Ph20 Assignment 1

Ung Shu Fay

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1 Lissajous Figures

$$X(t) = A_x \cos(2\pi f_x t) \tag{1}$$
$$Y(t) = A_y \sin(2\pi f_y t + \phi) \tag{2}$$
$$Z(t) = X(t) + Y(t) \tag{3}$$

If f_x/f_y is a rational number, the graph of $X(t)$ against $Y(t)$ is a closed curve.

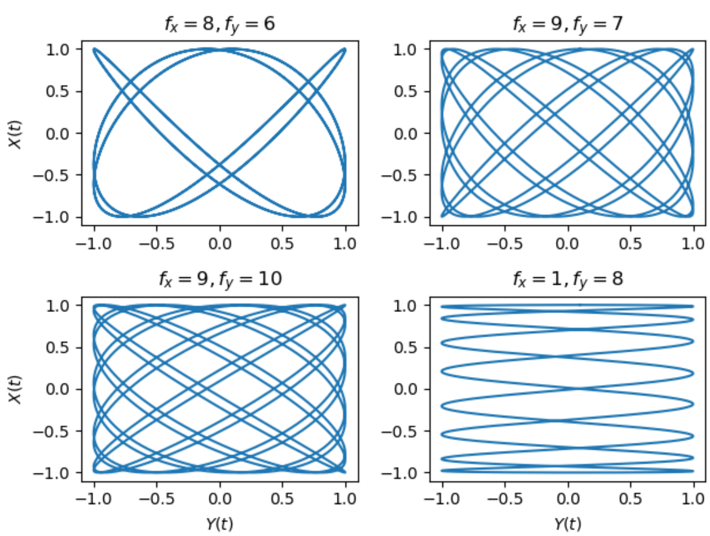

Figure 1: Plots of $X(t)$ against $Y(t)$ for rational f_x/f_y. The values of f_x and f_y were selected randomly. Graphs were generated with $A_x = A_y = 1$, $\phi = 0.1$, $\Delta t = 0.001$ and $N = 1000$.

1.1 f_x/f_y and the Shape of the Curve

For $f_x/f_y < 1$, the ratio of the number of times $X(t)$ achieves an extrema to the number of times $Y(t)$ achieves an extrema is equal to f_x/f_y. This is because in a fixed time interval, where f_x/f_y is an irreducible fraction, $X(t)$ oscillates f_x times while $Y(t)$ oscillates f_y times.

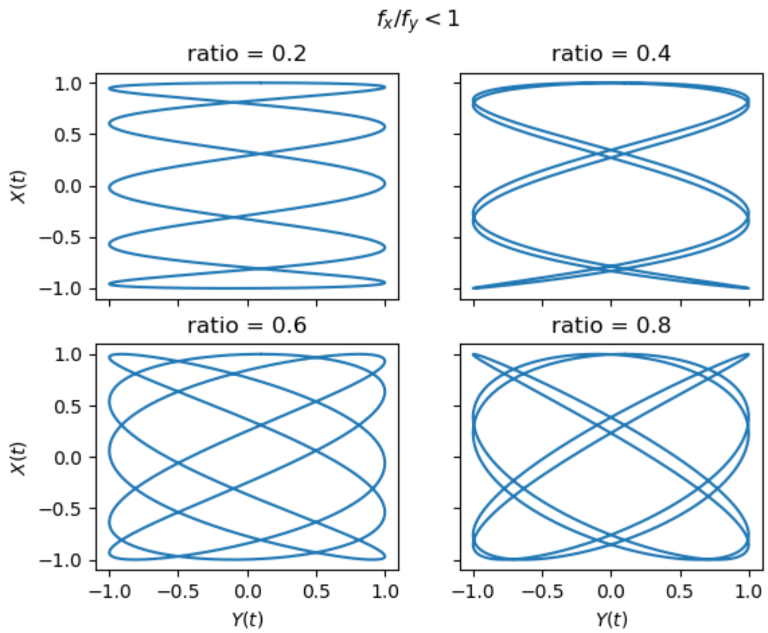

Figure 2: Plots of $X(t)$ against $Y(t)$ for $f_x/f_y < 1$. Graphs were generated with $f_y = 5$, $A_x = A_y = 1$, $\phi = 0.1$, $\Delta t = 0.001$ and $N = 1000$ for $f_x/f_y = 0.2, 0.4, 0.6, 0.8$.

