

# Chapter 6

```
library(tidyverse)

## Loading tidyverse: ggplot2
## Loading tidyverse: tibble
## Loading tidyverse: tidyr
## Loading tidyverse: readr
## Loading tidyverse: purrr
## Loading tidyverse: dplyr

## Conflicts with tidy packages -----

## filter(): dplyr, stats
## lag():      dplyr, stats

library(AzureML)
ws <- workspace()

mlbook_red <- tbl_df(download.datasets(
  dataset = ws,
  name     = "mlbook2_r.csv"))

# what are the available variables
names(mlbook_red)

## [1] "schoolnr"      "pupilNR_new"  "langPOST"     "ses"          "IQ_verb"
## [6] "sex"           "Minority"     "denomina"     "sch_ses"      "sch_iqv"
## [11] "sch_min"

# Attach library
library(nlme)

##
## Attaching package: 'nlme'

## The following object is masked from 'package:dplyr':
##
## collapse
```

## 0.1 Table 6.1.1

```
mlb611 <- lme(langPOST ~ IQ_verb + sch_iqv + ses,
              random =~ IQ_verb|schoolnr, data = mlbook_red,
              method="ML")

summary(mlb611)

## Linear mixed-effects model fit by maximum likelihood
## Data: mlbook_red
##      AIC      BIC    logLik
## 24677.29 24727.14 -12330.64
##
## Random effects:
## Formula: ~IQ_verb | schoolnr
## Structure: General positive-definite, Log-Cholesky parametrization
##              StdDev    Corr
```

```

## (Intercept) 3.0127238 (Intr)
## IQ_verb      0.4438311 -0.609
## Residual     6.1174655
##
## Fixed effects: langPOST ~ IQ_verb + sch_iqv + ses
##              Value Std.Error   DF   t-value p-value
## (Intercept) 41.15476 0.23440566 3545 175.57067  0.0000
## IQ_verb      2.26474 0.06483327 3545  34.93183  0.0000
## sch_iqv      0.64688 0.26390839  209   2.45117  0.0151
## ses          0.16150 0.01115911 3545  14.47215  0.0000
## Correlation:
##      (Intr) IQ_vrb sch_qv
## IQ_verb -0.271
## sch_iqv -0.001 -0.152
## ses      0.009 -0.231 -0.102
##
## Standardized Within-Group Residuals:
##      Min      Q1      Med      Q3      Max
## -4.25711851 -0.63394646  0.07372982  0.70574143  2.87358145
##
## Number of Observations: 3758
## Number of Groups: 211

# Deviation variable
IQ_dev <- mlbook_red$IQ_verb - mlbook_red$sch_iqv
mlb612 <- lme(langPOST ~ IQ_dev + sch_iqv + ses,
              random =~ IQ_verb|schoolnr, data = mlbook_red,
              method="ML")

summary(mlb612)

## Linear mixed-effects model fit by maximum likelihood
## Data: mlbook_red
##      AIC      BIC    logLik
## 24677.29 24727.14 -12330.64
##
## Random effects:
## Formula: ~IQ_verb | schoolnr
## Structure: General positive-definite, Log-Cholesky parametrization
##              StdDev   Corr
## (Intercept) 3.0127231 (Intr)
## IQ_verb      0.4438311 -0.609
## Residual     6.1174656
##
## Fixed effects: langPOST ~ IQ_dev + sch_iqv + ses
##              Value Std.Error   DF   t-value p-value
## (Intercept) 41.15476 0.23440562 3545 175.57070    0
## IQ_dev      2.26474 0.06483327 3545  34.93183    0
## sch_iqv      2.91163 0.26199765  209  11.11318    0
## ses          0.16150 0.01115911 3545  14.47215    0
## Correlation:
##      (Intr) IQ_dev sch_qv
## IQ_dev -0.271
## sch_iqv -0.068  0.094
## ses      0.009 -0.231 -0.160
##

```

```
## Standardized Within-Group Residuals:
##      Min      Q1      Med      Q3      Max
## -4.25711851 -0.63394645  0.07372983  0.70574140  2.87358140
##
## Number of Observations: 3758
## Number of Groups: 211
```

