IQ_verb:ses     -0.02227 0.00457409 3543  -4.86942  0.0000
## sch_iqv:sch_ses -0.10689 0.03258257  207  -3.28073  0.0012
##  Correlation:
##                 (Intr) IQ_vrb ses    sex    sch_qv sch_ss IQ_vr:
## IQ_verb         -0.308
## ses              0.024 -0.245
## sex             -0.371  0.036 -0.024
## sch_iqv         -0.078 -0.185  0.062  0.002
## sch_ses          0.052  0.062 -0.247 -0.011 -0.501
## IQ_verb:ses     -0.063  0.006 -0.240 -0.036 -0.014 -0.123
## sch_iqv:sch_ses -0.355 -0.013  0.044  0.034  0.172 -0.025 -0.152
##
## Standardized Within-Group Residuals:
##        Min         Q1        Med         Q3        Max
## -4.4049404 -0.6328478  0.0914108  0.6934011  2.8579901
##
## Number of Observations: 3758
## Number of Groups: 211
```

A more satisfactory approach that does not assume previous knowledge, is to use a categorized version of IQ_verb: First, look at the distribution to find reasonable category bounds

```
hist(mlbook_red$IQ_verb)
```




```
IQ_cat <- cut(mlbook_red$IQ_verb, c(-10,-2,-1,0,1,2,10))
table(IQ_cat)
```

```
## IQ_cat
## (-10,-2]  (-2,-1]   (-1,0]    (0,1]    (1,2]   (2,10]
##      488      426      747      901      607      589
```

Now we fit a model where the variance depends on categorized IQ

```
mod83c <- lme(langPOST ~ IQ_verb*ses + sex + sch_iqv*sch_ses,
              random=~ IQ_verb|schoolnr,
              weights=varIdent(form=~1|IQ_cat),
              data = mlbook_red, method="ML")
summary(mod83c)
```

```
## Linear mixed-effects model fit by maximum likelihood
##  Data: mlbook_red
##        AIC      BIC    logLik
##   24460.44 24566.38 -12213.22
##
## Random effects:
##  Formula: ~IQ_verb | schoolnr
##  Structure: General positive-definite, Log-Cholesky parametrization
##             StdDev    Corr
## (Intercept) 2.8564633 (Intr)
## IQ_verb     0.3691566 -0.718
## Residual    5.0157347
##
```

