Gradient Boosting

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Link to the Github repo: https://github.com/shufanhe/BoostingLearner.git

Overview

All the content bellow follow the method proposed by Jerome H. Friedman's paper on *Gradient Boosting (Friedman, 2002)*.

Our goal is to find a function $F^*(\mathbf{x})$ that maps x to y such that loss function $\Psi(y, F(\mathbf{x}))$ is minimized over the joint distribution of all (y, x) values

$$F^*(\mathbf{x}) = \arg\min_{F(\mathbf{x})} E_{y,\mathbf{x}}\, \Psi(y, F(\mathbf{x}))$$

Boosting approximates $F^*(\mathbf{x})$ with an additive expansion with base learner $h(\mathbf{x}; \mathbf{a}_m)$ of the form

$$F(\mathbf{x}) = \sum_{m=0}^{M} \beta_m h(\mathbf{x}; \mathbf{a}_m)$$

where the base learners are usually chosen to be simple functions of x with parameters $a = \{a_1, a_2, \dots\}$.

The expansion coefficients $\{\beta_m\}_0^M$ and the parameters $\{a_m\}_0^M$ are jointly fit to the training data in a forward "stage-wise" manner. We start with an initial guess $F_0(x)$, and then for stages $m = 1, 2, \dots, M$ we find β_m, a_m such that the loss between y and $F_m(x)$ is minimized,

$$(\beta_m, a_m) = \arg\min_{\beta, a} \sum_{i=1}^{N} \Psi(y_i, F_{m-1}(x) + \beta h(x_i; a))$$

and

$$F_m(x) = F_{m-1}(x) + \beta_m h(x; a_m)$$

.

Gradient Boosting approximately solves for β_m and a_m with a two step procedure.

First, the base learner is fit by least squares

$$a_m = \arg\min_{a,p} \sum_{i=1}^{N} [\acute{y}_{im} - ph(x_i; a)]^2$$

to the current "pseudo"-residuals (i.e. negative gradient of the loss w.r.t current predictor)

$$\acute{y}_{im} = -[\frac{\partial \Psi(y_i, F(x_i))}{\partial F(x_i)}]_{F(x)=F_{m-1}(x)}$$

Second, given the base learner $h(x; a_m)$, the optimal value of the expansion coefficient β_m is determined (i.e. by choosing it such that minimizes the "total" loss w.r.t the true y and new predicted value $F_m(x)$)

$$\beta_m = \arg\min_{\beta} \sum_{i=1}^{N} \Psi(y_i, F_{m-1}(x_i) + \beta h(x_i; a_m))$$

Gradient Tree Boosting specializes this approach to the case where the base learner $h(x; a)$ is an L-terminal node regression tree.

At each iteration m, (*first step*) a regression tree is fitted to the current pseudo-residuals using a top-down "best-first" manner with a least-squares splitting criterion. It will partition the x-space into L-disjoint regions $\{R_{lm}\}_{l=1}^{L}$ and predicts a separate constant value in each one (region)

$$h(x; \{R_{lm}\}_1^L) = \sum_{l=1}^{L} \bar{y}_{lm} 1_{x \in R_{lm}}$$

Here the constant value $\bar{y} = mean_{x_i \in R_{lm}}(\acute{y}_{im})$ is the mean of pseudo-residuals in each region R_{lm}. The parameters (a_m) of this base learner are the splitting variables and corresponding split points defining the tree, which in turn define the corresponding regions $\{R_{lm}\}_1^L$ of the partition at the $m\text{th}$ iteration.

Then (*second step*), with the regression tree defined (with $\{R_{lm}\}_1^L$ defined), the coefficient β_m can be solved separately within each region R_{lm}. Because the tree predicts a constant value within each region, the solution to β_m reduces to a simple "location" estimate based on the criterion of the loss function

$$\gamma_{lm} = \arg\min_{\gamma} \sum_{x_i \in R_{lm}} \Psi(y_i, F_{m-1}(x_i) + \gamma)$$

The current approximation is then separately updated in each corresponding region

$$F_m(x) = F_{m-1}(x) + lr \cdot \gamma_{lm} 1_{x \in R_{lm}}$$

where lr is the learning rate.