## 1 Example 6.2

### 1.1 Table 4.2

```
mlb42 <- lme(langPOST ~ IQ_verb,
             random =~ 1|schoolnr, data = mlbook_red,
             method="ML")
# OLS result, Table 4.3
mlb42.ols <- lm(langPOST ~ IQ_verb, data = mlbook_red)
-2*logLik(mlb42)
## 'log Lik.' 24912.17 (df=4)
-2*logLik(mlb42.ols)
## 'log Lik.' 25350.98 (df=3)
anova(mlb42, mlb42.ols)

##      Model df      AIC      BIC    logLik    Test  L.Ratio p-value
## mlb42      1  4 24920.17 24945.10 -12456.08
## mlb42.ols   2  3 25356.98 25375.67 -12675.49 1 vs 2 438.8096 <.0001
```

## 2 Example 6.4

### 2.1 Table 4.4

```
mlb44 <- lme(langPOST ~ IQ_verb + sch_iqv,
             random =~ 1|schoolnr, data = mlbook_red,
             method="ML")
summary(mlb44)

## Linear mixed-effects model fit by maximum likelihood
## Data: mlbook_red
##      AIC      BIC    logLik
## 24898.02 24929.18 -12444.01
##
## Random effects:
## Formula: ~1 | schoolnr
##      (Intercept) Residual
## StdDev:      2.946136 6.358435
##
## Fixed effects: langPOST ~ IQ_verb + sch_iqv
##      Value Std.Error DF  t-value p-value
## (Intercept) 41.11378 0.23189955 3546 177.29132      0
## IQ_verb      2.45361 0.05551067 3546 44.20073      0
```

```

## sch_iqv      1.31242 0.26170160 209 5.01493      0
## Correlation:
##      (Intr) IQ_vrb
## IQ_vrb -0.007
## sch_iqv 0.043 -0.210
##
## Standardized Within-Group Residuals:
##      Min      Q1      Med      Q3      Max
## -4.22204624 -0.64110095 0.06344872 0.70589967 3.21900337
##
## Number of Observations: 3758
## Number of Groups: 211

# Table 5.1
mlb51 <- lme(langPOST ~ IQ_vrb + sch_iqv,
             random =~ IQ_vrb|schoolnr, data = mlbook_red,
             method="ML")

summary(mlb51)

## Linear mixed-effects model fit by maximum likelihood
## Data: mlbook_red
##      AIC      BIC    logLik
## 24878.87 24922.49 -12432.44
##
## Random effects:
## Formula: ~IQ_vrb | schoolnr
## Structure: General positive-definite, Log-Cholesky parametrization
##      StdDev    Corr
## (Intercept) 2.9794996 (Intr)
## IQ_vrb      0.4415748 -0.634
## Residual    6.2996400
##
## Fixed effects: langPOST ~ IQ_vrb + sch_iqv
##      Value Std.Error   DF   t-value p-value
## (Intercept) 41.12748 0.2337154 3546 175.97252 0e+00
## IQ_vrb      2.47974 0.0643225 3546 38.55163 0e+00
## sch_iqv     1.02849 0.2623010 209 3.92103 1e-04
## Correlation:
##      (Intr) IQ_vrb
## IQ_vrb -0.279
## sch_iqv -0.003 -0.188
##
## Standardized Within-Group Residuals:
##      Min      Q1      Med      Q3      Max
## -4.26147885 -0.63351470 0.06768349 0.70332081 2.76194566
##
## Number of Observations: 3758
## Number of Groups: 211

-2*logLik(mlb44)

## 'log Lik.' 24888.02 (df=5)

-2*logLik(mlb51)

## 'log Lik.' 24864.87 (df=7)

```

```

2*logLik(mlb51) - 2*logLik(mlb44)
## 'log Lik.' 23.14879 (df=7)
# Averaging chi-squared distributions with df=1 and df=2 yields
1 - 0.5*pchisq(2*(logLik(mlb51) - logLik(mlb44)), df=1) -
    0.5*pchisq(2*(logLik(mlb51) - logLik(mlb44)), df=2)
## 'log Lik.' 5.451585e-06 (df=7)
# To check Table 6.2, e.g.,
1 - 0.5*pchisq(8.27, df=1) - 0.5*pchisq(8.27, df=2)
## [1] 0.01001661

