Ph20 Assignment 1
[CDW: needs correction]

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[CDW: comment] Good plots—legible labels, etc. Thank you. I suggest using pdf instead of png for the plots themselves; that tends to be much clearer when you put them in a pdf.

[CDW: needs correction] You don't use numpy to its full advantage.

1   Lissajous Figures

$$X(t) = A_x \cos(2\pi f_x t) \tag{1}$$
$$Y(t) = A_y \sin(2\pi f_y t + \phi) \tag{2}$$
$$Z(t) = X(t) + Y(t) \tag{3}$$

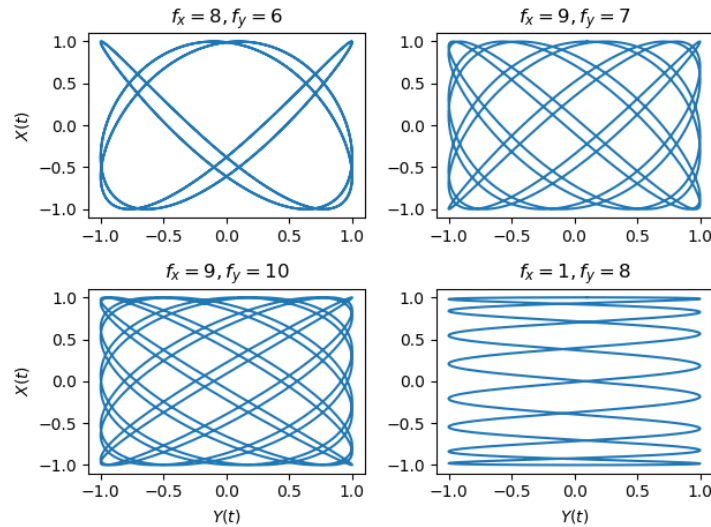If f_x/f_y is a rational number, the graph of $X(t)$ against $Y(t)$ is a closed curve.

Figure 1: Plots of $X(t)$ against $Y(t)$ for rational f_x/f_y. The values of f_x and f_y were selected randomly. Graphs were generated with $A_x = A_y = 1$, $\phi = 0.1$, $\Delta t = 0.001$ and $N = 1000$.

1.1 f_x/f_y and the Shape of the Curve

For $f_x/f_y < 1$, as the ratio increases, the number of points where the curve intersects itself increase. [CDW: needs correction] There's something more specific you can say. What about f_x/f_y an irreducible fraction?
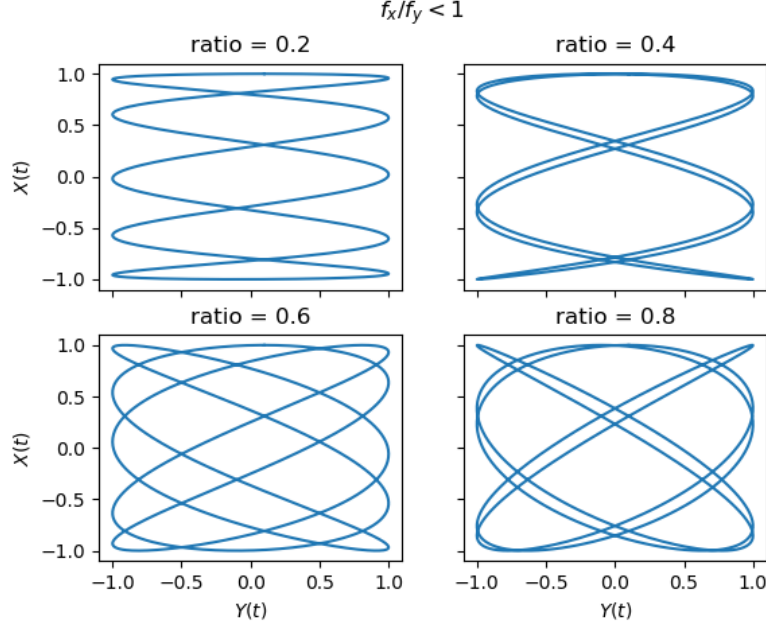

Figure 2: Plots of $X(t)$ against $Y(t)$ for $f_x/f_y < 1$. Graphs were generated with $f_y = 5$, $A_x = A_y = 1$, $\phi = 0.1$, $\Delta t = 0.001$ and $N = 1000$ for $f_x/f_y = 0.2, 0.4, 0.6, 0.8$.

