Homework 5

2.1 Suppose a parent and child play the following game, first analyzed by Becker (1974). First, the child takes an action, A, that produces income for the child, $I_C(A)$, and income for the parent $I_P(A)$. (Think of $I_C(A)$ as the child$'$ income net of any costs of the action A.) Second, the parent observes the incomes I_C and I_P and then chooses a bequest, B, to leave to the child. The child$'$ payoff is $U(I_C + B)$; the parent$'$s is $V(I_P - B) + kU(I_C + B)$, where $k > 0$ reflects the parent$'$s concern for the child$'$s well-being. Assume that: the action is a nonnegtive number, $A \geq 0$; the income functions $I_C(A)$ and $I_P(A)$ are strictly concave and are maximized at $A_C > 0$ and $A_P > 0$, respectively; the bequest B can be positive or negative; and the utility functions U and V are increasing and strictly concave. Prove the "Rotten Kid" Theorem: in the bacwards induction outcome, the child chooses the action that maximizes the family$'$ aggregate income, $I_C(A) + I_P(A)$, even though only the parent$'$s payoff exhibits altruism.

2.2 Now suppose the parent and child play a different game, first analyzed by Buchanan (1975). Let the incomes I_C and I_P be fixed exogenously. First, child decides how much of the income I_C to save (S) for the future, consuming the rest ($I_C - S$) today. Second, the parent observes the child$'$s choice of S and chooses a bequest, B. The child$'$s payoff is the sum of current and future utilities: $U_1(I_C - S) + U_2(S + B)$. The parent$'$s payoff is $V(I_P - B) + k[U_1(I_C - S) + U_2(S + B)]$. Assume that the utility functions U_1, U_2, and V are increasing and strictly concave. Show that there is a "Samaritan$'$s Dilemma": in the backwards-induciton outcome, the child saves too little, so as to induce the parent to leave a larger bequest (i.e., both the parent$'$s and child$'$s payoffs could be increased if S were suitably larger and B suitably smaller).