```

## 2.2 Table 5.4

```

mlb54 <- lme(langPOST ~ IQ_verb * ses + sch_iqv * sch_ses,
             random =~ IQ_verb|schoolnr, data = mlbook_red,
             method="ML")
mlb54r <- lme(langPOST ~ IQ_verb * ses + sch_iqv * sch_ses,
             random =~ 1|schoolnr, data = mlbook_red,
             method="ML")
summary(mlb54)
## Linear mixed-effects model fit by maximum likelihood
## Data: mlbook_red
##      AIC      BIC    logLik
## 24648.82 24717.37 -12313.41
##
## Random effects:
## Formula: ~IQ_verb | schoolnr
## Structure: General positive-definite, Log-Cholesky parametrization
##              StdDev   Corr
## (Intercept) 2.8928434 (Intr)
## IQ_verb      0.4053715 -0.792
## Residual     6.1137490
##
## Fixed effects: langPOST ~ IQ_verb * ses + sch_iqv * sch_ses
##              Value Std.Error DF t-value p-value
## (Intercept)  41.61218 0.24771021 3544 167.98735 0.0000
## IQ_verb      2.23109 0.06340304 3544 35.18901 0.0000
## ses          0.17444 0.01167064 3544 14.94658 0.0000
## sch_iqv      0.76017 0.29606815 207 2.56755 0.0109
## sch_ses     -0.08861 0.04231906 207 -2.09383 0.0375
## IQ_verb:ses -0.01734 0.00490566 3544 -3.53491 0.0004
## sch_iqv:sch_ses -0.11972 0.03329134 207 -3.59601 0.0004
## Correlation:
##              (Intr) IQ_vrb ses    sch_qv sch_ss IQ_vr:
## IQ_verb      -0.305
## ses           0.009 -0.251
## sch_iqv      -0.092 -0.167 0.061
## sch_ses       0.051 0.054 -0.269 -0.496
## IQ_verb:ses  -0.099 0.076 -0.123 -0.014 -0.140
## sch_iqv:sch_ses -0.374 -0.010 0.024 0.181 -0.017 -0.126
##

```

```

## Standardized Within-Group Residuals:
##      Min      Q1      Med      Q3      Max
## -4.26882474 -0.62952818  0.07825312  0.70358490  2.86972069
##
## Number of Observations: 3758
## Number of Groups: 211

summary(mlb54r)

## Linear mixed-effects model fit by maximum likelihood
## Data: mlbook_red
##      AIC      BIC    logLik
## 24673.09 24729.17 -12327.54
##
## Random effects:
## Formula: ~1 | schoolnr
##      (Intercept) Residual
## StdDev:      2.845786 6.165415
##
## Fixed effects: langPOST ~ IQ_verb * ses + sch_iqv * sch_ses
##              Value Std.Error   DF   t-value p-value
## (Intercept)  41.53488 0.24455372 3544 169.83950  0.0000
## IQ_verb      2.20799 0.05627212 3544  39.23776  0.0000
## ses          0.17406 0.01171237 3544  14.86133  0.0000
## sch_iqv      0.95456 0.30990668  207   3.08015  0.0023
## sch_ses     -0.09639 0.04371744  207  -2.20489  0.0286
## IQ_verb:ses  -0.01771 0.00467410 3544  -3.78998  0.0002
## sch_iqv:sch_ses -0.09233 0.03272793  207  -2.82127  0.0052
## Correlation:
##              (Intr) IQ_vrb ses    sch_qv sch_ss IQ_vr:
## IQ_verb      -0.016
## ses           0.012 -0.284
## sch_iqv      -0.092 -0.176  0.049
## sch_ses       0.016  0.071 -0.264 -0.487
## IQ_verb:ses   -0.094  0.089 -0.115  0.000  0.011
## sch_iqv:sch_ses -0.370 -0.008  0.015  0.281  0.024 -0.137
##
## Standardized Within-Group Residuals:
##      Min      Q1      Med      Q3      Max
## -4.24677706 -0.63934925  0.07294916  0.70529367  3.18543694
##
## Number of Observations: 3758
## Number of Groups: 211

-2*logLik(mlb54)

## 'log Lik.' 24626.82 (df=11)

-2*logLik(mlb54r)

## 'log Lik.' 24655.09 (df=9)

2*logLik(mlb54) - 2*logLik(mlb54r)

## 'log Lik.' 28.26471 (df=11)

# Averaging chi-squared distributions with df=1 and df=2 yields
1 - 0.5*pchisq(2*(logLik(mlb54) - logLik(mlb54r)), df=1) -