For $f_x/f_y > 1$, the curves resemble overlapping sinusoids with the endpoints connected together. As the ratio increases, the number of peaks and the number of points of intersection increase.
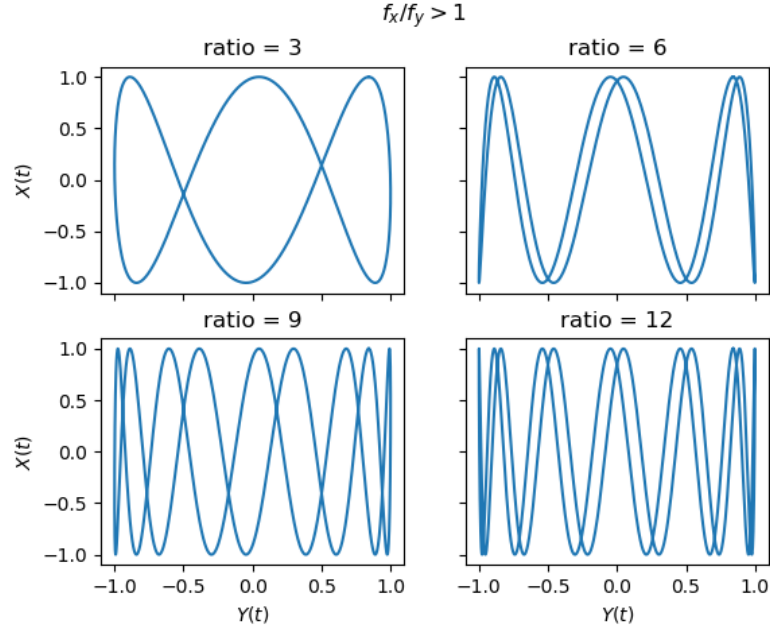
2

Figure 3: Plots of $X(t)$ against $Y(t)$ for $f_x/f_y > 1$. Graphs were generated with $f_y = 5$, $A_x = A_y = 1$, $\phi = 1$, $\Delta t = 0.0001$ and $N = 2000$ for $f_x/f_y = 3, 6, 9, 12$.

[CDW: needs correction] What about irrational f_x/f_y?