Stochastic Gradient Boosting draws a subsample of the training data at random (without replacement). This randomly selected subsample is then used to fit the base learner (a_m) and compute the model update for the current iteration (β_m).

Specifically, let $\{y_i, x_i\}_1^N$ be the entire training data sample and $\{\pi(i)\}_1^N$ be a random permutation of the integers $\{1, \ldots, N\}$. Then, a random subsample of size $\acute{N} < N$ is given by $\{y_{\pi(i)}, x_{\pi(i)}\}$.

Our Implementation takes the *loss function* of (half) squared loss (i.e. $0.5 * \sum_{i=1}^{N} (y_i - F(x_i))^2$) and used stochastic gradient tree boosting for regression (*representation* is described above -- as stochastic gradeint tree boosting). The *optimizer* is described in the pseudo code for stochastic gradient tree boosting below.

(Advantages) By injecting the randomness of subsampling into the function estimation procedures, it would be less prune to overfitting and more robust. By gradient boosting, it directly fits each base learner to the residual to the negative gradient of the loss thus converges faster and is more efficient.

(Disadvantages) Nevertheless, it still suffers from the following disadvantages more or less -- 1. still potential overfitting, 2. higher computation cost dues to sequential training, 3. harder interpretability due to complex structures of trees invovled with stochastic subsampling.

Stochastic Gradient Tree Boosting Pseudocode

- Init input
  - learning_rate = 0.1
  - n_esitmators = 20
    - number of estimators (weak learners), same as the number of stages (M)
  - subsample = 0.5
    - fraction for subsampling, if subsample<1, it is stochastic
  - min_samples = 1
    - the minimum number of samples to split for each tree
  - max_depth = 3
    - the maximum depth for each tree
- Train(X, Y)
  - start with an initial guess, say F(x) = mean(Y)
  - for m = 1 to M do:
    - X_batch, Y_batch = random_sampling(X, Y, fraction for subsampling (batch size)) # fraction < 1 leads to stochastic gradient boosting
    - Y_residual = negative gradient of loss between (true) Y and current F based on the batch
      - since we use (half of) sum of squared loss here
      - the gradient is thus F_m_batch-Y_batch
    - weak learner Tree $\{R_{lm}\}_1^L$ = L-terminal node tree fitting on (X_batch, Y_residual)
      - here $\{R_{lm}\}_1^L$ represents a set of L regions (leaf nodes) in stage m
      - R_{lm} represents the l-th region (leaf node) in stage m
    - $\gamma_{lm} = argmin_\gamma \sum_{x-batch \in R_{lm}}$ loss(Y_batch, F(x)+γ)
      - that is, for each region l at stage m, find γ such that it minimizes the loss for the new F(x) = F(x) + γ
      - which is the mean of the sample values at region l at stage m
    - update F(x) = F(x) + learning_rate * $\gamma * 1(x \in R_{lm})$
      - that is, F(x) = F(x) + learning_rate * new tree
      - note here we are multiplying the new tree with learning rate (different from how we obtain the best γ)
  - end for
- Predict(X)
  - Given input X, predict their values
  - which is the sum of initial guess and all the weak learners' prediction
  - where each weak learner's prediction is learning_rate * $\gamma * 1(x \in R_{lm})$
- Loss(X, Y)
  - Given input examples and their true values, calculate the squared loss

