Draft version December 18, 2020
Typeset using LaTeX **modern** style in AASTeX63

Revisit the Oort constants from *Gaia DR2* observations and simulations

Shufan Xia,[1] Karens Masters,[1] and Zhao-Yu Li[2]

[1]*Haverford College, Department of Physics and Astronomy*
[2]*Shanghai Jiaotong University, Department of Astronomy*

ABSTRACT

The Oort constants parameterize stellar motion in the Milky Way Galaxy(MW) It describes relative motions in radial, longitudinal, and latitude directions near the Sun. They are the first proof of Milky Way differential rotation and a flat MW rotation curve. An accurate determination of this set of constants helps us derive the MW rotation curve in the Sun's immediate neighborhood. They can be used to find local Galactic parameters, such as orbit ellipticity, solar distance from the Galaxy's center, etc. *Gaia* provides massive and exquisite data on parallax and proper motion of stars in the Milky Way. These data make it possible to determine the Oort constants in the solar vicinity with unprecedented accuracy. This work re-examines the influence of different sampling criterion on the stellar distance and latitude ranges in the Galactic coordinate for deriving the Oort constants based on *Gaia* DR2 data. We apply proper motion and line-of-sight velocity data from *Gaia* DR2 and compare the result with an idealized test particle simulation under an axisymmetric potential to determine the best stellar subsets for calculating the Oort constants.

1. INTRODUCTION

The simplest model of the Milky Way (MW) indicates that our galaxy is a flat circular disk, and stars move on perfectly circular orbits around the galactic center. However, the structure and the dynamics of our galaxy is much more complicated. Our galaxy is a barred- spiral galaxy with non-axisymmetric and time-dependent perturbation to the potential. Due to perturbation in the MW potential, all stars have non-circular components to their orbital energy, so they move in elliptic and non-closed orbits (Binney & Merrifield 1998; Bovy 2017). The rotational kinematics of the MW is complicated to quantify. The Oort constants are a set of empirically derived kinematic parameters A, B, C, and K that characterizes the local rotation properties in the Milky Way. They have been used extensively in the study of the rotational kinematics of our galaxy.

Corresponding author: Shufan Xia
sxia1@haverford.edu

1.1. *The Milky Way Rotation Curve*

The measurements of the rotation curves of galaxies, a function of circular rotational velocity, v_{circ} about the galactic center over distance, has given us a proof for the existence of dark matters. From the flat but slightly declining rotation curve of the Milky Way, we have derived the mass components and their distribution in our Galaxy. Fig.1 shows the recent measurement of the MW rotation curve (Eilers et al. 2019). The overall rotation curve comes from adding contributions from different components of mass - the central bulge, stellar disc, other stellar components and the dark matter halo (Olling & Merrifield 1998).

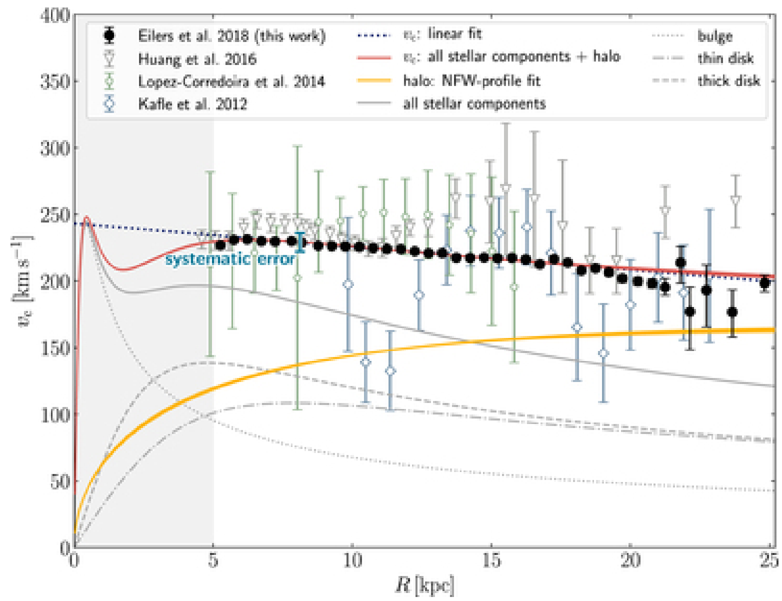

Figure 1. Recent measurements of the circular velocity curve of the Milky Way. The red curve is an approximate overall rotation curve (Eilers et al. 2019).