```

```

0.5*pchisq(2*(logLik(mlb54) - logLik(mlb54r)), df=2)
## 'log Lik.' 4.171258e-07 (df=11)

```

### 3 Example 6.5

```

# Use package lme4
detach("package:nlme")
library(lme4)

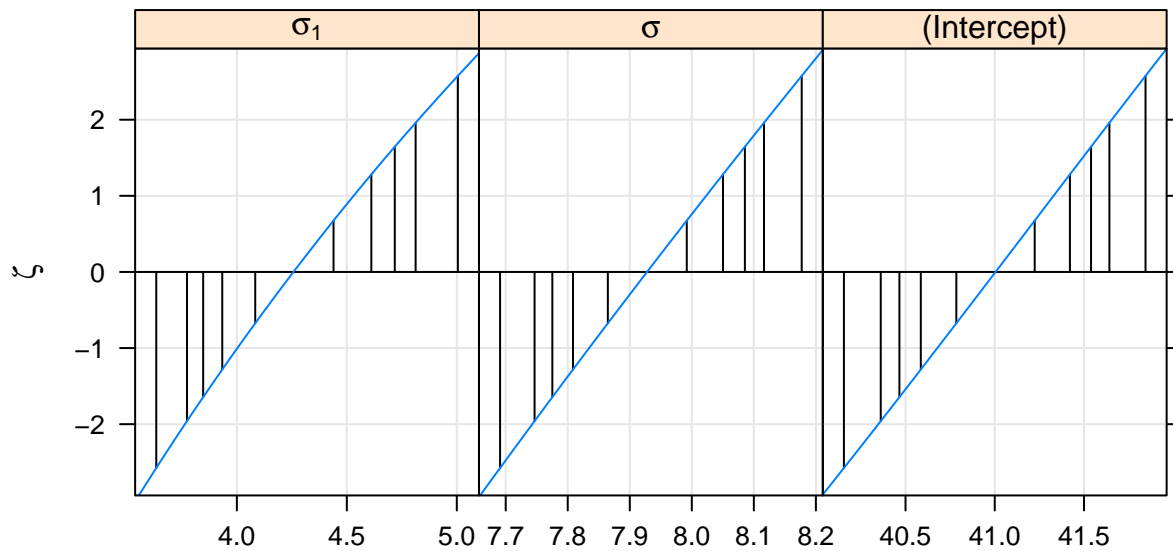
## Loading required package: Matrix
##
## Attaching package: 'Matrix'
## The following object is masked from 'package:tidyr':
##
##      expand
library(lattice)

mlb0 <- lmer(langPOST ~ (1|schoolnr), data = mlbook_red,
             REML = FALSE)
summary(mlb0)

## Linear mixed model fit by maximum likelihood ['lmerMod']
## Formula: langPOST ~ (1 | schoolnr)
## Data: mlbook_red
##
##      AIC      BIC   logLik deviance df.resid
## 26601.3 26620.0 -13297.6 26595.3     3755
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -4.1848 -0.6416  0.0909  0.7227  2.5277
##
## Random effects:
## Groups Name Variance Std.Dev.
## schoolnr (Intercept) 18.13  4.257
## Residual 62.85  7.928
## Number of obs: 3758, groups: schoolnr, 211
##
## Fixed effects:
## Estimate Std. Error t value
## (Intercept) 41.0046 0.3249 126.2

# profile likelihood
pr0 <- profile(mlb0)
xyplot(pr0, aspect=1.3)

```



```

confint(pr0)

##           2.5 %    97.5 %
## .sig01      3.773251  4.812830
## .sigma      7.746500  8.116355
## (Intercept) 40.361220 41.642399

confint(pr0, level=0.90)

##           5 %     95 %
## .sig01      3.846889  4.718028
## .sigma      7.775184  8.085545
## (Intercept) 40.465802 41.539440

# sig01 is the random intercept standard deviation
# lsig is the logarithm of the level-one standard deviation
# Understand the object returned:
str(confint(pr0))

## num [1:3, 1:2] 3.77 7.75 40.36 4.81 8.12 ...
## - attr(*, "dimnames")=List of 2
## ..$ : chr [1:3] ".sig01" ".sigma" "(Intercept)"
## ..$ : chr [1:2] "2.5 %" "97.5 %"

# Transform to confidence intervals for other functions
sqr <- function(a){a*a}
sqr(confint(pr0)[1,])

##    2.5 %    97.5 %
## 14.23742 23.16334

