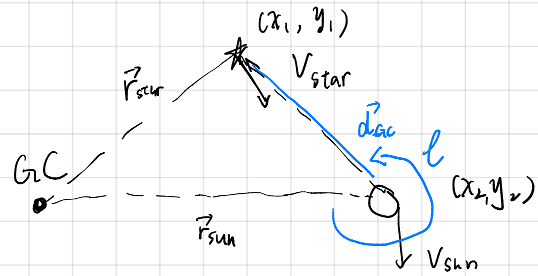


- $V_c = 240$ km/s
- $\vec r_{star} = (x_1, y_1)$

  $\vec r_{sun} = (x_2, y_2)$

① $\vec V_{star} \perp \vec r_{star}$: $\vec V_{star}$ direction $\left(\frac{y_1}{\sqrt{x_1^2+y_1^2}}, \frac{-x_1}{\sqrt{x_1^2+y_1^2}}\right)$

$$V_{star} = V_c\left(\frac{y_1}{|\vec r_{star}|}, \frac{-x_1}{|\vec r_{star}|}\right)$$

Similarly : $V_{sun} = V_c\left(\frac{y_2}{|\vec r_{sun}|}, \frac{-x_2}{|\vec r_{sun}|}\right)$

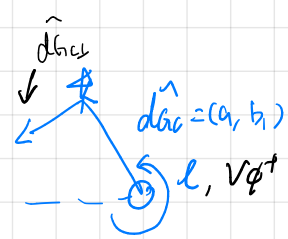
② distance from the Sun to the star $\vec d_{GC} = \vec r_{star} - \vec r_{sn} = (x_1 - x_2, y_1 - y_2)$

$$\hat d_{GC} = \frac{\vec d_{GC}}{|d_{GC}|} = (a_1, b_1), \quad a_1 = \frac{x_1 - x_2}{|\vec d_{GC}|} \text{ and } b_1 = \frac{y_1 - y_2}{|\vec d_{GC}|}$$

③ $V_{los} = (\vec V_{star} - \vec V_{sun}) \cdot \hat d_{GC}$

$$= V_c\left(\left(\frac{y_1}{|\vec r_{star}|}, \frac{-x_1}{|\vec r_{star}|}\right) - \left(\frac{y_2}{|\vec r_{sun}|}, \frac{-x_2}{|\vec r_{sun}|}\right)\right) \cdot \hat d_{GC}$$

$$\boxed{\frac{V_{los}}{d} = \frac{\vec V_{los}}{|\vec d_{GC}|} = \frac{(\vec V_{star} - \vec V_{sun}) \cdot \hat d_{GC}}{|\vec d_{GC}|}}$$



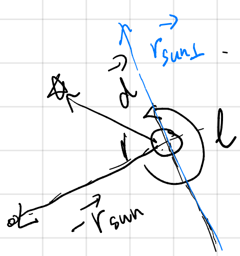
④ μ_l: $\hat d_{GC} = (a_1, b_1)$

$\hat d_{GC\perp} = (-b_1, a_1)$ → correction.

$\Rightarrow \vec v_\phi = (\vec V_{star} - \vec V_{sun}) \cdot \hat d_{GC\perp}$

$$\boxed{\mu_l = \frac{\vec v_\phi}{|\vec d_{GC}|} = \frac{(\vec V_{star} - \vec V_{sun}) \cdot \hat d_{GC\perp}}{|\vec d_{GC}|}}$$

⑤ l: l = angle between $\vec d$ and Sun to GC, $-\vec r_{sun}$. $-\vec r_{sun} = (-x_2, -y_2)$



$$\vec d \cdot (-\vec r_{sun}) = |\vec d||\vec r_{sun}| \sin l$$

$$\boxed{l = \arcsin \frac{\vec d \cdot (-\vec r_{sun})}{|\vec d||\vec r_{sun}|} \quad (0 \le l \le 180°)}$$

arcsin(θ) in python numpy gives $0 \le l \le 180°$.

To correct for θ between $-180° \le l \le 0$.

define $\vec r_{sun\perp}$: $\vec r_{sun} = (x_2, y_2) \Rightarrow \vec r_{sun\perp} = (-y_2, x_2)$

$$\boxed{\text{If } \vec r_{sun\perp} \cdot \vec d > 0,\ -180° \le l \le 0 \text{ and } l = -\arcsin\left(\frac{\vec d \cdot (-\vec r_{sun})}{|\vec d||\vec r_{sun}|}\right)}$$