With Newtonian Mechanics, we can connect velocity, potential, and mass distribution (Binney & Merrifield 1998); therefore, kinematics can serve as a mass probe. The implications of the rotation curve are multiple: it can be used to study kinematics, evolutionary histories of galaxies, and departures from Keplerian form due to dark matter (Sofue & Rubin 2001). However, measuring the Milky Way rotation curve is challenging because of the observation constraints from our location in the Milky Way. We are relatively close to the MW center and in one of the spiral arms of the Galaxy (Binney & Merrifield 1998). This means that our line of sight is blocked by dust, so we can not observe the other side of the Galaxy. The fact that we co-moving with the Galaxy also makes it challenging to determine relative motion. Therefore, measuring the whole rotation curve remains a challenging task.

Various objects and methods have been used to track the rotation curve. Determining distances is quite difficult because of extinction from interstellar medium. For the inner region within the sun, the MW rotation curve has been inferred via the tangentpoint method using the line of sight velocity of HI or CO emission line measured from Doppler's effect (Levine et al. 2008). Based on geometric and trigonometric derivations, this method is able determine distances and rotation velocity just from the radial velocities of gas clouds in a fixed direction but at different distances with respect to the observer. The rotation curve of the outer region beyond the Sun has been measured by other tracers, such as classical Cepheid (Joy 1939; Metzger et al. 1998; Mróz et al. 2019), RR lyrae stars (Wegg et al. 2019), and luminous red giant stars (Eilers et al. 2019). Both classical Cepheids and RR lyrae stars are better for determining rotation curve because they are intrinsically bright and their distance-period relationship makes distance measurement more reliable. (Metzger et al. 1998).

1.2. *Motivation*

While these methods require difficult observational measurements across a very large space, we can use the Oort constants to describe local rotational properties. Oort constants A and B (which are explained more in section 1.3 below) tell us the local rotational velocity around the galactic center and the local slope of the rotational curve (Binney & Merrifield 1998). The power of the Oort constants is that they are local parameters, but it enables us to test against different rotational models from their predicted rotation curves. This work looks at the Oort constants in the vicinity of our Sun. Accurate measurement of Oort constants in this region will help us determine the local rotation curve, galactic radius, and rotational velocity of the sun (Olling & Merrifield 1998), as well as the eccentricity of the solar orbit (Kuijken & Tremaine 1994; Metzger et al. 1998).

1.3. *Oort constants*

To study the local rotation near the Sun and the Oort constants, the following discussion is based on the Galactic coordinate. It is a spherical coordinate, with the Sun at its center and its plane parallel to the Milky Way midplane. Galactic longitude l is the counter-clockwise azimuthal measured from the Galactic center at $l = 0$, and the Galactic longitude, b, is the elevation angle (Fig.2). A nuance to this coordinate frame is that the Sun, in reality, does not stay in a rest frame called LSR which follows a perfect circular orbit. Its motion relative to the LSR is called peculiar motion (Binney & Merrifield 1998).

Assuming the Milky Way has axisymmetric potential and stars are on circular orbits, Oort derived Oort constants A and B via radial velocities and tangential proper motions as a function of Galactic longitude l by Taylor-expanding the local velocity field to the first order about the local rotational center (Oort 1927). It is shown that the radial velocity and tangential velocity are proportional to $cos2l$, $sin2l$:

$$v_r = Adsin2l \quad \text{and} \quad v_\perp = d(Acos2l + B) \tag{1}$$

where $A = -\frac{1}{2}(\frac{dv_{circ}}{dR} - \frac{v_{circ}}{R})\Big|_{R_\odot}$ and $B = -\frac{1}{2}(\frac{dv_{circ}}{R} + \frac{v_{circ}}{R})\Big|_{R_\odot}$.