For $f_x/f_y > 1$, the curves resemble overlapping sinusoids with the endpoints connected together. As the ratio increases, the number of peaks and the number of points of intersection increase.

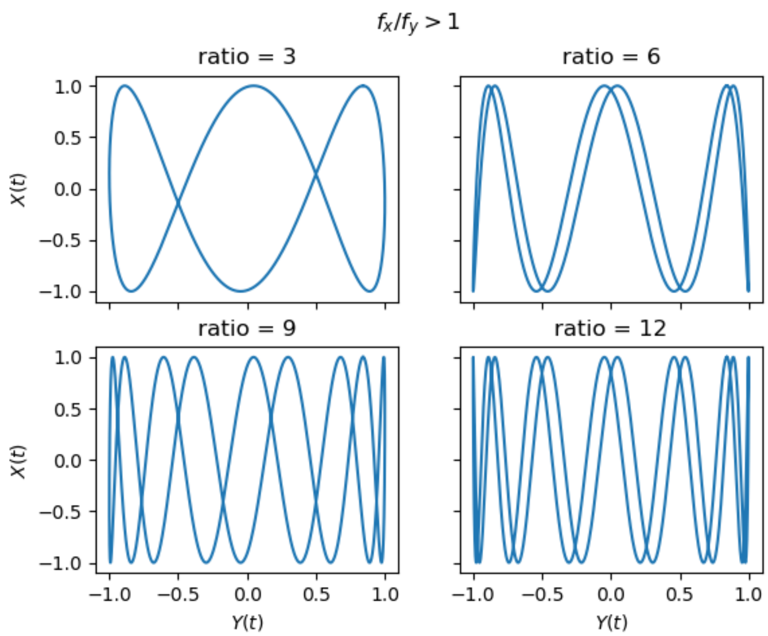

Figure 3: Plots of $X(t)$ against $Y(t)$ for $f_x/f_y > 1$. Graphs were generated with $f_y = 5$, $A_x = A_y = 1$, $\phi = 1$, $\Delta t = 0.0001$ and $N = 2000$ for $f_x/f_y = 3$, 6, 9, 12.

For f_x/f_y irrational, the curves are not closed.