Model

In [2]:
```python
import numpy as np
import math

# Helper Functions

def squaErr(data, result, sequence, parameter, divide):
    """
    This function computes the sum of squared errors by splitting the data
    based on the specified parameter and dividing point.

    :param data : numpy.ndarray
        The dataset with samples as rows and features as columns.
    :param result : numpy.ndarray
        The target values corresponding to the data.
    :param sequence : list or numpy.ndarray
        Indices of the data points to consider for the split.
    :param parameter : int
        The index of the feature to split on.
    :param divide : float
        The value used to split the data for the parameter.

    :return err1+err2 : float
        The sum of squared errors for the split.
    """
    left = []
    right = []

    for i in sequence:
        if data[i, parameter] < divide:
            left.append(i)
        else:
            right.append(i)

    if len(left) == 0 or len(right) == 0:  # If either subset is empty, return positive
        return float('inf')

    c1 = np.mean(result[left])
    err1 = np.sum((result[left] - c1) ** 2)

    c2 = np.mean(result[right])
    err2 = np.sum((result[right] - c2) ** 2)

    return err1 + err2


def bestdivide(data, result, sequence, num_bins=256):
    """
    Find the best splitting parameter and point using histogram-based binning.

    :param data: numpy.ndarray
        The dataset with samples as rows and features as columns.
    :param result: numpy.ndarray
        The target values corresponding to the data.
    :param sequence: list or numpy.ndarray
        Indices of the data points to consider for splitting.
    :param num_bins: int, optional
        Number of bins for histogram-based binning (default is 256).
```

```
    :return min_para: int
        The index of the best feature for splitting.
    :return min_divide: float
        The optimal split point for the feature.
    :return min_error: float
        The squared error for the optimal split.
    """
    min_para = None
    min_divide = None
    min_error = float('inf')

    for para in range(data.shape[1]):
        # Extract feature values for the current parameter
        feature_values = data[sequence, para]

        # Compute histogram bins
        bins = np.linspace(feature_values.min(), feature_values.max(), num_bins + 1)
        digitized = np.digitize(feature_values, bins) - 1  # Bin indices

        # Bin boundaries as possible split points
        bin_boundaries = (bins[:-1] + bins[1:]) / 2  # Midpoints between bin edges

        for boundary in bin_boundaries:
            error = squaErr(data, result, sequence, para, boundary)
            if error < min_error:
                min_error = error
                min_para = para
                min_divide = boundary

    return min_para, min_divide, min_error


def squared_loss(predict, target):
    '''
    Calculates the sum of squared loss between predicted values and true values

    :param predict: a 1-d numpy array containing the predicted values
    :param target: a 1-d numpy array containing the true values
    :return loss: squared loss
    '''
    loss = 0.5*np.sum(np.power(predict-target, 2))

    return loss

# Base Learner
class DecisionTreeRegressor:
    def __init__(self, min_samples=1, max_depth=3):
        """
        Initialize the decision tree regressor.
        :param min_samples: int, optional (default=1)
            The minimum number of samples required to split a node.
        :param max_depth: int, optional (default=3)
            The maximum depth of the tree.
        """
        self.min_samples = min_samples  # Minimum number of samples
        self.max_depth = max_depth  # Maximum depth
        self.root = None  # Root node of the decision tree

    class RegressionTree:
        def __init__(self, sequence, depth=0, max_depth=3):
            """
```

```
        Initialize the regression tree node.
        :param sequence: list of int
            Indices of the samples at the current node.
        :param depth: int, optional (default=0)
            The depth of the current node in the tree.
        :param max_depth: int, optional (default=3)
            The maximum depth allowed for the tree.
        """
        self.isLeaf = True  # Whether this node is a leaf
        self.left = None  # Left subtree
        self.right = None  # Right subtree
        self.output = None  # Prediction value for the current node
        self.sequence = sequence  # Indices of samples at the current node
        self.parameter = None  # Splitting feature
