# Ph20 Assignment 3 (Part 2)
## [CDW: comment] PASS

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[CDW: comment] You've put a whole bunch of compilation products (e.g. the pdf of this report, but also some files minted/pygments created) under version control. This has upsides and downsides: it makes it easy for a collaborator to see results without having to pull and recompile, but it also makes diffs very hard to understand.

1 Phase Space Trajectories
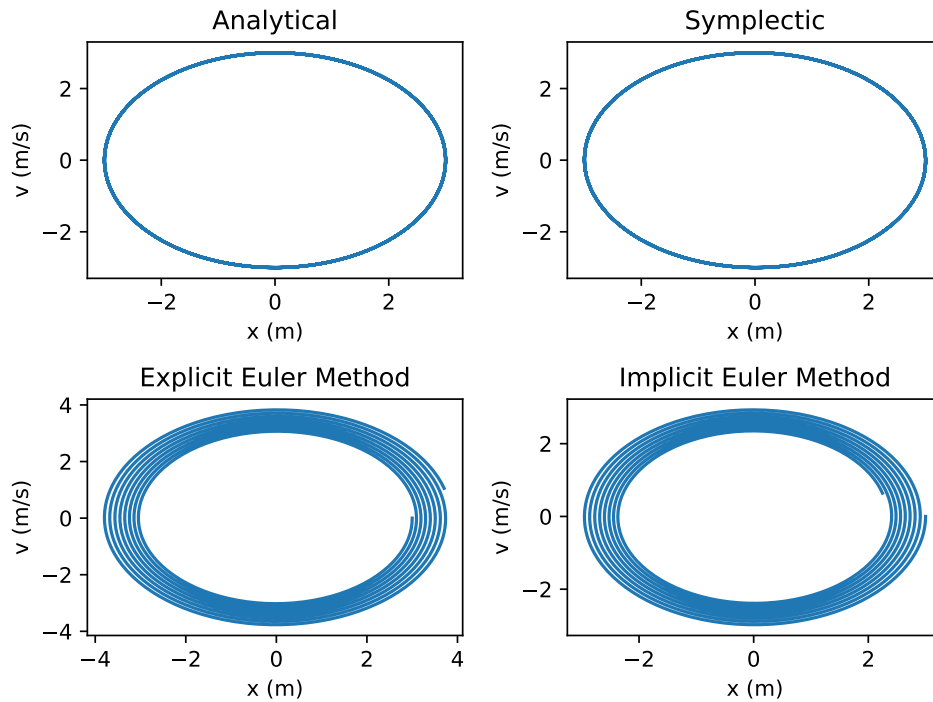
Phase space trajectories of solutions for the spring given by the analytical, symplectic, explicit and implicit Euler s method. The analytical and symplectic solutions trace out a circle in phase space while the Euler s methods do not trace out closed curves. The parameters were:
$x_0 = 3, v_0 = 3, h = 0.01, N = 5000$.

2   Energy
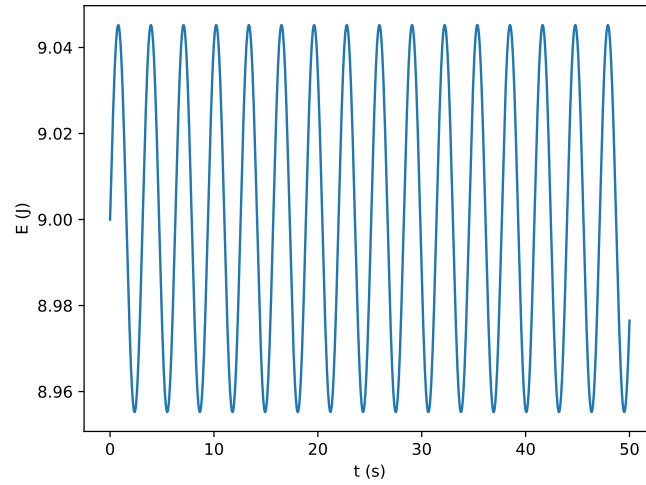
Evolution of the total energy obtained with the symplectic Euler method. The total energy as a function of time is sinusoidal. The average total energy $E = 9\,\mathrm{J}$ is constant and produces the circle $x^2 + v^2 = E$ traced out in phase space. The plot was generated with $h = 0.01$.

