Ph21 Assignment 4

Ung Shu Fay

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1 Part I

1.1 Uniform Prior

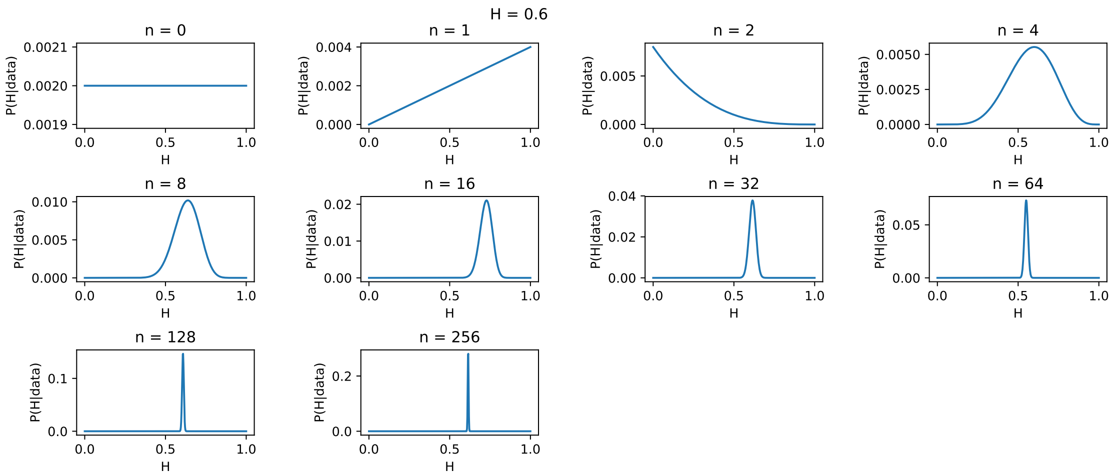

Figure 1: Posterior distributions for $n = 0,\ 1,\ 2,\ 4, ..., 256$ with a true H value of 0.60.

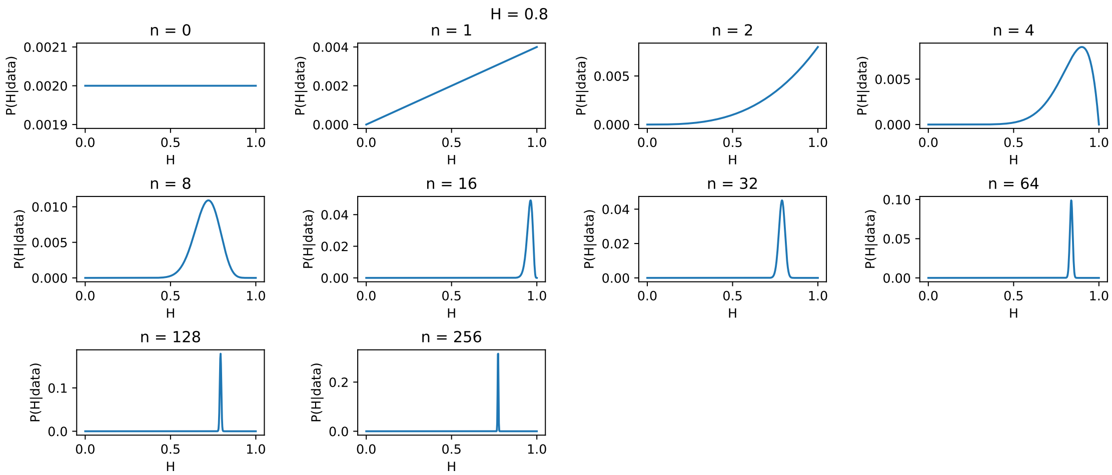

Figure 2: Posterior distributions for $n = 0,\ 1,\ 2,\ 4, ..., 256$ with a true H value of 0.80.