        self.divide = None  # Splitting point
        self.depth = depth  # Current depth
        self.max_depth = max_depth  # Maximum depth
        self.leaf_index = id(self)  # Unique identifier for the leaf

    def grow(self, data, result, minnum):
        """
        Performs recursive tree growth by finding the best split for
        the data based on the given parameters. It stops if the sample size is
        below a minimum threshold or the maximum tree depth is reached. The split
        is based on minimizing the sum of squared errors.

        :param data : numpy.ndarray
            The dataset used to train the tree.
        :param result : numpy.ndarray
            The target values corresponding to the dataset.
        :param minnum : int
            The minimum number of samples required in a node to perform a split.
        :return None
        """
        if len(self.sequence) <= minnum or self.depth >= self.max_depth:  # Stop spl
            self.output = np.mean(result[self.sequence])  # Set the prediction value
            return

        # Find the best splitting feature and splitting point
        parameter, divide, err = bestdivide(data, result, self.sequence)
        left = []
        right = []

        # Split data
        for i in self.sequence:
            if data[i, parameter] < divide:
                left.append(i)
            else:
                right.append(i)

        # Update node information
        self.parameter = parameter
        self.divide = divide
        self.isLeaf = False
        self.left = DecisionTreeRegressor.RegressionTree(left, depth=self.depth + 1,
        self.right = DecisionTreeRegressor.RegressionTree(right, depth=self.depth +

        # Recursively grow left and right subtrees
        self.left.grow(data, result, minnum)
        self.right.grow(data, result, minnum)

    def predict_single(self, x):
```

```
            """
            Traverses the decision tree and makes a prediction for a
            single input sample. If the current node is a leaf, the prediction value
            is returned. Otherwise, the method moves left or right based on the split.

            :param x : numpy.ndarray
                A single sample for which the prediction is made.
            :return self.right.predict_single(x): float
                The predicted output for the input sample.
            """
            if self.isLeaf:  # If this is a leaf node, return the prediction value
                return self.output
            if x[self.parameter] < self.divide:  # If less than the splitting point, go
                return self.left.predict_single(x)
            else:  # Otherwise, go to the right subtree
                return self.right.predict_single(x)

        def predict_leaf_index_single(self, x):
            """
            Traverses the decision tree and returns the index of the
            leaf node where the sample would end up. If the current node is a leaf,
            the leaf index is returned.

            :param x : numpy.ndarray
                A single sample for which the leaf index is predicted.
            :return self.right.predict_leaf_index_single(x): int
                The index of the leaf node for the input sample.
            """
            if self.isLeaf:  # If this is a leaf node, return the leaf index
                return self.leaf_index
            if x[self.parameter] < self.divide:  # If less than the splitting point, go
                return self.left.predict_leaf_index_single(x)
            else:  # Otherwise, go to the right subtree
                return self.right.predict_leaf_index_single(x)

    def fit(self, X, y):
        """
        Fit the model to the training data.
        :param X: numpy.ndarray
            The feature matrix (training data) where each row is a sample and each
            column is a feature.
        :param y: numpy.ndarray
            The target values corresponding to the training data samples.
        :return: None
        """
        self.root = self.RegressionTree(sequence=range(len(y)), max_depth=self.max_depth
        self.root.grow(X, y, self.min_samples)

    def predict(self, X):
        """
        Predict the output for the input data.
        :param X: numpy.ndarray
            The input data for which predictions are to be made.
        :return: numpy.ndarray
            The predicted outputs for each sample in the input data.
        """
        return np.array([self.root.predict_single(sample) for sample in X])

    def predict_leaf_indices(self, X):
        """