3   Global Error
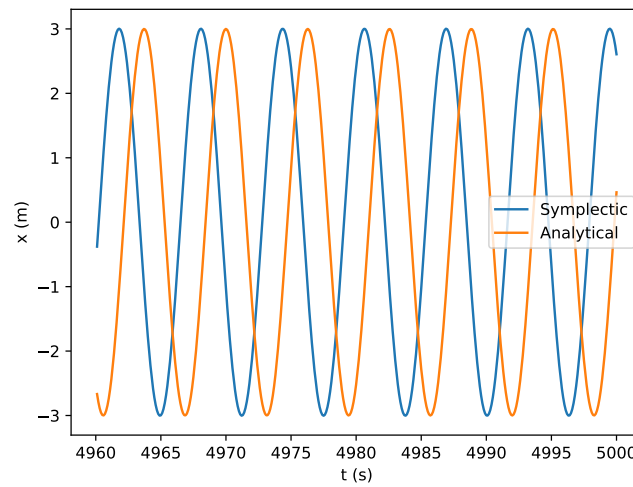
Global error in phase produced by the symplectic Euler method. The plot was produced for the last 6 oscillations of 750 oscillations with $h = 0.1$ and $N = 50000$.

4   Makefile

```makefile
# Generate plots
# phase space

SRC = phase.py
TEX = main.tex

DO_SRC = python $(SRC)
COMPILE = pdflatex -shell-escape $(TEX)

TXT = $(wildcard *.txt)
ARGS = phase_space symplectic_spring energy error


.PHONY : all
all : $(SRC) $(TEX) $(TXT)
        git log > git.txt

        for arg in $(ARGS) ; do \
                $(DO_SRC) $$arg > $$arg.pdf ; \
        done

        $(COMPILE)

.PHONY : clean
clean :
        rm -rf *.pdf *.aux *.log git.txt _minted-main main.mintedcmd
```

5   Source Code

```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.backends.backend_pdf import PdfPages
import sys

def x_anal(t):
        return 3 * np.cos(t)

def v_anal(t):
        return -3 * np.sin(t)

def spring_im(x0, v0, h, N):
        t = np.linspace(0, h*N, N)
        x = np.zeros(N)
        v = np.zeros(N)
        x[0] = x0
        v[0] = v0

        for i in range(1, N):
```

```python
            x[i] = (x0 + h * v0) / (1 + h**2)
            v[i] = (v0 - h * x0) / (1 + h**2)
            x0 = x[i]
            v0 = v[i]

        return t, x, v

def spring_ex(x0, v0, h, N):
        t = np.linspace(0, h*N, N)
        x = np.zeros(N)
        v = np.zeros(N)
        x[0] = x0
        v[0] = v0

        for i in range(1, N):
                x[i] = x0 + h * v0
                v[i] = v0 - h * x0
                x0 = x[i]
                v0 = v[i]

        return t, x, v

def symplectic(x0, v0, h, N):
    t = np.linspace(0, h*N, N)
    x = np.zeros(N)
    v = np.zeros(N)
    x[0] = x0
    v[0] = v0

    for i in range(1, N):
        x[i] = x0 + h * v0
        v[i] = v0 - h * x[i]
        x0 = x[i]
        v0 = v[i]

    return t, x, v

def energy_sym(x0, v0, h0, N):
    sym = symplectic(x0, v0, h0, N)
    return sym[0], np.square(sym[1]) + np.square(sym[2])

def plot_phase_space(x0, v0, h0, N):
    with PdfPages('phase_space.pdf') as pdf:
        # Spring (explicit)
        spring_x = spring_ex(x0, v0, h0, N)

        # Spring (implicit)
        spring_m = spring_im(x0, v0, h0, N)

        # Spring (symplectic)
        spring_sym = symplectic(x0, v0, h0, N)
```

```python
        # Spring (analytic)
        spring_anx = x_anal(spring_x[0])
        spring_anv = v_anal(spring_x[0])

        fig, ((an, sym), (ex, im)) = plt.subplots(2, 2)

        # Spring (analytical)
        an.plot(spring_anx, spring_anv)
        an.set_xlabel('x (m)')
        an.set_ylabel('v (m/s)')
        an.set_title('Analytical')

        # Spring (symplectic)
        sym.plot(spring_sym[1], spring_sym[2])
        sym.set_xlabel('x (m)')
        sym.set_ylabel('v (m/s)')
        sym.set_title('Symplectic')

        # Spring (explicit)
        ex.plot(spring_x[1], spring_x[2])
        ex.set_xlabel('x (m)')
        ex.set_ylabel('v (m/s)')
        ex.set_title('Explicit Euler Method')

        # Spring (implicit)
        im.plot(spring_m[1], spring_m[2])
        im.set_xlabel('x (m)')
        im.set_ylabel('v (m/s)')
        im.set_title('Implicit Euler Method')

        plt.tight_layout()
        pdf.savefig()

def plot_sym_spring(x0, v0, h0, N):
    with PdfPages('symplectic_spring.pdf') as pdf:
        spring_sym = symplectic(x0, v0, h0, N)
        fig2, (sym_x, sym_v) = plt.subplots(1, 2)

        sym_x.plot(spring_sym[0], spring_sym[1])
        sym_x.set_xlabel('t (s)')
        sym_x.set_ylabel('x (m)')

        sym_v.plot(spring_sym[0], spring_sym[2])
        sym_v.set_xlabel('t (s)')
        sym_v.set_ylabel('v (m/s)')

        plt.tight_layout()
        pdf.savefig()

def plot_e_sym(x0, v0, h0, N):
```

```python
    with PdfPages('energy.pdf') as pdf:
        energy = energy_sym(x0, v0, h0, N)
        plt.figure()
        plt.plot(energy[0], energy[1])
        plt.xlabel('t (s)')
        plt.ylabel('E (J)')
        pdf.savefig()

def plot_error(x0, v0, h0, N):
    with PdfPages('error.pdf') as pdf:
        error_sym = symplectic(x0, v0, h0, N)
        error_x = x_anal(error_sym[0])

        plt.figure()
        err_sym, = plt.plot(error_sym[0][N-400:], error_sym[1][N-400:], label='Symplectic')
        err_an, = plt.plot(error_sym[0][N-400:], error_x[N-400:], label='Analytical')
        plt.legend(handles=[err_sym, err_an])
        plt.xlabel('t (s)')
        plt.ylabel('x (m)')

        pdf.savefig()

# Constants
x0 = 3
v0 = 0
h0 = 0.01
N = 5000

if len(sys.argv) != 2:
    usage = 'python phase.py <command>'
    print(usage)

elif sys.argv[1] == 'phase_space':
    plot_phase_space(x0, v0, h0, N)

elif sys.argv[1] == 'symplectic_spring':
    plot_sym_spring(x0, v0, h0, N)

elif sys.argv[1] == 'energy':
    plot_e_sym(x0, v0, h0, N)

elif sys.argv[1] == 'error':
    h0 = 0.1
    N = 50000
    plot_error(x0, v0, h0, N)
```

