

Ph20 Assignment 3 (Part 2)

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1 Phase Space Trajectories

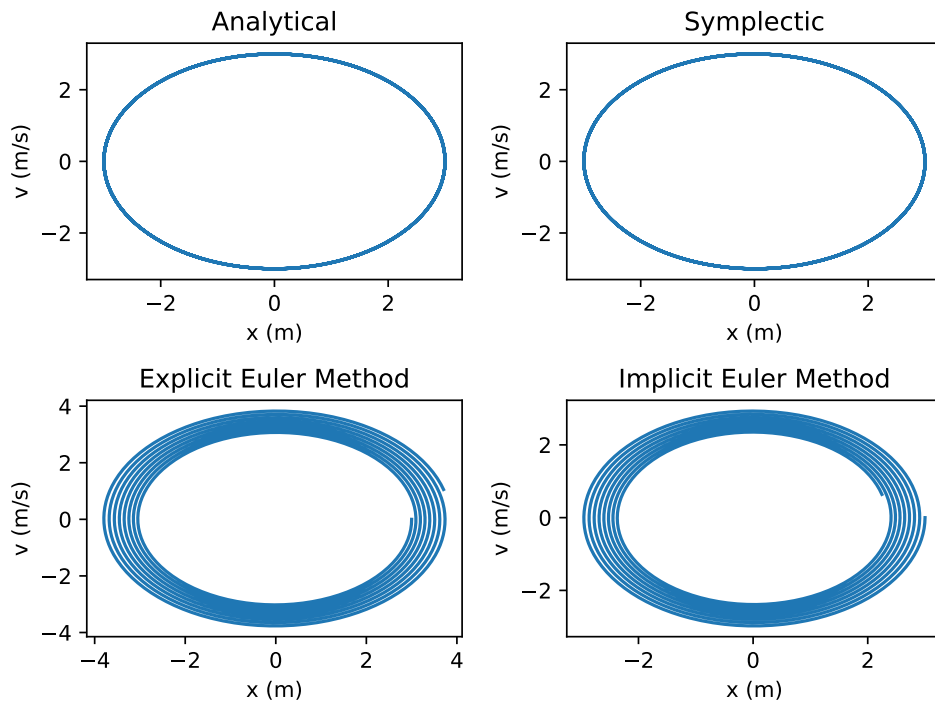


Figure 1: Phase space trajectories of solutions for the spring given by the analytical, symplectic, explicit and implicit Euler's method. The analytical and symplectic solutions trace out a circle in phase space while the Euler's methods do not trace out closed curves. The parameters were: $x_0 = 3, v_0 = 3, h = 0.01, N = 5000$.

The circular trajectory produced by the symplectic Euler method shows that its solution replicates the behaviour of the analytical solution in phase space, unlike the explicit and implicit Euler methods.

2 Energy

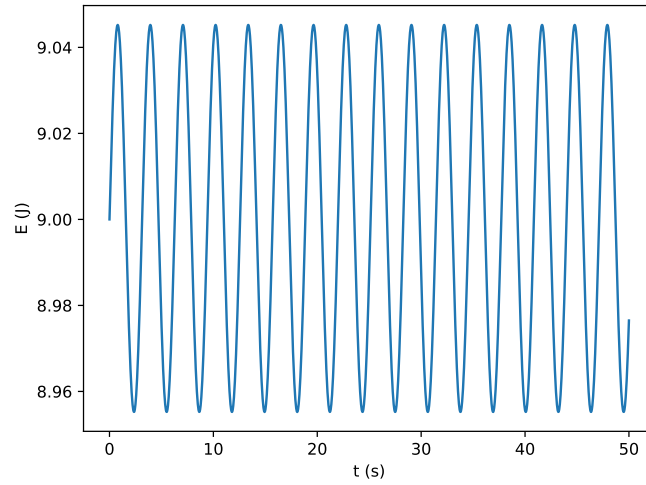


Figure 2: Evolution of the total energy obtained with the symplectic Euler method. The total energy as a function of time is sinusoidal. The average total energy $E = 9$ J is constant and produces the circle $x^2 + v^2 = E$ traced out in phase space. The plot was generated with $h = 0.01$.

3 Global Error

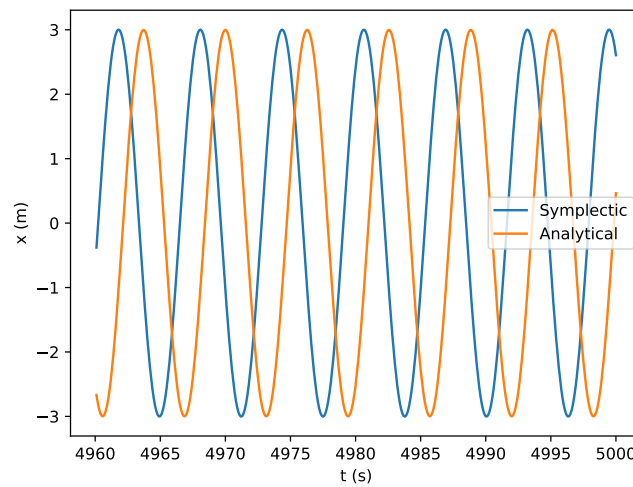


Figure 3: Global error in phase produced by the symplectic Euler method. The plot was produced for the last 6 oscillations of 750 oscillations with $h = 0.1$ and $N = 50\,000$.