        Predict the leaf indices for the input data.
        :param X: numpy.ndarray
```

```
            The input data for which leaf indices are to be predicted.
        :return: numpy.ndarray
            The indices of the leaf nodes for each sample in the input data.
        """
        return np.array([self.root.predict_leaf_index_single(sample) for sample in X])


# Stochastic Gradient Boosting
class StochasticGradientBoosting:
    def __init__(self, learning_rate=0.1, n_estimators=20, subsample=0.5, min_samples=1,
        """
        :param learning_rate: learning rate, default 0.1
        :param n_estimators: number of weak learners, default 20 (same as M)
        :param subsample: fraction for subsampling, default 0.5
        :param min_samples: the minimum number of samples for each tree, default 1
        :param max_depth: the maximum depth for each tree (weak learner), default 3
        """
        self.learning_rate = learning_rate
        self.n_estimators = n_estimators
        self.subsample = subsample
        self.min_samples = min_samples
        self.max_depth = max_depth
        self.models = []    # models is a list of weak learners (decision trees)
        self.gammas = []    # gammas is a list of lists of gamma value for each tree's r
        self.initial_prediction = None  # will be initialized in train to be mean
        self.leaf_indices_dict = []     # this will store a list of leaf indices for eac

    def train(self, X, Y):
        """
        Train the Gradient Boosting model by iteratively fitting weak learners.
        :param X: ndarray of shape (n_samples, n_features)
            The input feature matrix for training.
        :param Y: ndarray of shape (n_samples,)
            The target values for training.
        :return: None
        """
        # initial guess F_m=mean(Y)
        self.initial_prediction = np.mean(Y)
        F_m = np.full(Y.shape, self.initial_prediction) # current F

        for m in range(self.n_estimators):
            # Random sampling for stochastic gradient boosting
            batch_size = math.floor(self.subsample*len(Y))
            indices = np.random.choice(len(Y), batch_size, replace=False)
            X_batch, Y_batch = X[indices], Y[indices]

            # Calculate residuals (negative gradient of the loss)
            residuals = Y_batch - F_m[indices]

            # Train a weak learner on the residuals
            weak_learner = DecisionTreeRegressor(min_samples=self.min_samples, max_depth
            weak_learner.fit(X_batch, residuals)
            self.models.append(weak_learner)

            # Update F_m for all samples
            leaf_indices = weak_learner.predict_leaf_indices(X)
            unique_leaves = np.unique(leaf_indices)
            self.leaf_indices_dict.append(unique_leaves)
            gamma_m = []    # the gammas for m'th tree, where each region (leaf) will ha
            for leaf_index in unique_leaves:
                region_mask = (leaf_indices == leaf_index)
                gamma = np.mean(residuals[region_mask[indices]])
```

```
                gamma_m.append(gamma)
                F_m[region_mask] += self.learning_rate * gamma
            self.gammas.append(gamma_m)

    def predict(self, X):
        """
        Predict the target values for the input data based on the trained model.
        :param X: ndarray of shape (n_samples, n_features)
            The input feature matrix for prediction.
        :return F_m: ndarray of shape (n_samples,)
            The predicted target values for the input samples.
        """
        # Start with the initial prediction
        F_m = np.full(X.shape[0], self.initial_prediction)

        # Add contributions from each weak learner
        for m, model in enumerate(self.models):
            leaf_indices = model.predict_leaf_indices(X)
            unique_leaves = self.leaf_indices_dict[m]
            for i, leaf_index in enumerate(unique_leaves):
                region_mask = (leaf_indices == leaf_index)
                F_m[region_mask] += self.learning_rate * self.gammas[m][i]

        return F_m

    def loss(self, X, Y):
        """
        Calculate the squared loss between predicted and true target values.
        :param X: ndarray of shape (n_samples, n_features)
            The input feature matrix.
        :param Y: ndarray of shape (n_samples,)
            The true target values.
        :return loss: float
            The squared loss between the predictions and the true target values.
        """
        pred = self.predict(X)
        loss = squared_loss(pred, Y)

        return loss
```