6   Git Log

```
commit 5311a33f38a4709b2f3a7d55950628c7fcb8170f
Author: Ung Shu Fay <shufay.ung@gmail.com>
```

```
Date:    Fri Feb 23 18:19:20 2018 -0800

    make

commit db150e39384418afe6c5976437d2a7d6bc906920
Author: Ung Shu Fay <shufay.ung@gmail.com>
Date:    Fri Feb 23 18:17:43 2018 -0800

    Makefile

commit 6467098c9dd5bcf38034809343a28c80109d8369
Author: Ung Shu Fay <shufay.ung@gmail.com>
Date:    Fri Feb 23 01:11:16 2018 -0800

    added tex file

commit 043b1d9d2f43e5e4c27d93f9232d992a5381909a
Author: Ung Shu Fay <shufay.ung@gmail.com>
Date:    Wed Feb 14 13:57:09 2018 -0800

    changed

commit aba8b22205279e4f97598f6840a125936f77a214
Author: Ung Shu Fay <shufay.ung@gmail.com>
Date:    Wed Feb 14 13:55:48 2018 -0800

    added assignment2

commit 01f16c69a65a4900a4576e2c1b6bc7b484d8404f
Author: Ung Shu Fay <shufay.ung@gmail.com>
Date:    Wed Feb 14 13:55:24 2018 -0800

    added file to assignment1

commit 2f206c8195430cc17dab1c4c50c0f97fef091d29
Author: Ung Shu Fay <shufay.ung@gmail.com>
Date:    Wed Feb 14 13:49:57 2018 -0800

    changed constants

commit 726bcc7a0ef4e3ecf46644987916de4e4ee1ddd8
Merge: 391cfbe e40bf51
Author: Ung Shu Fay <shufay.ung@gmail.com>
Date:    Wed Feb 14 13:46:27 2018 -0800

    Merge branch 'master' of https://github.com/shufay/Ph20

commit e40bf51a453e70fb72899b1098109ccedc8c58c6
Author: shufay <shufay@yahoo.com>
Date:    Wed Feb 14 13:43:42 2018 -0800
```

Initial commit

commit 391cfbef13e6d575849d025c34e5375b2a39f112
Author: Ung Shu Fay <shufay.ung@gmail.com>
Date: Wed Feb 14 13:43:41 2018 -0800

        added file

commit 92adfea2d8c961a0b61a305b8fb5f13358a52a11
Author: Ung Shu Fay <shufay.ung@gmail.com>
Date: Thu Jan 18 01:24:02 2018 -0800

        add files