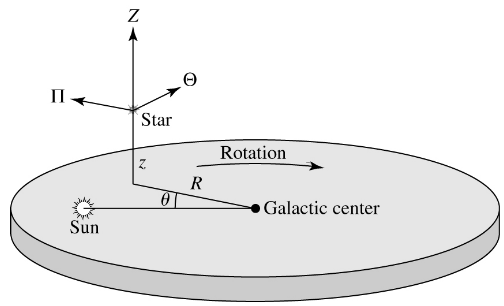

Figure 2. Galactic coordinate system. Figure adapted from Mihos unknown year.

In Oort's original measurement, he determined that near the Sun $A \approx 19$(km/s/kpc) and $B \approx 24$ (km/s/kpc) (Oort 1927). A describes the local shearing, and B describes local vorticity. The result of a non-zero A suggests that stars near the Sun have varying rotation angular velocity, providing the first strong evidence that the Milky Way is rotating differentially. This result supported an earlier rotation model of the MW hypothesized by Lindblad. In Lindblad's hypothesis that the Galactic systems may be divided into subsystems (globular clusters). Each subsystem rotates around the Galactic center, and within each subsystem, objects rotate around a common axis but at different rotational speeds (Oort 1927).

The Oort constants were generalized to consider the Sun's peculiar motion with respect to nearby stars. It is derived analytically that the double sine (or cosine) trend is distinguishable from the pattern due to the Sun's peculiar motion about nearby stars. And two additional constants were introduced to the function of radial and tangential velocity against l, namely C, and K (Ogrodnikoff 1932). These four constants, A, B, C, and K describe transverse shear, vorticity, radial shear, and divergence in the local velocity field from Galactic rotation accordingly. The radial velocity, v_{los}, is:

$$v_{los} = d(K + Ccos2l + Asin2l) \tag{2}$$

The proper motion in longitude and latitude direction, μ_l and μ_b, have the following dependence on $2l$ (Olling & Dehnen 2003):

$$\mu_l = (Acos2l - Csin2l + B)cosb + \varpi(u_0 sinl - v_0 cosl) \tag{3}$$

$$\mu_b = -(Asin2l + Ccos2l + K)sinb\, cosb + \varpi[(u_0 cosl + v_0 sinl)sinb - w_0 cosb]. \tag{4}$$

In equations above, the single *sin* and *cos* terms represent the effect due to the sun's peculiar motion.

The local slope of the rotation curve, $\frac{d\Theta}{dr}|_{R_\odot}$, is given by $-(A + B)$, and rotational velocity, $\Theta_\odot$ is $R_\odot(A - B)$. If the galaxy is not axis-symmetric, C and K are nonzero (Binney & Merrifield 1998, Ogrodnikoff 1932). In the Method section, I will present the full derivation of Oort constants. The Fourier series approximation approach has also been used to derive Oort constants. The Fourier coefficients of the 0 and 2 modes correspond to Oort Constants (Lin et al. 1978).

It must be emphasized that Oort constants can be extended to a set of functions depending on the distances from the Galactic center as we expand velocity fields about different distances. Typically, the Oort functions vary at a rate of a few $km/s/kpc^2$ (Olling & Merrifield 1998). For distant stars in the disk, a higher order of the streaming velocity field equation must be taken into consideration (Siebert et al. 2011). The Oort constants in the first order of the streaming velocity field equation are restrained to the rotation in the solar vicinity because the higher-order contributions will become significant at large distances (Bovy 2017). Due to the contribution from interstellar gas in the MW which density varied non-monotonically with distance from the Galactic center, between $0.9R_{\odot}$ and $1.2R_{\odot}$, the Oort functions $A(R)$ and $B(R)$ differ significantly from the general angular frequency dependence (Olling & Merrifield 1998). This suggests that when measuring the Oort constants, it is crucial to constrain the range of galactic distance.

1.4. *Measuring Oort constants*

In order to measure the Oort constants based on Eq.4, we need to have locations and velocities of stars in the Galactic coordinate. From our vantage point on Earth, we can get radial velocities from the Doppler shift, and proper motions are measured optically by tracing the stars moving across the plane of the sky. The positions or the distances of stars are measured from parallax optically. Measurements of positions, radial velocities, and proper motions are then converted to the Galactic coordinate.