Check Model

Testing individual functions with dummy data.

In [3]:

```
from sklearn.ensemble import GradientBoostingRegressor
from sklearn.datasets import make_regression
import numpy as np
import pytest


X = np.array([[1], [2], [3], [4], [5]])
Y = np.array([1.1, 2.0, 2.9, 4.1, 5.0])

model = StochasticGradientBoosting(
    learning_rate=0.1,
    n_estimators=10,
    subsample=0.8,
    min_samples=1,
    max_depth=2
)
```

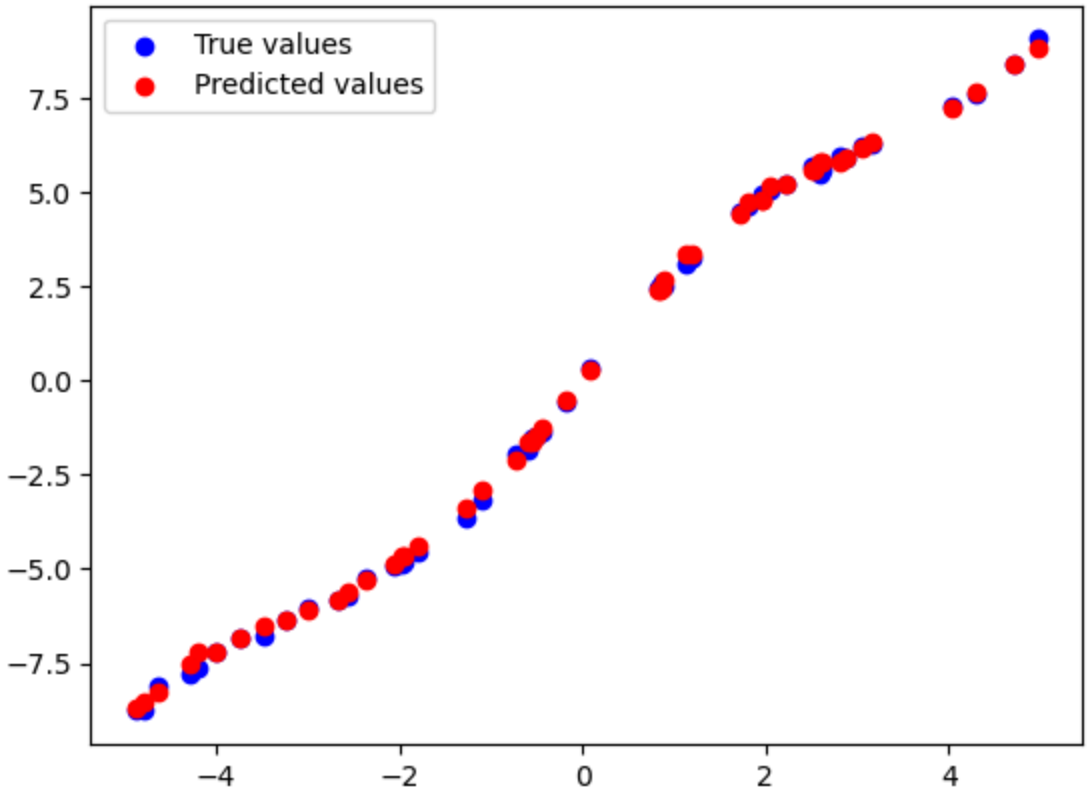
```
# Train the model
model.train(X, Y)

# Test initial prediction
assert np.isclose(model.initial_prediction, np.mean(Y)), "Initial prediction should be t

# Test the number of weak learners (decision trees) trained
assert len(model.models) == model.n_estimators, "Number of models should match n_estimat

# Test that gammas have been computed for each tree
assert len(model.gammas) == model.n_estimators, "Gammas should be computed for each tree
for gamma_list in model.gammas:
    assert len(gamma_list) > 0, "Each tree should have at least one gamma value"

# Test predictions
predictions = model.predict(X)
assert predictions.shape == Y.shape, "Predictions should have the same shape as Y"

# Test if predictions improve with training
assert np.mean((predictions - Y) ** 2) < np.mean((np.mean(Y) - Y) ** 2), \
    "Predictions should reduce mean squared error compared to baseline"

# Test loss
loss = model.loss(X, Y)
assert loss >= 0, "Loss should be non-negative"
expected_loss = 0.5 * np.sum(np.power(predictions - Y, 2))
assert np.isclose(loss, expected_loss), "Loss should match the defined loss function (sc
```

Testing against sklearn on dummy data.

In [4]:
```
# toy test for the whole model -- GradientBoostingRegressor

from sklearn.ensemble import GradientBoostingRegressor

np.random.seed(1)
# set some param for testing
learning_rate=0.1
n_estimators=50     # generally, more estimators lead to better resutls
# subsample=0.5   # subsample of 1 achieves almost the same as sklearn, but subsample le
subsample=1
min_samples=5
max_depth=3

def generate_synthetic_data(n_samples=100, noise=0.1):
    X = np.random.rand(n_samples, 1) * 10 - 5  # Random features between [-5, 5]
    Y = 2 * X.squeeze() + np.sin(X).squeeze() + np.random.randn(n_samples) * noise  # Li
    return X, Y

X_train, Y_train = generate_synthetic_data(n_samples=500)
X_test, Y_test = generate_synthetic_data(n_samples=50)

# Train the model
model = StochasticGradientBoosting(learning_rate=learning_rate, n_estimators=n_estimator
model.train(X_train, Y_train)
predictions = model.predict(X_test)

# Also the sklearn model
sklearn_model = GradientBoostingRegressor(learning_rate=learning_rate, n_estimators=n_es
sklearn_model.fit(X_train, Y_train)
sklearn_pred = sklearn_model.predict(X_test)
```

```
sklearn_loss = squared_loss(sklearn_pred, Y_test)

# print loss
print(f"model loss: {squared_loss(predictions, Y_test)}")
print(f"sklearn model loss: {squared_loss(sklearn_pred, Y_test)}")
print(f"model vs sklearn model: {squared_loss(predictions, sklearn_pred)}")

# Plot the results
import matplotlib.pyplot as plt
plt.scatter(X_test, Y_test, color='blue', label='True values')
plt.scatter(X_test, predictions, color='red', label='Predicted values')
plt.legend()
plt.show()
```

```
model loss: 0.5607269200106376
sklearn model loss: 0.38524973632554715
model vs sklearn model: 0.1464148251085622
```



Main

In this section, we are comparing our model with the sklearn GradientBoostingRegressor. Our goal is to test and compare their results on a public dataset to demonstrate that our model was able to reproduce the results of sklearn. Since the two models should work the same way, we will not add additional explanation of the sklearn model here, to read more about it please follow the link in the reference section.

