Ph20 Assignment 3 (Part 1)

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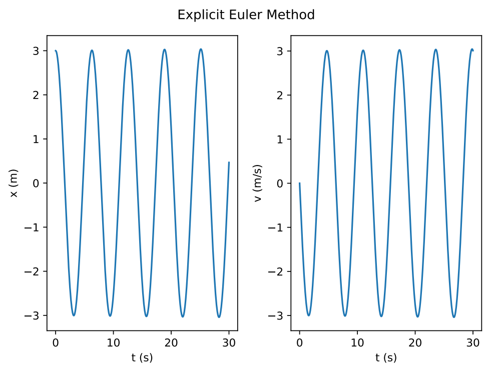
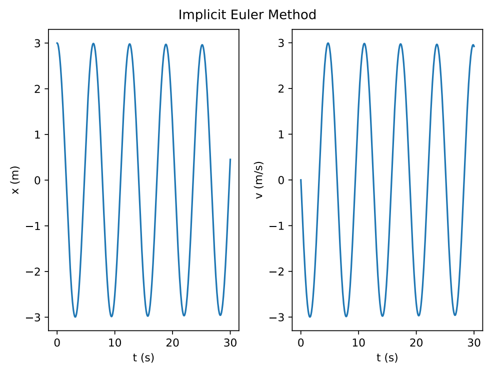
1 Simple Harmonic Oscillator

1.1 Analytic Solution

We let the initial condition be $x_0 = 3$ m and $v_0 = 0$ ms^{-1}. Then for a spring with $\omega^2 = k/m = 1$ Nm^{-1}, the solution for the equation $\ddot{x} = -x$ is $x(t) = 3\cos t$ and $v = \dot{x} = -3\sin t$.



(a) Explicit Euler method.



(b) Implicit Euler method.

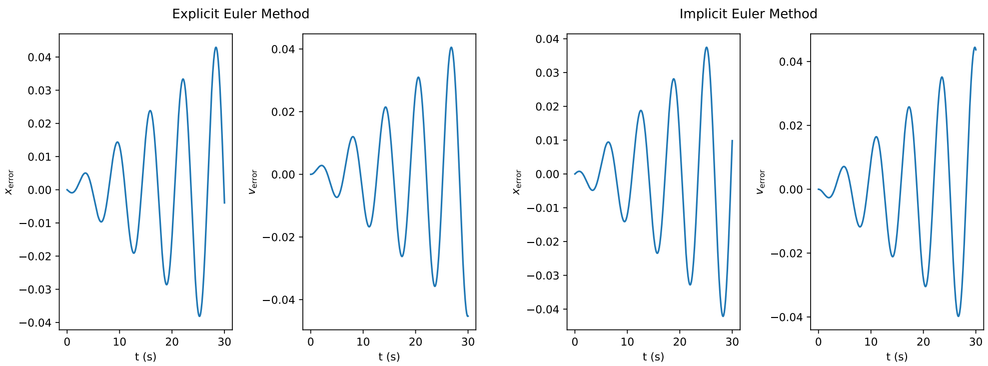
Figure 1: (a) Plots of x and v solved for by the explicit Euler method. Parameters were set with $x_0 = 3, v_0 = 0, h = 0.001, N = 30000$. (b) Plots of x and v solved for by the implicit Euler method. Parameters were set with $x_0 = 3, v_0 = 0, h = 0.001, N = 30000$.

1.2 Global Errors

The global errors x_{error} and v_{error} are given as follows:

$$x_{\text{error}} = x_{\text{analytic}}(t_i) - x_i$$

$$v_{\text{error}} = v_{\text{analytic}}(t_i) - v_i$$



(a) Explicit Euler method

(b) Implicit Euler method.

Figure 2: (a) Plot of the global errors x_{error} and v_{error} against time generated by the explicit Euler method with $h = 0.001$. The amplitudes of the errors increase linearly with time. (b) Plot of the global errors x_{error} and v_{error} against time generated by the implicit Euler method with $h = 0.001$. The amplitudes of the errors also increase linearly with time, but they are in antiphase with the errors for the explicit method.

1.3 Truncation Error

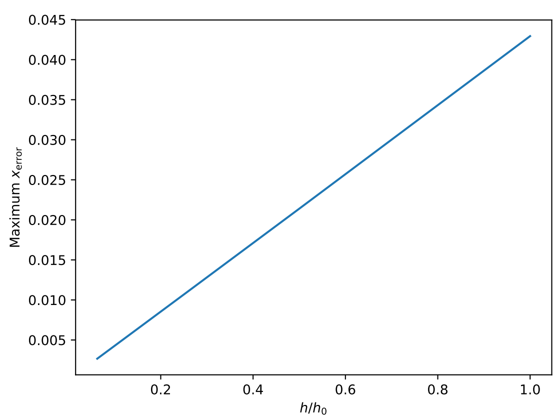
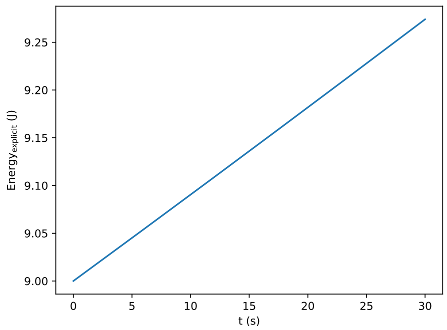
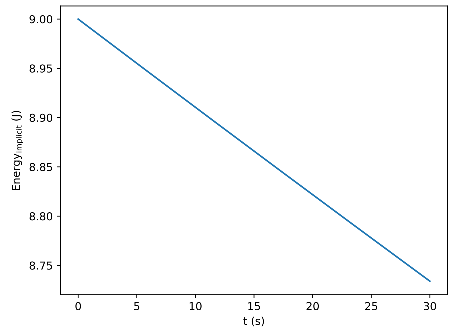


Figure 3: Plot of the maximum value of x_{error} evaluated at $h/h_0 = 1/2, 1/4, 1/8, 1/16$, where $h_0 = 0.001$. The graph shows a linear relationship between the truncation error and h.

1.4 Normalized Total Energy, $E = x^2 + v^2$



(a) Explicit Euler method.



(b) Implicit Euler method.

Figure 4: (a) Plot of the normalized total energy against time generated by the explicit Euler method. The energy increases linearly with time, which follows the same trend as the linearly increasing amplitudes of the global errors. (b) Plot of the normalized total energy against time generated by the implicit Euler method. The energy decreases linearly with time instead.