1.2 ϕ and the Shape of the Curve

Setting $f_x = f_y$, the shape of the curve was observed while the phase ϕ was varied. The plots trace out ellipses for $n \neq k/2$ where k is odd, and straight lines when $n = k/2$. This is due to the fact that the graphs of sin and cos are shifted by a phase of $\pi/2$.
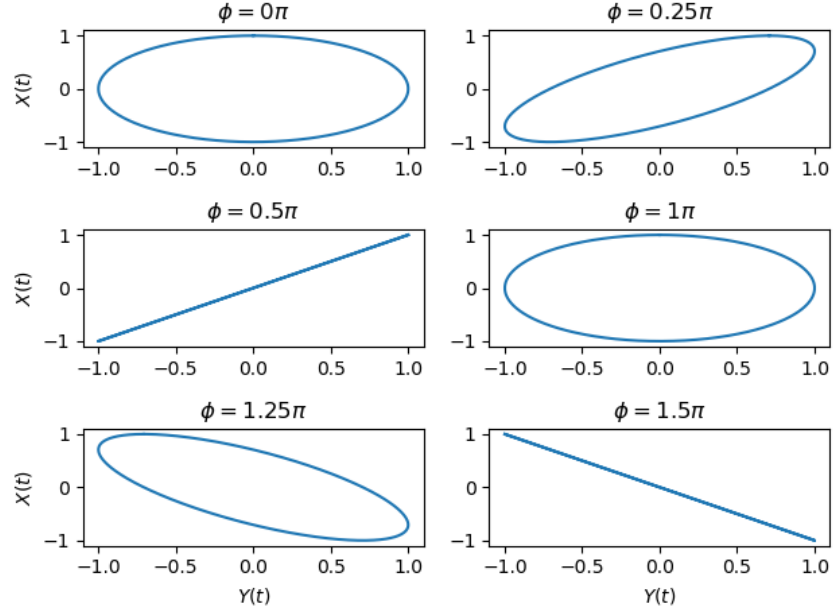
Figure 4: Plots of $X(t)$ against $Y(t)$ for different values of phase ϕ. Graphs were generated with $f_x = f_y = 1$, $A_x = A_y = 1$, $\phi = 1$, $\Delta t = 0.001$ and $N = 1000$ for $\phi = n\pi$ where $n = 0$, $\frac{1}{4}$, $\frac{1}{2}$, 1, $\frac{5}{4}$, $\frac{3}{2}$.

In electronic circuits, if two alternative currents are out of phase by ϕ, plotting the currents against each other and adjusting them until a horizontal ellipse is seen on the oscilloscope means they are in phase or in antiphase with each other.
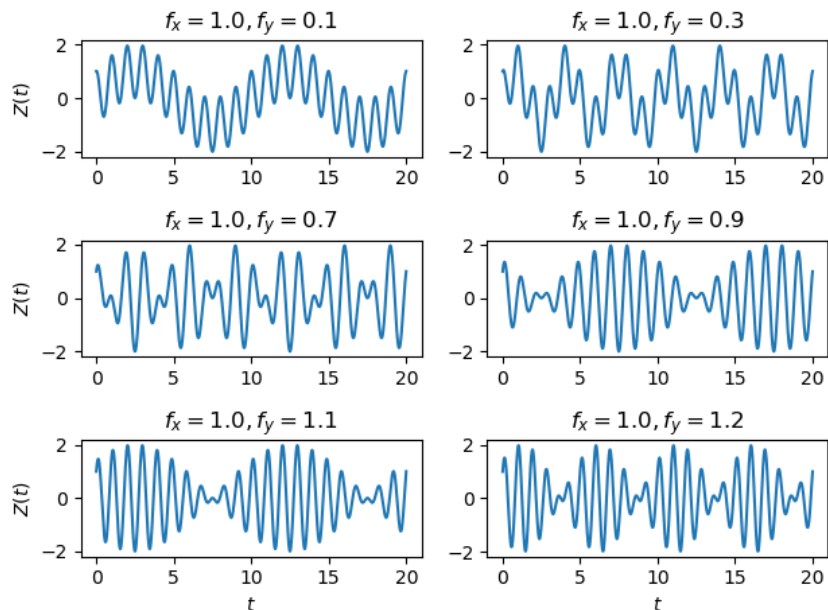
2   Beats

Figure 5: Plots of $Z(t)$ against t. Beats produced by setting similar values for f_x and f_y. Graphs were generated with $A_x = A_y = 1$, $\phi = 0$, $\Delta t = 0.01$ and $N = 2000$ for $f_x = 1$ and $f_y = 0.1$, 0.3, 0.7, 0.9, 1.1, 1.2.[CDW: needs correction] What frequency does one naïvely expect? Did you see it? Why not?

3   Thoughts

The programming was rather fun to do, especially in investigating the properties of the graphs. The assignments were not too difficult.

I havn't really programmed in other languages other than Python, but taking CS2 this term in C++ made me appreciate the simplicity and level of abstraction Python provides for the user. I do agree with Guido Van Rossum - Python is incredibly powerful and is pleasurable work with (no segmentation faults!).