In [6]:
```
from sklearn.model_selection import train_test_split
from sklearn.ensemble import GradientBoostingRegressor
from sklearn.metrics import mean_squared_error
import os


def squared_loss(predictions, targets):
    """Custom squared loss calculation."""
    return np.mean((predictions - targets) ** 2)
```

```python
def test_boosting_models(dataset, test_size=0.2, random_states=10):
    """
    Compares the performance of our StochasticGradientBoosting with sklearn's GradientBo
    :param dataset: The path to the dataset
    :param test_size: The proportion of the dataset to include in the test split
    :return: None
    """
    # Check if the file exists
    if not os.path.exists(dataset):
        print(f"The file {dataset} does not exist")
        exit()

    # Load in the dataset
    data = np.loadtxt(dataset, skiprows=1)
    X, Y = data[:, 1:], data[:, 0]

    # Normalize the features
    X = (X-np.mean(X, axis=0))/np.std(X, axis=0)

    my_train_losses, my_test_losses = [], []
    sklearn_train_losses, sklearn_test_losses = [], []

    print('Running models on {} dataset'.format(dataset))

    for i in range(random_states):
        print(f"----- Random State {i} -----")
        X_train, X_test, Y_train, Y_test = train_test_split(X, Y, test_size=test_size, r
        #### StochasticGradientBoosting ######
        print("----- StochasticGradientBoosting -----")
        my_model = StochasticGradientBoosting(
            learning_rate=0.1,
            n_estimators=100,
            subsample=0.8,
            min_samples=2,
            max_depth=3
        )

        my_model.train(X_train, Y_train)
        my_train_predictions = my_model.predict(X_train)
        my_test_predictions = my_model.predict(X_test)

        my_train_loss = squared_loss(my_train_predictions, Y_train)
        my_test_loss = squared_loss(my_test_predictions, Y_test)
        print(f"Random State {i}: My Model Train Loss = {my_train_loss:.4f}, Test Loss =


        my_train_losses.append(my_train_loss)
        my_test_losses.append(my_test_loss)

        #### sklearn GradientBoostingRegressor ######
        print("----- sklearn GradientBoostingRegressor -----")
        sklearn_model = GradientBoostingRegressor(
            learning_rate=0.1,
            n_estimators=100,
            subsample=0.8,
            min_samples_split=2,
            max_depth=3,
            random_state=0
        )
        sklearn_model.fit(X_train, Y_train)
        sklearn_train_predictions = sklearn_model.predict(X_train)
```

```
        sklearn_test_predictions = sklearn_model.predict(X_test)

        sklearn_train_loss = squared_loss(sklearn_train_predictions, Y_train)
        sklearn_test_loss = squared_loss(sklearn_test_predictions, Y_test)
        print(f"Random State {i}: Sklearn Model Train Loss = {sklearn_train_loss:.4f}, T

        sklearn_train_losses.append(sklearn_train_loss)
        sklearn_test_losses.append(sklearn_test_loss)

    #### Compare results ######
    print("----- Results Summary -----")
    print("My Model:")
    print(f"Training Loss - Mean: {np.mean(my_train_losses):.4f}, Std: {np.std(my_train_
    print(f"Testing Loss - Mean: {np.mean(my_test_losses):.4f}, Std: {np.std(my_test_los
    print("Sklearn Model:")
    print(f"Training Loss - Mean: {np.mean(sklearn_train_losses):.4f}, Std: {np.std(skle
    print(f"Testing Loss - Mean: {np.mean(sklearn_test_losses):.4f}, Std: {np.std(sklear

    print("----- Comparison -----")
    print(f"Training Loss Difference - Mean: {np.mean(my_train_losses) - np.mean(sklearn
    print(f"Testing Loss Difference - Mean: {np.mean(my_test_losses) - np.mean(sklearn_t