```



```

exp(2*confint(pr0)[2,])
##      2.5 %    97.5 %
## 5352098 11214399

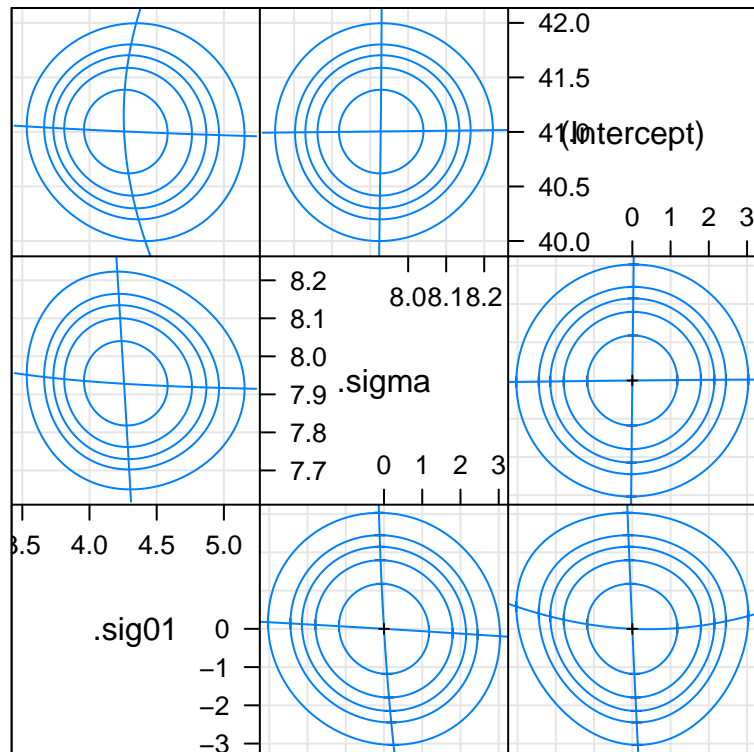
# Symmetric confidence intervals:

s <- sigma(mlb0)
t2 <- VarCorr(mlb0)$schoolnr[1,1]
s2 <- s*s
t <- sqrt(t2)
c <- qnorm(0.975)

set2 <- 2.16
ses2 <- 1.49
ses <- ses2/(2*s)
set <- set2/(2*t)

t2+1.96*set2
## [1] 22.35868
t2-1.96*set2
## [1] 13.89148
s2 + 1.96*ses2
## [1] 65.77094
s2 - 1.96*ses2
## [1] 59.93014
t+1.96*set
## [1] 4.754566
t-1.96*set
## [1] 3.760146
s+1.96*ses
## [1] 8.112019
s-1.96*ses
## [1] 7.743646
splom(pr0)