When $n = 0$, the posterior PDF is equal to our prior PDF since we do not have any data to update our prior. With $n = 1$ we obtain a head, so the posterior PDF rises linearly and is greatest at $H = 1$ and zero at $H = 0$ - we do not yet know if the coin has a tail. At $n = 2$, Figure 1 shows the distribution when a tail is obtained on the second flip. The PDF at $H = 1$ falls to zero since we now know the coin has a tail. Figure 2 shows the distribution when a heads is obtained on the second flip. The PDF becomes more peaked towards $H = 1$ since there is more evidence that the coin only has a head. With increasing n, the PDF eventually peaks at the true value of H with decreasing uncertainty.

1.2 Gaussian Prior

The Gaussian distribution used has $\mu = 0.50$ and $\sigma = 0.25$.

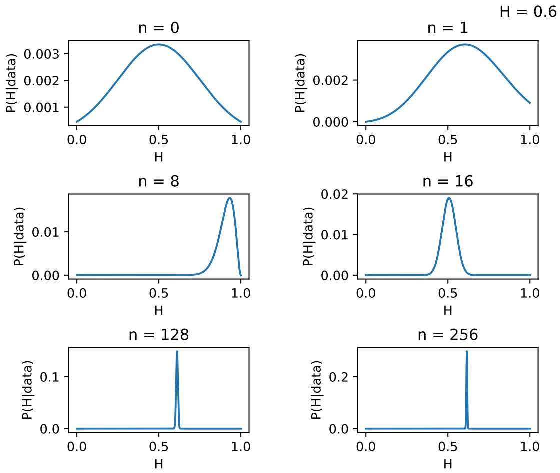

Figure 3: Posterior distributions for $n = 0,\ 1,\ 2,\ 4, ..., 256$ with a true H value of 0.60 (within 1σ of μ).

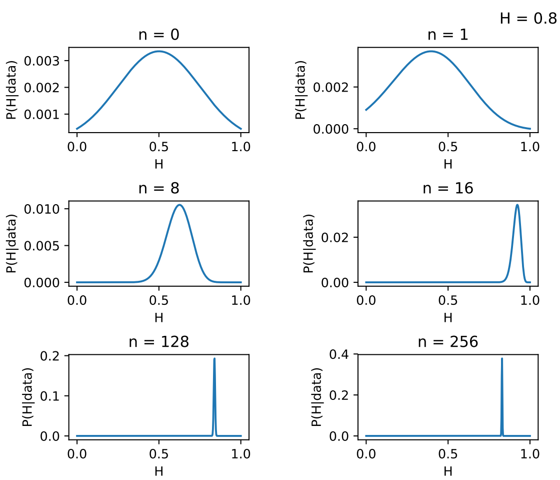

Figure 4: Posterior distributions for $n = 0,\ 1,\ 2,\ 4, ..., 256$ with a true H value of 0.80 (within 3σ of μ).

When $n = 0$, the posterior PDF is equal to our prior PDF since we do not have any data to update our prior. With $n = 1$ Figure 3 shows the distribution when a head is obtained. The peak of the Gaussian shifts towards the right since for all we know the coin could have a high H value. Figure 4 shows the distribution when a tail is obtained, with the peak shifting to the left. With increasing n, the PDF eventually peaks at the true value of H with decreasing uncertainty.

For large n, the contributions from the initially chosen priors become negligible and the posterior PDFs become more similar, i.e. they depend less on the chosen priors.