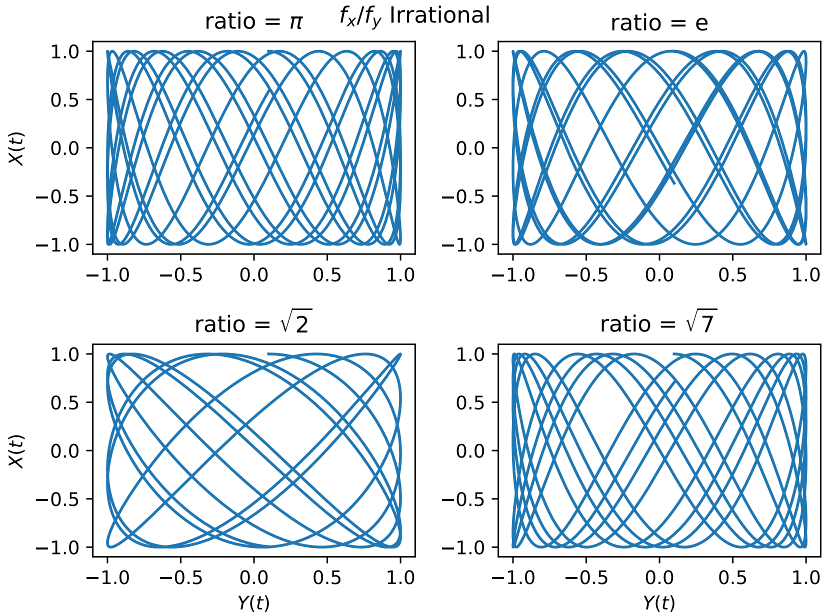

Figure 4: Plots of $X(t)$ against $Y(t)$ for $f_x/f_y = \pi, e, \sqrt{2}, \sqrt{7}$. Graphs were generated with $f_y = 2$, $A_x = A_y = 1$, $\phi = 0.1$, $\Delta t = 0.001$ and $N = 3000$.

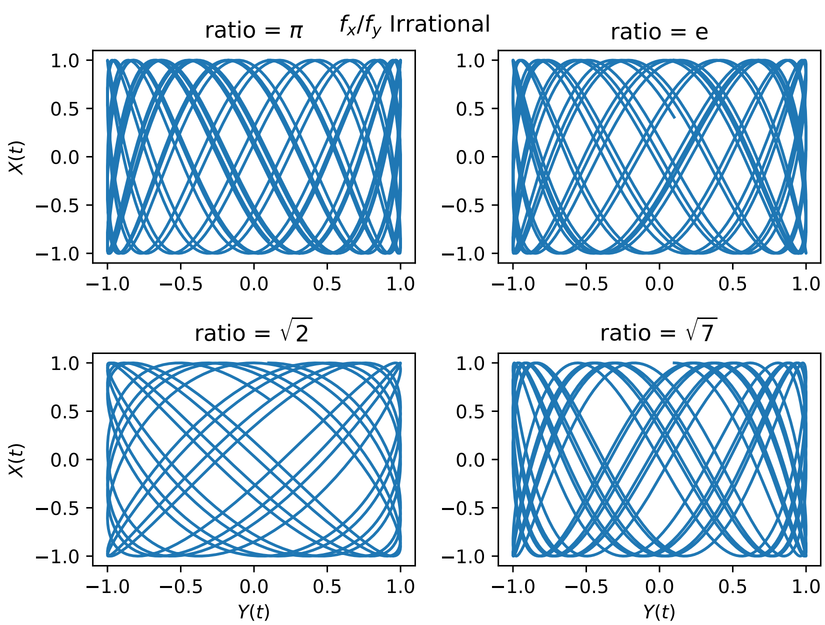

Figure 5: Same as Figure 4, but with $N = 5000$.

1.2 ϕ and the Shape of the Curve

Setting $f_x = f_y$, the shape of the curve was observed while the phase ϕ was varied. The plots trace out ellipses for $n \neq k/2$ where k is odd, and straight lines when $n = k/2$. This is due to the fact that the graphs of sin and cos are shifted by a phase of $\pi/2$.

