Assignment7

Shufan Xia
(Dated: Mar.31th, 2020)

Collaborated with Nathan Wolthuis on the pendulum problem

1. PENDULUM

(a)

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{l}sin\theta \tag{1}$$

Turn this second-order equation into two first-order equations be defining a new variable $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$. Eq1 becomes:

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega, \qquad and \ \ \frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{g}{l}sin\theta \tag{2}$$

Then use forth-order Runge-Kutta method to solve this system of DEQs based on the code from Newman p345. Figure 1 shows the result $\theta(t)$ to DEQ 1 using 10000 steps:

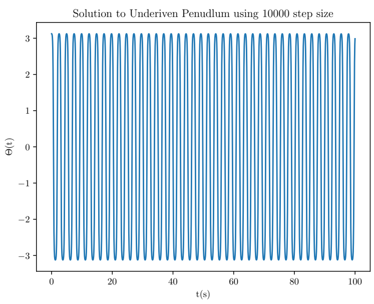

FIG. 1: results from using 10000 steps to solve DEQ1

Using a smaller step size in Runge-Kutta method has very significant computational error which accumulates even larger over time. Figure 2 shows the result using not enough steps.

(b) Driven pendulum The motion of the pendulum system is characterized by

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{l}sin\theta + Ccos\theta sin\Omega t \tag{3}$$

where C and D are two constant Rewrite Eq 3 as:

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega, \qquad and \ \ \frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{g}{l}sin\theta + Ccos\theta sin\Omega t \tag{4}$$

Using the same algorithm of fourth-order Runge-Kutta method in part a), I found the solution when $C = 2s^{-2}$

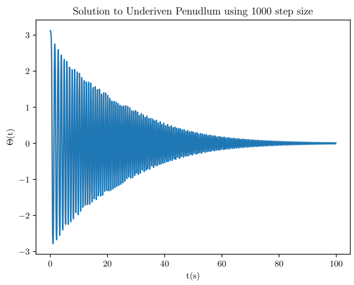

FIG. 2: solution to DEQ 1 using forth order Runge-Kutta method with step size of 1000

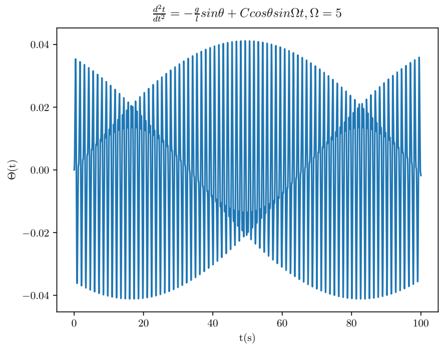

FIG. 3: Solution to $\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{g}{l}sin\theta + 2cos\theta sin5t$

and $D = 5s^{-1}$: When $\Omega = \sqrt{\frac{g}{l}}$, that is the frequency of the driving force is the same as the natural frequency of the pendulum, the driving force resonates with the pendulum. The maximum amplitude is largest and the motion is periodically. Figure 4 shows $\theta(t)$ in this scenario:

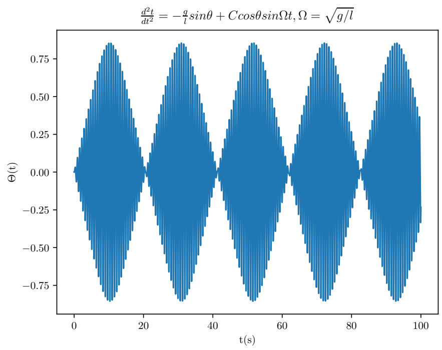

FIG. 4: $\theta(t)$ when the driving force is in resonance with the pendulum

Animation

Run **xia_hw7_animate.py**, type "undriven" in Command line to see the animation for the undriven pendulum motion in part (a), type "driven" to see the animation for the resonant case of driven pendulum motion in part (b)

2. THE LOTKA-VOLTERRA EQUATIONS

The populations of rabbits(x) and foxes(y) are described by this following system of equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \alpha x - \beta xy \tag{5}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \gamma xy - \delta y \tag{6}$$

Similarly, use forth-order Runge-Kutta method to solve the system of equations.

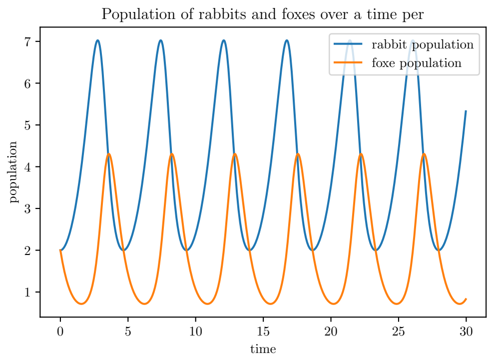

FIG. 5: Predator and prey population

The populations of both rabbits and foxes predicted by the model are cyclical, but they are out off phase. At first the population of rabbits increases because there are not many foxes that eat them. On the same time, with a low population of prey, more foxes die than they are born. However, as rabbits population grows, foxes population also grows because they have enough food to sustain. As the foxes population becomes big enough that more rabbits are killed than they are born, the rabbit population decreases. When the rabbit population is to low to provide enough food for foxes, the population of foxes becomes smaller. Finally, with fewer predators, rabbit population start to increase again. And the population of rabbits and foxes begin to follow the cycle described above. The rabbit population peaks before the foxes population, and the foxes population hits the trough before the rabbit population does.

3. SURVEY QUESTIONS

The homework took me 7 hours. I learned to implement the significance of choosing the right step sizes when implementing Runge-Kutta algorithm to solve DEQs. Making animation for pendulum motion was the most interesting one. This problem set is just right.