    print(f"Training Loss Difference - Std: {np.std(my_train_losses) - np.std(sklearn_tr
    print(f"Testing Loss Difference - Std: {np.std(my_test_losses) - np.std(sklearn_test


# Set random seeds for reproducibility
np.random.seed(0)

test_boosting_models('wine.txt')
```

```
Running models on wine.txt dataset
----- Random State 0 -----
----- StochasticGradientBoosting -----
Random State 0: My Model Train Loss = 0.3800, Test Loss = 0.5691
----- sklearn GradientBoostingRegressor -----
Random State 0: Sklearn Model Train Loss = 0.3744, Test Loss = 0.5681
----- Random State 1 -----
----- StochasticGradientBoosting -----
Random State 1: My Model Train Loss = 0.3917, Test Loss = 0.4786
----- sklearn GradientBoostingRegressor -----
Random State 1: Sklearn Model Train Loss = 0.3869, Test Loss = 0.4796
----- Random State 2 -----
----- StochasticGradientBoosting -----
Random State 2: My Model Train Loss = 0.3893, Test Loss = 0.4795
----- sklearn GradientBoostingRegressor -----
Random State 2: Sklearn Model Train Loss = 0.3852, Test Loss = 0.4722
----- Random State 3 -----
----- StochasticGradientBoosting -----
Random State 3: My Model Train Loss = 0.3788, Test Loss = 0.5201
----- sklearn GradientBoostingRegressor -----
Random State 3: Sklearn Model Train Loss = 0.3769, Test Loss = 0.5251
----- Random State 4 -----
----- StochasticGradientBoosting -----
Random State 4: My Model Train Loss = 0.3899, Test Loss = 0.4709
----- sklearn GradientBoostingRegressor -----
Random State 4: Sklearn Model Train Loss = 0.3917, Test Loss = 0.4799
----- Random State 5 -----
----- StochasticGradientBoosting -----
Random State 5: My Model Train Loss = 0.3944, Test Loss = 0.4472
----- sklearn GradientBoostingRegressor -----
Random State 5: Sklearn Model Train Loss = 0.3975, Test Loss = 0.4462
----- Random State 6 -----
----- StochasticGradientBoosting -----
Random State 6: My Model Train Loss = 0.3918, Test Loss = 0.4710
----- sklearn GradientBoostingRegressor -----
Random State 6: Sklearn Model Train Loss = 0.3904, Test Loss = 0.4676
----- Random State 7 -----
----- StochasticGradientBoosting -----
Random State 7: My Model Train Loss = 0.3870, Test Loss = 0.4781
----- sklearn GradientBoostingRegressor -----
Random State 7: Sklearn Model Train Loss = 0.3895, Test Loss = 0.4807
----- Random State 8 -----
----- StochasticGradientBoosting -----
Random State 8: My Model Train Loss = 0.3917, Test Loss = 0.4624
----- sklearn GradientBoostingRegressor -----
Random State 8: Sklearn Model Train Loss = 0.3915, Test Loss = 0.4693
----- Random State 9 -----
----- StochasticGradientBoosting -----
Random State 9: My Model Train Loss = 0.3854, Test Loss = 0.5214
----- sklearn GradientBoostingRegressor -----
Random State 9: Sklearn Model Train Loss = 0.3775, Test Loss = 0.5084
----- Results Summary -----
My Model:
Training Loss - Mean: 0.3880, Std: 0.0049
Testing Loss - Mean: 0.4898, Std: 0.0344
Sklearn Model:
Training Loss - Mean: 0.3861, Std: 0.0072
Testing Loss - Mean: 0.4897, Std: 0.0334
----- Comparison -----
Training Loss Difference - Mean: 0.0019
Testing Loss Difference - Mean: 0.0001
```

```
Training Loss Difference - Std: -0.0023
Testing Loss Difference - Std: 0.0011
```

References

1. Friedman, J.H., 2002. Stochastic gradient boosting. Computational statistics & data analysis, 38(4), pp.367-378.
2. Scikit-learn documentation: GradientBoostingRegressor. Available at: https://scikit-learn.org/dev/modules/generated/sklearn.ensemble.GradientBoostingRegressor.html [Accessed 1 Dec. 2024].

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