Assignment3

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Collaborated with Hongyou Lin and Linda Yuan on 5.14 and 5.21.

1. 5.13 QUANTUM UNCERTAINTY IN THE HARMONIC OSCILLATOR

a) The definition of the Hermite polynomial suggests a recursion function will be useful. I used a *for* loop to implement the recursion, specifically, I created a list of $H_n(x)$ to store $H_0, H_1, ...H_n$. The base cases of the recursion are defined as $H_0 = 1$ and $H_1 = 2x$. For $n > 1$, I used the following definition and use indices in the list to obtain the corresponding previous H_ns:

$$H_n(x) = 2xH_{n-1}(x) - 2(n-1)H_{n-2}(x) \tag{1}$$

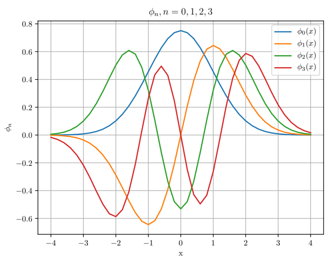

FIG. 1: Harmonic oscillator wave functions for $n = 0, 1, 2$ and 3

b) This question is relatively straightforward. I just defined the function of $\phi_n(n, x)$ based on $H_n(n, x)$ by:

$$\phi_n(n, x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} e^{-\frac{x^2}{2}} H_n(n, x) \tag{2}$$

I then evaluated $\phi_30(30, x)$ for x from -10 to 10.

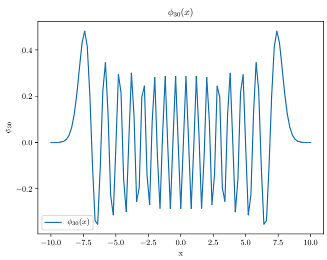

FIG. 2: Harmonic oscillator wave functions for $n = 30$, ϕ_{30}.

c) My program gives: $\sqrt{< x^2 >}$ =2.3452078737858177. To evaluate the integral over x from $-\infty$ to ∞, I used the method of substitution of variable. I substituted x with $\frac{z}{1-z}$, which gives:

$$dx = \frac{1+z^2}{(-1+z^2)^2} dz \tag{3}$$

$$< x^2 > = \int_0^1 (\frac{z}{1-z})^2 |\phi_5(\frac{z}{1-z})|^2 \frac{1+z^2}{(-1+z^2)^2} dz \tag{4}$$

I used gaussian quardauture integration method of 100 sample points to evaluate this integral over z, and then take the square root of the integral result.

2. 5.14 GRAVITATIONAL PULL ON A UNIFORM SHEET

a) This configuration has x and y symmetry, so the x and y components due to each infinitesimal mass dm cancel out overall.

$$dF_z = G\frac{m_p dm}{r^2} cos\theta \tag{5}$$

where r = $\sqrt{x^2 + y^2 + z^2}$, $cos\theta = \frac{z}{r}$, $dm = \sigma dxdy$ and m_p =1.

$$F_z = \iint_{sheet} G\frac{m_p dm}{r^2} cos\theta \tag{6}$$

$$= G\sigma z \iint_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dxdy}{(x^2 + y^2 + z^2)^{3/2}} \tag{7}$$

b) To evaluate the double integral of Eq 7, I used the Gauss–Legendre product formula:

$$I \approx \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j f(x_i, y_j) \tag{8}$$

I used a nested *for* loop to evaluate the sum over 100×100 values of $f(x_i, y_j)$. See 3.

c) The drop is due to two reasons 2) division by zero at when $x, y, z = 0$ 1) the expression for F_z does not converge at z=0. To smoothen the drop, I plotted more sample points:

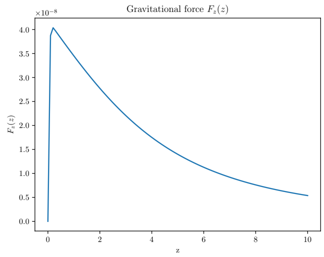

FIG. 3: $F_z z$

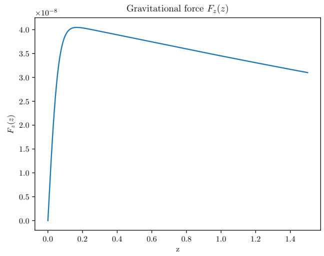

FIG. 4: Smoothened $F_z z$

3. 5.21 ELECTRIC FIELD OF A CHARGE DISTRIBUTION

a) The potential at V(x,y) is given by the sum of V(x,y) due to each point charge:

$$V(x,y) = k\frac{q}{\sqrt{x^2+y^2}} - k\frac{q}{\sqrt{(x-0.1)^2+y^2}} \tag{9}$$

To execute the program faster, I used *meshgrid()* to create a grid of (x,y) and evaluate V(x,y) at each (x,y). To deal with the issue that as (x,y) gets very close to the negative charge at x=0.1m, division by zero encountered. I replaced the -inf value with a large negative potential.

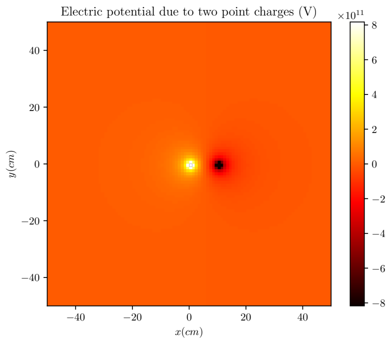

FIG. 5: Electric potential due to two point charge

b) I used central difference method to calculate the partial derivative. I used stream plot to plot the direction of the electric field and density plot to show the strength of the electric field. See fig 6.

c) The potential at (a,b) due to a continuous charge distribution is give by the integral sum of each infinitesimal point charge dq=:

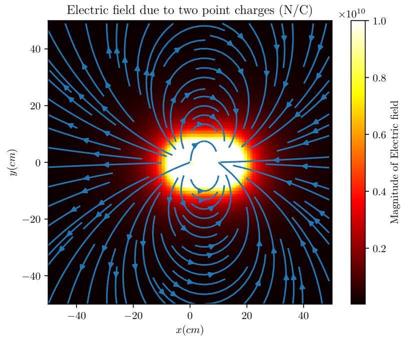

FIG. 6: Electric field due to two point charge

$$V(a,b) = \int k\frac{dq}{\sqrt{(a-x)^2+(b-y)^2}} \tag{10}$$

Since $dq = \sigma dxdy$,

$$V(a,b) = \int_{-\frac{L}{2}}^{\frac{L}{2}}\int_{-\frac{L}{2}}^{\frac{L}{2}} k\frac{q_0 sin\frac{2\pi x}{L} sin\frac{2\pi y}{L}}{\sqrt{(a-x)^2+(b-y)^2}}dxdy \tag{11}$$

Similarly, I used *meshgrid* to plot a 2D density plot for electric potential, see Fig 7: Then I used the same method in part b), central difference method, to calculate the partial derivative of $V(a,b)$ and plotted the results as vector field and density plot, I found the electric field is:

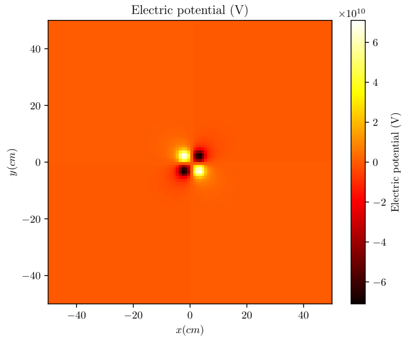

FIG. 7: Electric field due to two point charge

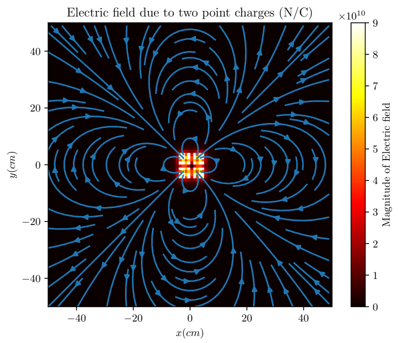

FIG. 8: Electric field due to two point charge

4. SURVEY QUESTIONS

It took me 14 hours to finish the problem set. I learned using *for* loop to implement is faster, plotting 2D plots using *meshgrid()* is much faster than a nested *for* loop. 5.14 and 5.21 are both interesting. 5.14 shows the limiting ability of calculation in python. 5.21 is interesting because I learned to plot vector field. This problem set is just right.