Many investigations have been conducted to attempt to use proper motion to determine the character of the non-uniformity of rotation and to measure the Oort constants. Fig.3 shows one example of measurement of Oort constants with proper motion. However, not all the results are in agreement (Kerr & Lynden-Bell 1986), which is mainly due to the absence of a complete proper motions catalog. A complete measurement of proper motion can give us stars distant enough that their random motions do not dominate their proper motions. It will also ensure sufficient sky coverage to allow the separation between the double sine curve of galactic proper motions from the single sine curve due to the solar motion in Eq.4 (Kerr & Lynden-Bell 1986). Although the observational precisions have been continuously developed, the values of A and B are still consistent with the values within uncertainties. However, for the values of C and K, it is another story (Li et al. 2019). Besides needing a complete proper motion catalog, another three factors complicate an accurate determination of values for the Oort constants.

First, the effect of non-axisymmetric potential associated with spiral or bar structure is not taken into account in Oort's analysis. Observation data suggests the necessity of looking at a non-axisymmetric mode (Metzger et al. 1998). From the radial velocity of Cepheid, a significant zero-point offset (the Oort constant K) in the radial velocities is suggested for a non-axisymmetric model. Metzger et al. (1998) also found a positive antisymmetric ellipticity component at R0. The deviations from the

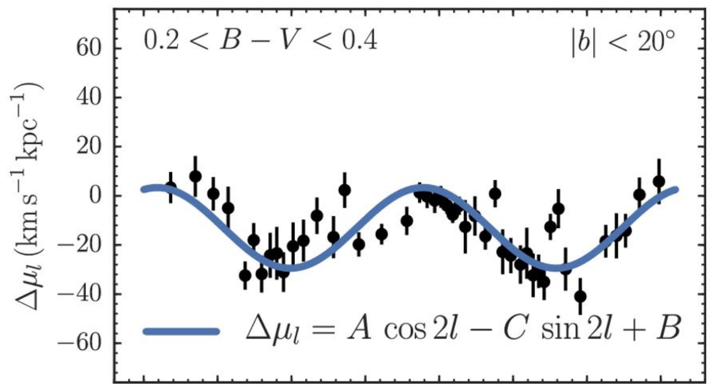

Figure 3. One example of historical measurement of Oort constants with observed proper motion in Galactic longitude: the value on $y - axis$ is corrected for the solar motion based on Eq.4. The averages $\mu_l(l)$ binned by l and their errors are plotted (Bovy 2017).

general axisymmetric velocity field could be used to study the non-circular motions in great detail and infer the mass associated with the spiral arms (Olling & Merrifield 1998).

Second, specifically, moderate strength spiral structure causes errors of order $5km/s/kpc$ in A and B(Minchev & Quillen 2007). The spiral structure can be understood in the light of density waves where the spirals have concentrated stars. The streaming motion caused by density waves depending on the local spiral structure is difficult to determine (Lin et al. 1978). Further, the spiral structure raises the level of random motions in the Galactic disk (Sellwood & Carlberg 2014). Determination of other structural parameters to account for the effects from the Milky Way spirals must be made after a basic circular model.

Third, the nonuniform distribution of parallax over longitudinal in conjunction with the solar peculiar motion contributes to significant 0 and 2 order terms deviated from the first order Oort analysis (Olling & Dehnen 2003). This distribution of x(l) may be due to intrinsic density nonuniformity and observational errors due to interstellar extinction. The components due to this systematic error in the longitudinal proper motions $\mu_l(l)$ are indistinguishable from the effect of double *sin* and *cos* dependency. Olling & Dehnen (2003) suggested using the latitudinal proper motions $\mu_l(l)$ of stars at low latitudes could correct for the errors from mode mixing.

1.5. *Oort constants measurements based on **Gaia** data*

In previous decades, the HIPPARCOS telescope had been used extensively in studying the MW kinematics, including deriving local Oort Constants and the rotation curve from classical cepheid (Feast & Whitelock 1997, Mignard 2000, Olling & Dehnen 2003). The *Gaia* mission (Gaia Collaboration et al. 2016a), since its launch by the European Space Agency in 2013, has collected astrometric parameters with unprecedented accuracy and multitude, aiming to build a three-dimensional map of our Galaxy. By the second release of *Gaia* data, we have the parallax and proper motions of over 1,600,000,000 stars (Andrae et al. 2018). This also includes stars in higher latitude with significant latitude proper motion. Therefore, *Gaia*'s large set of astrometric measurements allows the first truly local precise measurement of the Oort constant and investigation of fine local kinematic features.