```
## Variance function:
##  Structure: Different standard deviations per stratum
##  Formula: ~1 | IQ_cat
##  Parameter estimates:
##   (2,10] (-10,-2]   (-1,0]    (1,2]  (-2,-1]    (0,1]
## 1.000000 1.275106 1.298808 1.094673 1.326459 1.185924
## Fixed effects: langPOST ~ IQ_verb * ses + sex + sch_iqv * sch_ses
##                    Value  Std.Error   DF   t-value p-value
## (Intercept)     40.50986 0.26298405 3543 154.03921  0.0000
## IQ_verb          2.21973 0.05941152 3543  37.36191  0.0000
## ses              0.17266 0.01145585 3543  15.07173  0.0000
## sex              2.34700 0.19833644 3543  11.83344  0.0000
## sch_iqv          0.73007 0.29035690  207   2.51438  0.0127
## sch_ses         -0.08754 0.04130055  207  -2.11966  0.0352
## IQ_verb:ses     -0.02088 0.00460322 3543  -4.53503  0.0000
## sch_iqv:sch_ses -0.10202 0.03269125  207  -3.12085  0.0021
##  Correlation:
##                 (Intr) IQ_vrb ses    sex    sch_qv sch_ss IQ_vr:
## IQ_verb         -0.309
## ses              0.026 -0.256
## sex             -0.372  0.032 -0.021
## sch_iqv         -0.082 -0.173  0.063  0.003
## sch_ses          0.052  0.060 -0.251 -0.011 -0.500
## IQ_verb:ses     -0.075  0.058 -0.238 -0.038 -0.009 -0.118
## sch_iqv:sch_ses -0.359 -0.006  0.040  0.037  0.178 -0.022 -0.134
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -4.21533798 -0.64363434  0.08025082  0.69012048  2.96378961
##
## Number of Observations: 3758
## Number of Groups: 211
```

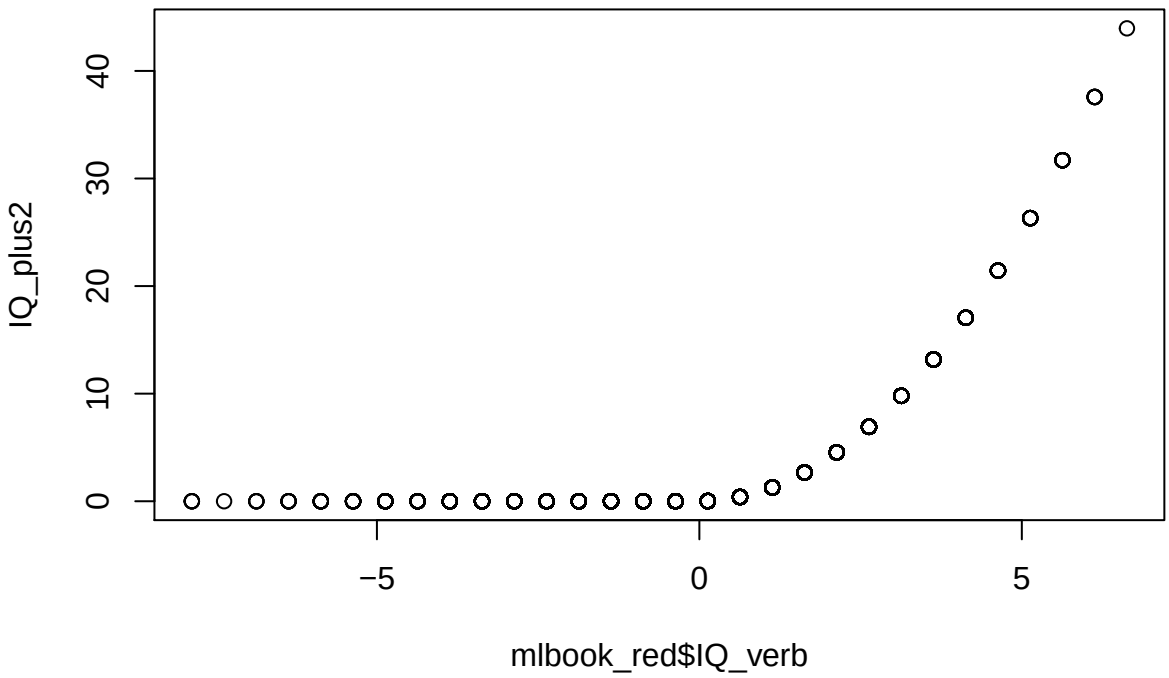
Parameter const is estimated close to 0 and power close to 0.5, with the log-likelihood -12215.1; this means that effectively the same model results as Model 3 in Table 8.2.

For Model 4 in Table 8.2, we first compute the transformed variables.

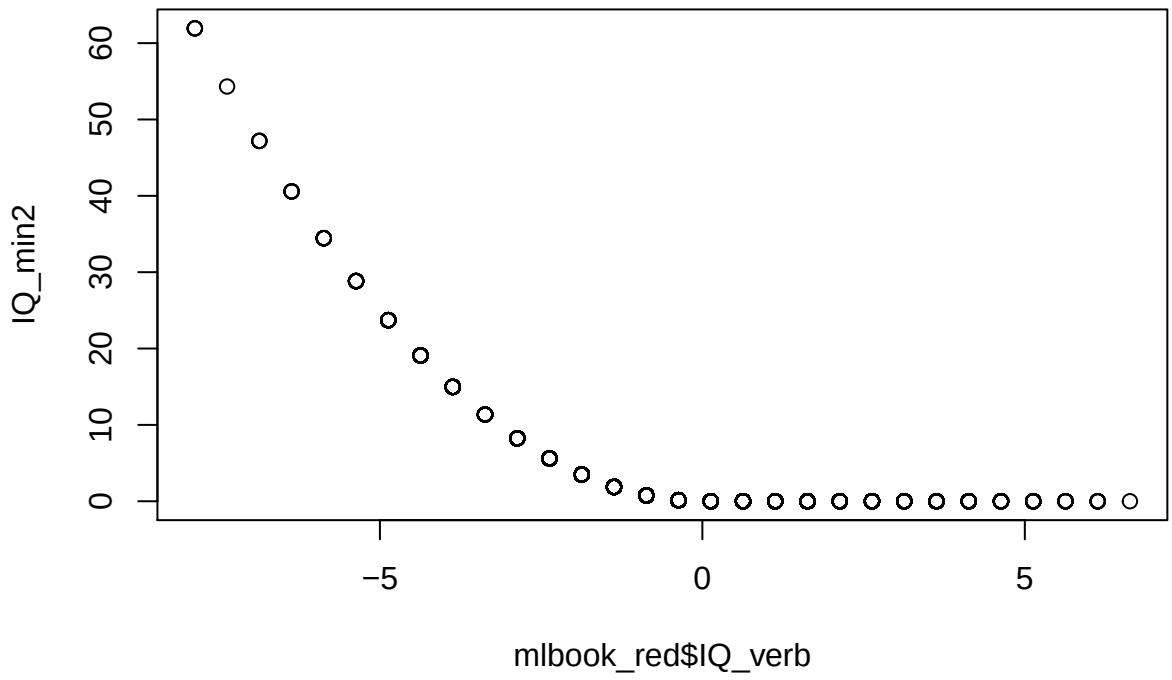
```
IQ_plus2 <- with(mlbook_red, ifelse(IQ_verb > 0, IQ_verb^2, 0))
IQ_min2 <- with(mlbook_red, ifelse(IQ_verb < 0, IQ_verb^2, 0))
```

Have a look at whether this does what we wanted:

```
plot(mlbook_red$IQ_verb, IQ_plus2)
```




```
plot(mlbook_red$IQ_verb, IQ_min2)
```



Again working toward the known answer, we transform IQ_verb to

```
IQ_trans2 <- 36.139 - 2*1.769*mlbook_red$IQ_verb
```

Estimating with varFixed turns out to require some higher control settings for the algorithm:

```
mod84 <- lme(langPOST ~ IQ_verb*ses + sex + sch_iqv*sch_ses +
                        IQ_plus2 + IQ_min2,
           random=~ IQ_verb|schoolnr, weights=varFixed(~IQ_trans2),
           control=lmeControl(maxIter=400,msMaxIter=400,msMaxEval=400,
           niterEM=400,nlmStepMax=400,msTol=1E-9,msVerbose=TRUE),
           data = mlbook_red, method="ML")
```

```
##   0:     22490.732:  1.61187  2.88586  39.0130
##   1:     22341.734: 0.827962  2.26520  39.0295
##   2:     22324.900: 0.524463  1.97372  39.0368
##   3:     22320.274: 0.882021  1.75234  39.0205
##   4:     22319.469:  1.17143  1.45069  39.0088
##   5:     22319.450:  1.24673  1.38131  39.0040
##   6:     22319.441:  1.21373  1.41592  39.0058
##   7:     22319.441:  1.21644  1.41233  39.0060
##   8:     22319.441:  1.21663  1.41225  39.0063
```

```
summary(mod84)
```

```
## Linear mixed-effects model fit by maximum likelihood
##  Data: mlbook_red
##        AIC      BIC    logLik
##   24397.11 24484.35 -12184.56
```

```
##
## Random effects:
##  Formula: ~IQ_verb | schoolnr
##  Structure: General positive-definite, Log-Cholesky parametrization
##             StdDev    Corr
## (Intercept) 2.8276561 (Intr)
## IQ_verb     0.2433749 -0.995
## Residual    0.9991031
##
## Variance function:
##  Structure: fixed weights
##  Formula: ~IQ_trans2
## Fixed effects: langPOST ~ IQ_verb * ses + sex + sch_iqv * sch_ses + IQ_plus2 +      IQ_min2
##                    Value  Std.Error   DF   t-value p-value
## (Intercept)     40.51311 0.26548153 3541 152.60239  0.0000
## IQ_verb          3.04701 0.12578961 3541  24.22305  0.0000
## ses              0.16770 0.01149100 3541  14.59444  0.0000
## sex              2.25127 0.19670762 3541  11.44473  0.0000
## sch_iqv          0.80070 0.28526656  207   2.80685  0.0055
## sch_ses         -0.08379 0.04071565  207  -2.05788  0.0409
## IQ_plus2        -0.25992 0.03307122 3541  -7.85939  0.0000
## IQ_min2          0.19310 0.03786231 3541   5.10014  0.0000
## IQ_verb:ses     -0.01571 0.00501028 3541  -3.13469  0.0017
## sch_iqv:sch_ses -0.08925 0.03196439  207  -2.79202  0.0057
##  Correlation:
##                 (Intr) IQ_vrb ses    sex    sch_qv sch_ss IQ_pl2 IQ_mn2
## IQ_verb         -0.185
## ses             -0.001 -0.152
## sex             -0.368 -0.009 -0.018
## sch_iqv         -0.058 -0.049  0.047 -0.003
## sch_ses          0.043  0.045 -0.236 -0.010 -0.502
## IQ_plus2        -0.025 -0.829  0.098  0.039 -0.074  0.002
## IQ_min2         -0.156  0.744  0.041 -0.007 -0.024  0.043 -0.546
## IQ_verb:ses      0.036  0.050 -0.285 -0.050  0.032 -0.138 -0.232 -0.213
## sch_iqv:sch_ses -0.336  0.047  0.037  0.031  0.173 -0.030 -0.078  0.017
##                 IQ_vr:
## IQ_verb
## ses
## sex
## sch_iqv
## sch_ses
## IQ_plus2
## IQ_min2
## IQ_verb:ses
## sch_iqv:sch_ses -0.117
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -4.37792295 -0.64085607  0.08891489  0.70081229  2.78148497
##
## Number of Observations: 3758
## Number of Groups: 211
```

Or, again with the categorized version of IQ:

```
mod84c <- lme(langPOST ~ IQ_verb*ses + sex + sch_iqv*sch_ses +
                         IQ_plus2 + IQ_min2,
           random=~ IQ_verb|schoolnr, weights=varIdent(form=~1|IQ_cat),
           data = mlbook_red, method="ML")
summary(mod84c)
```

```
## Linear mixed-effects model fit by maximum likelihood
##  Data: mlbook_red
##        AIC      BIC    logLik
##   24406.66 24525.06 -12184.33
##
## Random effects:
##  Formula: ~IQ_verb | schoolnr
##  Structure: General positive-definite, Log-Cholesky parametrization
##             StdDev    Corr
## (Intercept) 2.8218589 (Intr)
## IQ_verb     0.2876167 -0.887
## Residual    4.9719445
##
## Variance function:
##  Structure: Different standard deviations per stratum
##  Formula: ~1 | IQ_cat
##  Parameter estimates:
##   (2,10] (-10,-2]   (-1,0]    (1,2]  (-2,-1]    (0,1]
## 1.000000 1.279763 1.308092 1.094504 1.326499 1.195896
## Fixed effects: langPOST ~ IQ_verb * ses + sex + sch_iqv * sch_ses + IQ_plus2 +      IQ_min2
##                    Value  Std.Error   DF   t-value p-value
## (Intercept)     40.48811 0.26581745 3541 152.31545  0.0000
## IQ_verb          3.06454 0.12627484 3541  24.26878  0.0000
## ses              0.16749 0.01147842 3541  14.59195  0.0000
## sex              2.31031 0.19694474 3541  11.73073  0.0000
## sch_iqv          0.80631 0.28625199  207   2.81680  0.0053
## sch_ses         -0.08446 0.04077742  207  -2.07137  0.0396
## IQ_plus2        -0.26700 0.03564002 3541  -7.49171  0.0000
## IQ_min2          0.19815 0.03545232 3541   5.58930  0.0000
## IQ_verb:ses     -0.01658 0.00507468 3541  -3.26707  0.0011
## sch_iqv:sch_ses -0.08695 0.03207971  207  -2.71039  0.0073
##  Correlation:
##                 (Intr) IQ_vrb ses    sex    sch_qv sch_ss IQ_pl2 IQ_mn2
## IQ_verb         -0.182
## ses              0.000 -0.150
## sex             -0.369 -0.005 -0.017
## sch_iqv         -0.061 -0.055  0.050  0.001
## sch_ses          0.041  0.041 -0.242 -0.011 -0.502
## IQ_plus2        -0.039 -0.815  0.087  0.031 -0.067  0.008
## IQ_min2         -0.150  0.756  0.041 -0.005 -0.033  0.041 -0.547
## IQ_verb:ses      0.029  0.051 -0.271 -0.048  0.038 -0.138 -0.221 -0.226
## sch_iqv:sch_ses -0.339  0.042  0.033  0.035  0.181 -0.027 -0.070  0.014
##                 IQ_vr:
## IQ_verb
## ses
## sex
## sch_iqv
## sch_ses
```

```
## IQ_plus2
## IQ_min2
## IQ_verb:ses
## sch_iqv:sch_ses -0.100
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -4.29110406 -0.63793404  0.08608576  0.69427596  2.89597266
##
## Number of Observations: 3758
## Number of Groups: 211
```