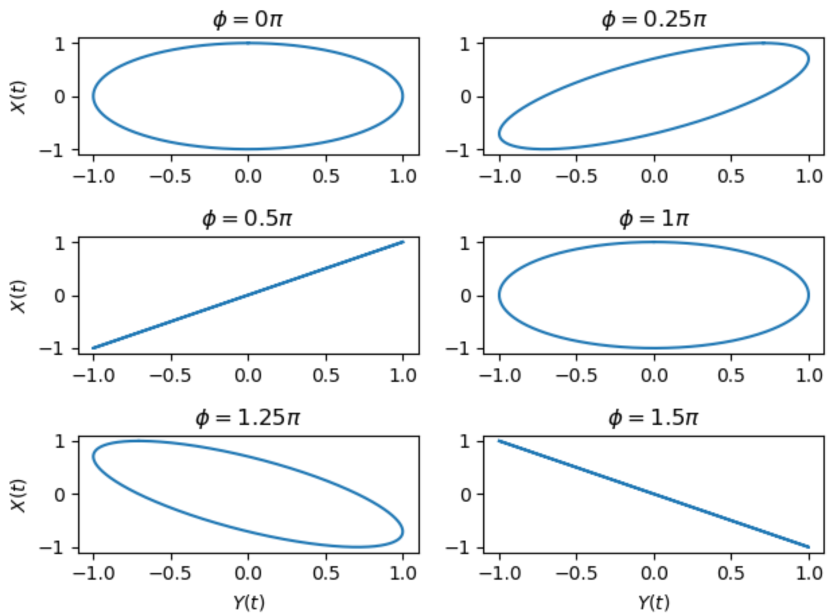

Figure 6: Plots of $X(t)$ against $Y(t)$ for different values of phase ϕ. Graphs were generated with $f_x = f_y = 1$, $A_x = A_y = 1$, $\phi = 1$, $\Delta t = 0.001$ and $N = 1000$ for $\phi = n\pi$ where $n = 0,\ \frac{1}{4},\ \frac{1}{2},\ 1,\ \frac{5}{4},\ \frac{3}{2}$.

In electronic circuits, if two alternative currents are out of phase by ϕ, plotting the currents against each other and adjusting them until a horizontal ellipse is seen on the oscilloscope means they are in phase or in antiphase with each other.

2 Beats

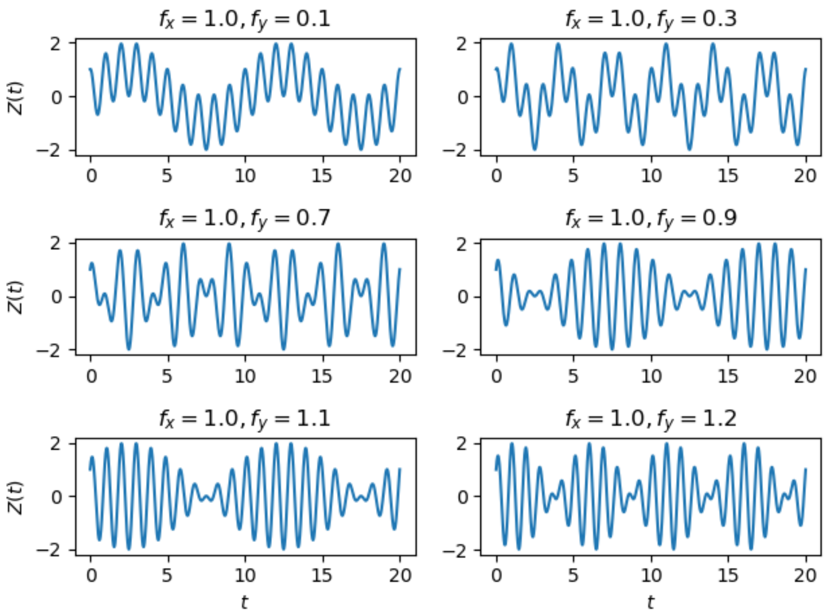

Figure 7: Plots of $Z(t)$ against t. Beats produced by setting similar values for f_x and f_y. Graphs were generated with $A_x = A_y = 1$, $\phi = 0$, $\Delta t = 0.01$ and $N = 2000$ for $f_x = 1$ and $f_y = 0.1$, 0.3, 0.7, 0.9, 1.1, 1.2. The modulation frequency seen in the beats is $w_1 - w_2$, not $(w_1 - w_2)/2$. This is because $(w_1 - w_2)/2$ is the frequency at which the sinusoid for the amplitude varies, but a beat occurs whenever the amplitude is a maximum or a minimum, hence the frequency of the beats is twice that of the frequency of the amplitude modulation.

3 Thoughts

The programming was rather fun to do, especially in investigating the properties of the graphs. The assignments were not too difficult.

I havn't really programmed in other languages other than Python, but taking CS2 this term in C++ made me appreciate the simplicity and level of abstraction Python provides for the user. I do agree with Guido Van Rossum - Python is incredibly powerful and is pleasurable work with (no segmentation faults!).