Two studies using *Gaia* DR1 and DR2 to calculate the Oorts Constant in solar vicinity yielded results in agreement (Bovy 2017; Li et al. 2019). The current values of the Oort constants based on DR2 is $A = 15.1 \pm 0.1$, $B = 13.4 \pm 0.1$ $C = 2.7 \pm 0.1$ $K = 1.7 \pm 0.2$, all in the unit of km/s/kpc (Li et al. 2019). The slope of the rotation curve is determined from $-(A + B)$ which is negative, thus confirming a slightly declining rotational curve in the solar vicinity. And significant non-zero C and K further indicate non-axisymmetric potential (Bovy 2017; Li et al. 2019). This suggests applying the axis-symmetric assumption of Oort analysis should be carefully examined. Both studies found the local Oort constants varied among different stellar populations based on their positions in the Hertzsprung-Russell Diagram. The result in Li et al. (2019) shows that the red giants deviate from the main-sequence stars in all four constants because they show more elliptical orbits and larger velocity dispersion. This finding is inconsistent with the suggestion in Olling & Dehnen (2003) that red giants are the "true" tracer of the Oort constants because they are old enough to be in equilibrium and distant enough to be unaffected by possible local anomalies. This disagreement suggests establishing an appropriate set of stars is critical to determine Oort constants accurately.

Previous work using HIPPARCOS and *Gaia* data to fit Oort constants used average proper motion binned by l, for example Fig.3, but fine features from each sample point in l vs proper motion are neglected (Bovy 2017, Li et al. 2019). Neither work used radial velocity information to determine the local Oort constants because the radial velocity measurement is only available for a fraction of the stars in the *Gaia* data. However, a closer look at the radial velocity results and their deviation from the predicted theoretical model potentially entails significant information.

1.6. *Proposed research*

This work will look at the Main Sequence and Red Giant stars from *Gaia* DR2 and will use their latitudinal and longitudinal proper motions, and their radial velocity to determine the solar local Oort constants to higher precision. Individual sample will also be considered in deriving Oort constants from proper motions.

As to measure Oort constants to higher precision and to understand the influence of sample selection on the final results, this work will examine the conventional constraints on data sampling. In previous measurements of Oort constants (Olling & Dehnen 2003; Bovy 2017; Li et al. 2019), the sampling limit on parallax and latitude were explained qualitatively without justification. For example, Olling & Dehnen (2003) suggested, using the μ_b of low latitudes stars, but both Bovy (2017) and Li et al. (2019) adopted the $40° < |b| < 50°$ sampling criteria. Therefore, it is not clear, especially in the case of analyzing μ_b, which subsets of stars in observation data should be used to derive the Oort constants and how using different subsets affects the final results.

I will use a simple 3D toy model with test particle simulation under the near circular orbital motion assumption to obtain a theoretical prediction of longitudinal and latitudinal proper motion, μ_l and μ_b, and radial velocity v_{los} as functions of Galactic longitude l. The model will simulate $> 10,000$ test particles moving under the MW potential specified by Bovy (2015). The goal of this model simulation and comparing it to observational data is twofold. First, this simple model will allow us to find the range of parallax and Galactic latitude that give the best stellar subset(s) for Oort constants measurement. Second, it will enable us to characterize the deviations in *Gaia*'s observational result from the expected model. The larger significance of this work is to provide a sample selection guidance for future Oort constants analysis.

In addition, to derive the Oort constants and their uncertainties, this work will apply the Monte Carlo Markov Chain(MCMC) method to fit both observational results from *Gaia* and the simulation results of the toy model to Eq.4. Detail of the sample selection, MCMC fitting, and simulation will be included in the following sections

Finally, the recent third release of *Gaia* in December 2020 adds 200 million new samples to our database (Gaia Collaboration et al. 2020). This work also looks forward to using this very new measurement to derive Oort constants.