2 Part II

2.1 Unknown α, Known β

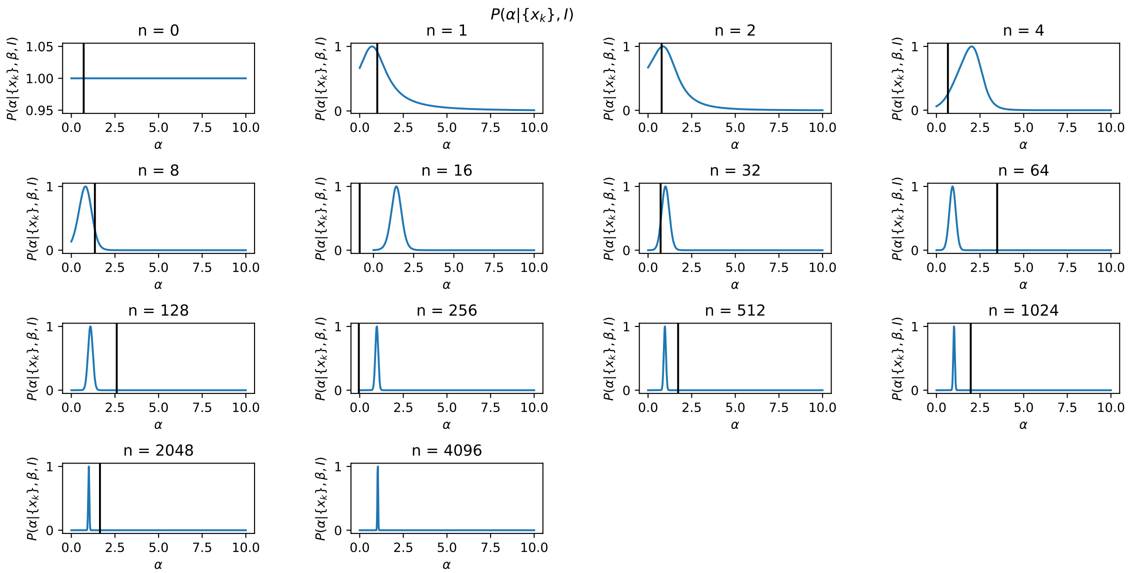

Figure 5: Posterior distributions for $n = 0,\ 1,\ 2,\ 4, ..., 4096$ with the true $\alpha = 1$ km and true $\beta = 1$ km. The mean value of x_k for this sample was 1.1372819372938014 km.

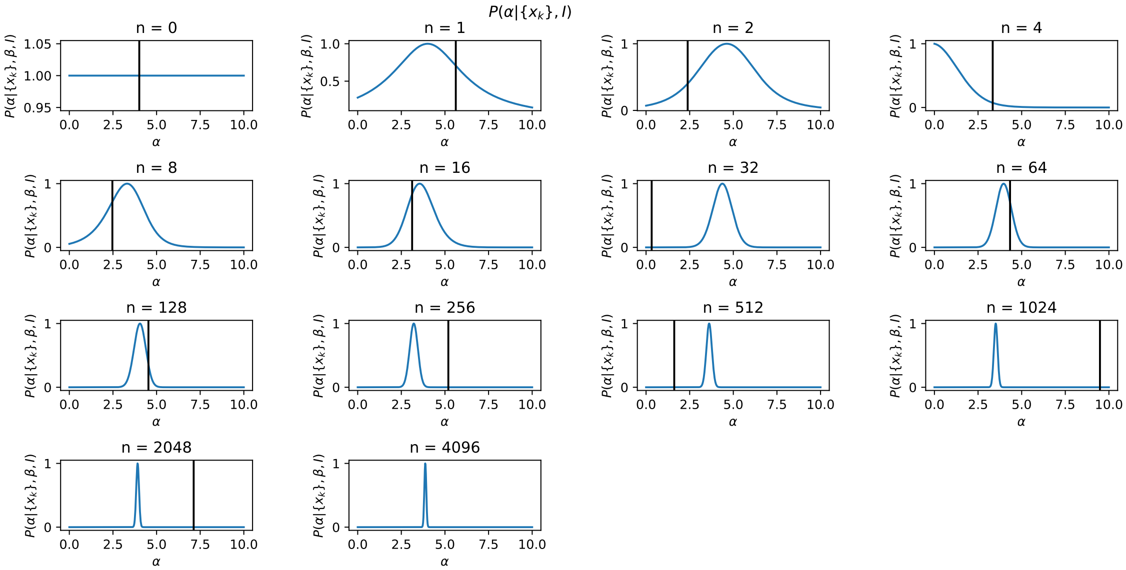

Figure 6: Posterior distributions for $n = 0,\ 1,\ 2,\ 4, ..., 4096$ with the true $\alpha = 3.8$ km and true $\beta = 2.5$ km. The black line shows mean value of x_k for each n.