```



Scatter Plot Matrix

```
mlb44r <- lmer(langPOST ~ IQ_verb + sch_iqv + (1|schoolnr), data = mlbook_red,
               REML = FALSE)
pr44 <- profile(mlb44r)
VarCorr(mlb44)

## schoolnr = pdLogChol(1)
##               Variance StdDev
## (Intercept)  8.679716 2.946136
## Residual    40.429696 6.358435

confint(pr44)

##               2.5 %   97.5 %
## .sig01         2.6013206 3.340226
## .sigma         6.2132799 6.509326
## (Intercept) 40.6558906 41.569208
## IQ_verb      2.3448269 2.562397
## sch_iqv       0.7983407 1.827612

sqr(confint(pr44)[1,])

##      2.5 %   97.5 %
## 6.766869 11.157108

exp(confint(pr44)[2,])

##      2.5 %   97.5 %
## 499.3364 671.3738

exp(2*confint(pr44)[2,])
```

```

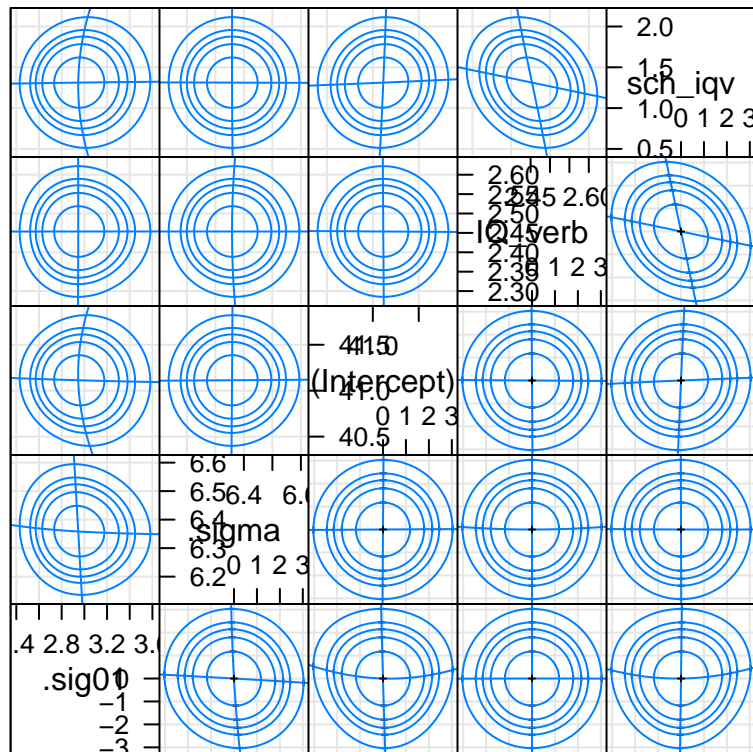
##      2.5 %    97.5 %
## 249336.8 450742.8

s <- sigma(mlb44r)
t2 <- VarCorr(mlb44r)$schoolnr[1,1]
s2 <- s*s
t <- sqrt(t2)
c <- qnorm(0.975)

# Now using the standard errors obtained from MLwiN:
set2 <- 1.0965
ses2 <- 0.9597
ses <- ses2/(2*s)
set <- set2/(2*t)

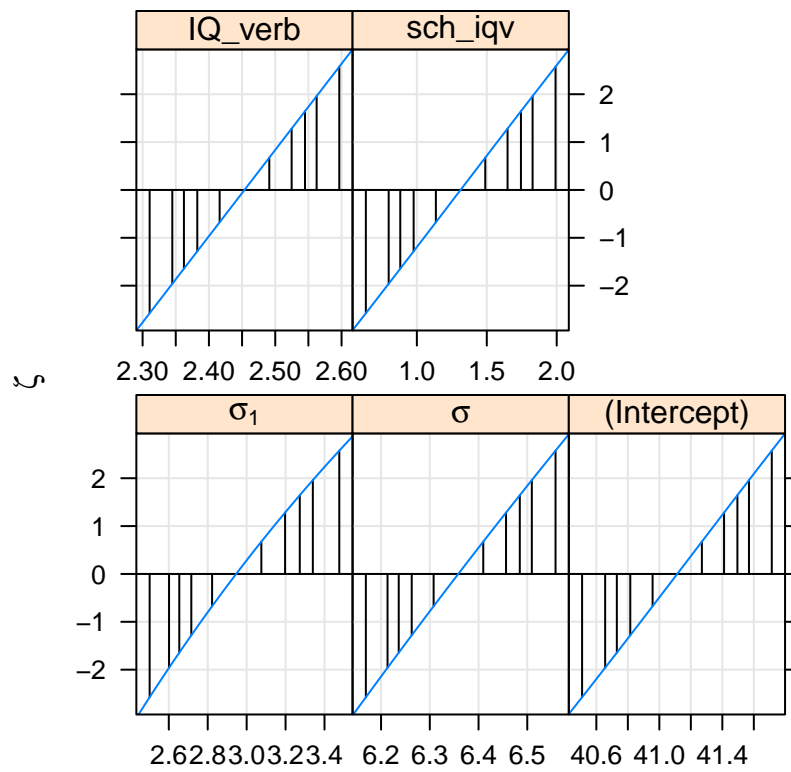
t2+1.96*set2
## [1] 10.82886
t2-1.96*set2
## [1] 6.530579
s2 + 1.96*ses2
## [1] 42.31071
s2 - 1.96*ses2
## [1] 38.54868
t+1.96*set
## [1] 3.310875
t-1.96*set
## [1] 2.581398
s+1.96*ses
## [1] 6.50635
s-1.96*ses
## [1] 6.21052
splom(pr44)

```



Scatter Plot Matrix

```
xyplot(pr44, aspect=1.3)
```



```
# The further analysis using mcmcscamp, which was listed here earlier, is withdrawn
# because mcmcscamp was withdrawn; see the help page
# ?pvalues
# for lme4.
```