ACKNOWLEDGMENTS

This work has made use of data from the European Space Agency (ESA) mission *Gaia* (https://www.cosmos.esa.int/gaia), processed by the *Gaia* Data Processing and Analysis Consortium (DPAC, https://www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the *Gaia* Multilateral Agreement.

I would like to acknowledge the tremendous opportunity to do this work offered by Dr.Zhaoyu Li and Dr.Juntai Shen from Shanghai Jiaotong University, as well as Prof.Karen Masters, and their continuous supports throughout the project.

Software: Astropy (Astropy Collaboration et al. 2013, 2018), galpy (Bovy 2015), emcee (Foreman-Mackey et al. 2013) .

REFERENCES

Andrae, R., Fouesneau, M., Creevey, O., et al. 2018, A&A, 616, A8, doi: 10.1051/0004-6361/201732516

Astropy Collaboration, Robitaille, T. P., Tollerud, E. J., et al. 2013, A&A, 558, A33, doi: 10.1051/0004-6361/201322068

Astropy Collaboration, Price-Whelan, A. M., SipHocz, B. M., et al. 2018, aj, 156, 123, doi: 10.3847/1538-3881/aabc4f

Binney, J., & Merrifield, M. 1998, Galactic Astronomy

Bovy, J. 2015, ApJS, 216, 29, doi: 10.1088/0067-0049/216/2/29

—. 2017, MNRAS, 468, L63, doi: 10.1093/mnrasl/slx027

Eilers, A.-C., Hogg, D. W., Rix, H.-W., & Ness, M. K. 2019, ApJ, 871, 120, doi: 10.3847/1538-4357/aaf648

Feast, M., & Whitelock, P. 1997, MNRAS, 291, 683, doi: 10.1093/mnras/291.4.683

Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, PASP, 125, 306, doi: 10.1086/670067

Gaia Collaboration, Smart, R. L., Sarro, L. M., et al. 2020, arXiv e-prints, arXiv:2012.02061. https://arxiv.org/abs/2012.02061

Joy, A. H. 1939, ApJ, 89, 356, doi: 10.1086/144060

Kerr, F. J., & Lynden-Bell, D. 1986, MNRAS, 221, 1023, doi: 10.1093/mnras/221.4.1023

Kuijken, K., & Tremaine, S. 1994, ApJ, 421, 178, doi: 10.1086/173635

Levine, E. S., Heiles, C., & Blitz, L. 2008, ApJ, 679, 1288, doi: 10.1086/587444

Li, C., Zhao, G., & Yang, C. 2019, ApJ, 872, 205, doi: 10.3847/1538-4357/ab0104

Lin, C. C., Yuan, C., & Roberts, W. W., J. 1978, A&A, 69, 181

Metzger, M. R., Caldwell, J. A. R., & Schechter, P. L. 1998, AJ, 115, 635, doi: 10.1086/300198

Mignard, F. 2000, A&A, 354, 522

Mihos, C. unknown year, The Solar Motionnbsp;. http://burro.case.edu/Academics/Astr222/Galaxy/Kinematics/solarmotion.html

Minchev, I., & Quillen, A. C. 2007, MNRAS, 377, 1163, doi: 10.1111/j.1365-2966.2007.11661.x

Mróz, P., Udalski, A., Skowron, D. M., et al. 2019, ApJL, 870, L10, doi: 10.3847/2041-8213/aaf73f

Ogrodnikoff, K. 1932, ZA, 4, 190

Olling, R. P., & Dehnen, W. 2003, ApJ, 599, 275, doi: 10.1086/379278

Olling, R. P., & Merrifield, M. R. 1998, MNRAS, 297, 943, doi: 10.1046/j.1365-8711.1998.01577.x

Oort, J. H. 1927, BAN, 3, 275

Sellwood, J. A., & Carlberg, R. G. 2014, ApJ, 785, 137, doi: 10.1088/0004-637X/785/2/137

Siebert, A., Famaey, B., Minchev, I., et al. 2011, MNRAS, 412, 2026, doi: 10.1111/j.1365-2966.2010.18037.x

Sofue, Y., & Rubin, V. 2001, ARA&A, 39, 137, doi: 10.1146/annurev.astro.39.1.137

Wegg, C., Gerhard, O., & Bieth, M. 2019, MNRAS, 485, 3296, doi: 10.1093/mnras/stz572