The mean x_k is not a good estimator for the most probable values of α since it often lies outside the range of values deemed most probable by the posterior PDF.

2.2 Unknown α, Unknown β

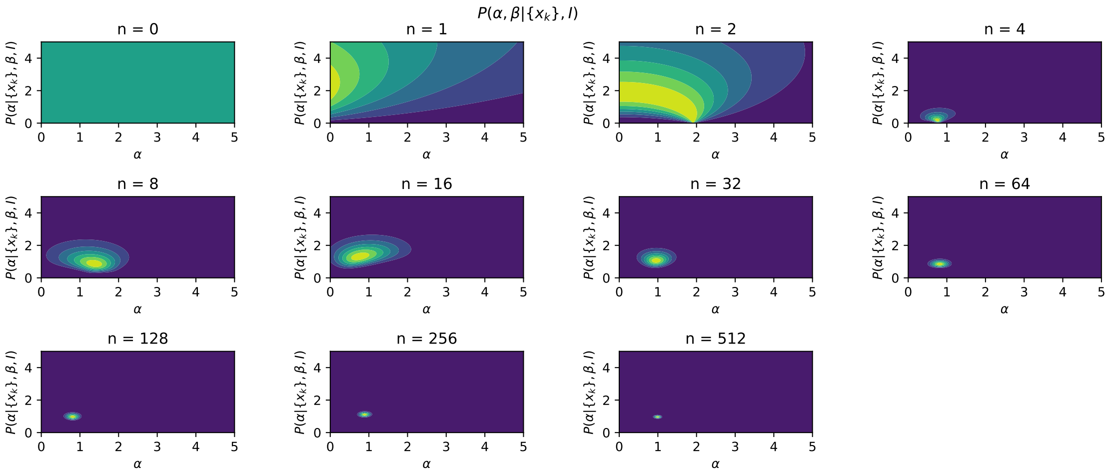

Figure 7: Posterior distributions for $n = 0,\ 1,\ 2,\ 4, ..., 512$ with the true $\alpha = 1$ km and true $\beta = 1$ km.

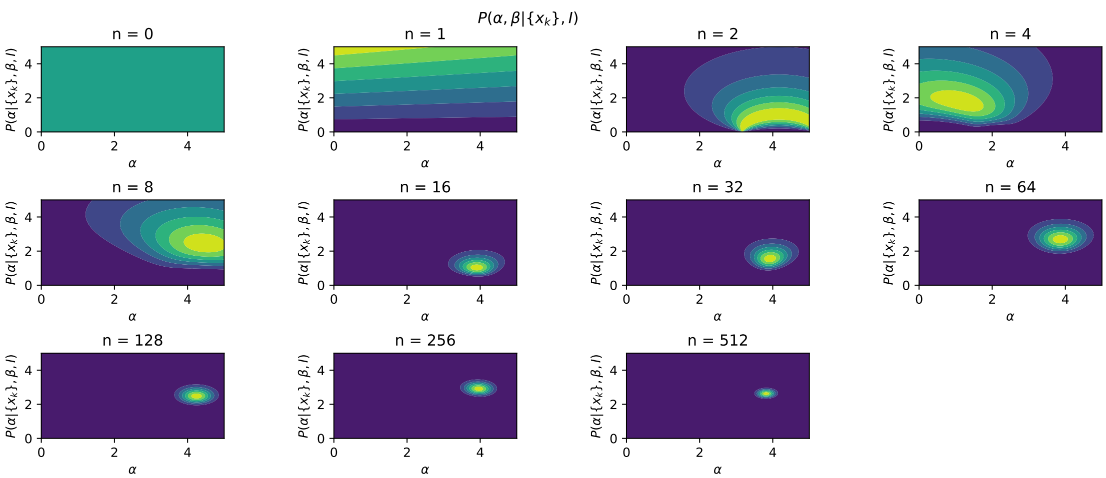

Figure 8: Posterior distributions for $n = 0,\ 1,\ 2,\ 4, ..., 512$ with the true $\alpha = 3.8$ km and true $\beta = 2.5$ km.