To Save Crowdsourcing from Cheap-talk: Strategic Learning from Biased Users

Shugang Hao
Pillar of Engineering Systems and Design
Singapore University of Technology and Design
Singapore
shugang_hao@sutd.edu.sg

Lingjie Duan
Pillar of Engineering Systems and Design
Singapore University of Technology and Design
Singapore
lingjie_duan@sutd.edu.sg

***Abstract*—Today many users are invited by a crowdsourcing platform (e.g., TripAdvisor, and Waze) to provide their anonymous reviews about service experiences (e.g., in hotels, restaurants, and trips), yet many reviews are found biased to be extremely positive or negative. It is difficult to learn from biased users' reviews to infer actual service state, as the service state can also be extreme and the platform cannot verify immediately. Further, due to the anonymity, reviewers can hide their bias types (positive or negative) from the platform and adaptively adjust their reviews against the platform's inference. To our best knowledge, we are the first to study how to save crowdsourcing from cheap-talk and strategically learn the actual service state from biased users' reviews. We formulate the problem as a dynamic Bayesian game, including the unknown users' messaging and the platform's follow-up inference. Through involved analysis, we provide closed-form expressions for the Perfect Bayesian Equilibrium (PBE). Our PBE shows that the platform's strategic learning can successfully prevent biased users from cheap-talk in most cases, where a user even with extreme bias still honestly messages to convince the platform of listening to his review. We prove that the price of anarchy (PoA) is 2, telling that the social cost can at most be doubled in the worst-case. As the user number becomes large, our platform always learns the actual service state. Perhaps surprisingly, the platform's expected cost may be worse off after adding one more user.**

***Index Terms*—crowdsourcing, cheap-talk, dynamic Bayesian game, strategic learning**

I. Introduction

Today platforms (e.g., TripAdvisor and Waze) invite many users to submit anonymous reviews to rate their experienced services (e.g., of hotels, restaurants and trips), yet many reviews are found biased to be extremely positive or negative. For example, the latest investigation shows that anonymous users having no other ratings posted 79% of the five-star fraudulent hotel reviews on TripAdvisor [1]. As another example, carpet-cleaning company Hadeed was targeted by many anonymous negative reviews on Yelp [2]. Given ever-increasing criticisms of biased reviews in crowdsourcing platforms [3], it is critical for crowdsourcing platforms to strategically learn from these biased reviews to infer actual service state. However, such learning is difficult. On the platform side, users' reviews are not easy to verify immediately and extremely positive/negative reviews can also be the truth [4].

On the user side, due to major platforms' privacy protection, reviewers can hide their identities and bias types (e.g., positive or negative in rating) from a platform, which may wrongly offset in final rating. Further, users are smart to adjust their reviews to mislead the platform's inference or recommendation [5]. For example, suburban-area residents in the traffic navigation platform Waze tend to send fake reports about a speed trap, a wreck and some other traffic snarl during rush hours to deflect the traffic near their homes [6]. Besides, some policemen purposely posted no congestion messages in Waze to attract drivers there to possibly catch speeders [7]. These biased users are very different from malicious data attackers in the literature of crowdsourcing systems, who send fake reports to maximally reduce systems' inference accuracy (e.g., [8]–[11]). Crowdsourcing systems thus cannot learn from malicious attackers and focus on detecting them to abandon in final rating. Differently, here a biased user aims to mislead the system to his preferred state and may honestly reveal his preferred state. It is important for the platform to strategically learn from such biased users.

Our study to save crowdsourcing is more related to cheap-talk games in the economics or game theory literature (e.g., [12]–[16]), in which senders (i.e., users in our problem) observe the nature state and adaptively send messages to affect the receiver's (platform's) inference of the actual state. There are some very recent works to analyze and tackle extreme bias (e.g., [17], [18]). However, they only consider a simple scenario of one or two users, and assumed that a user's bias type is fixed and already known to aid the receiver's state learning. In practice, a user may have extremely positive or negative bias type, which is unknown to the platform. He may be bribed to target at over-the-top praise for business owners [1], or message poor rating to maximally attack service competitors or deflect the traffic near home [6].

We summarize our key novelty and main results below.

- *Strategic learning from biased users to save crowdsourcing:* To our best knowledge, we are the first to study how to save crowdsourcing from cheap-talk and strategically learn from biased users. Unlike the cheap-talk literature, our model is not limited to one or two users and allows the users' biases to be extremely positive or negative. We also practically consider that users' extreme biases are private information uncertain to the platform. We present new analytical studies to provide guidance on *whether* *and how a platform can learn useful information from extremely biased users' reviews.*

- *Dynamic Bayesian game modelling and involved analysis for closed-form PBE:* We formulate a two-stage dynamic Bayesian game including an arbitrary number of users to crowdsource service reviews for the platform. In Stage I, users observing the service state send unverifiable rating messages to the platform, and then in Stage II the platform takes action to infer the actual service state according to all the users' messages. As users' messaging have many ways to interplay and mislead the platform's inference from their observed state to their privately biased states, the Bayesian game theoretic analysis becomes involved. Nonetheless, we successfully provide closed-form expressions for characterizing the Perfect Bayesian Equilibrium (PBE) in the one-user case, where we show that the platform can successfully prevent the biased user from cheap-talk in most cases. Due to our proper design of strategic learning, a user even with extreme bias may honestly message to convince the platform of listening to his review. We prove that the price of anarchy (PoA) is 2, telling that the social cost can at most be doubled in the worst-case.
- *Analysis extension for an arbitrary number of users:* We extend our PBE analysis to an arbitrary number N of users with hidden bias from the platform. In the two-user case, at the closed-form PBE we analytically show that the platform can still prevent cheap-talk in most cases, and we prove that PoA=2 still holds in the worst case. Perhaps surprisingly, the platform's expected cost may be worse off after adding one more user. As $N\rightarrow\infty$, our asymptotic analysis shows that the platform always learns the actual service state to incur no cost and keeps $PoA<2$.

II. System Model and Problem Formulation

A. Dynamic Bayesian Game Modelling

As shown in Figure 1, we consider that N users observe the actual service state and are invited to send review messages in Stage I, and then in Stage II the platform makes an inference for the actual state and posts to the public.[1]

In practice, binary rating systems are prevailing and widely deployed. For example, social platform sites like Reddit and Twitter utilize upvote and downvote as users' binary ratings for better highlighting relevant replies from a long conversation thread [19]. Netflix also substituted star ratings with binary like or dislike for users to evaluate movies [20]. Accordingly, we model that service-state information (e.g., like or dislike of a service) θ is realized from a binary set $\{\theta_H, \theta_L\}$, where $\theta_H \neq \theta_L$ can be any real values to tell high or low rating.[2] The

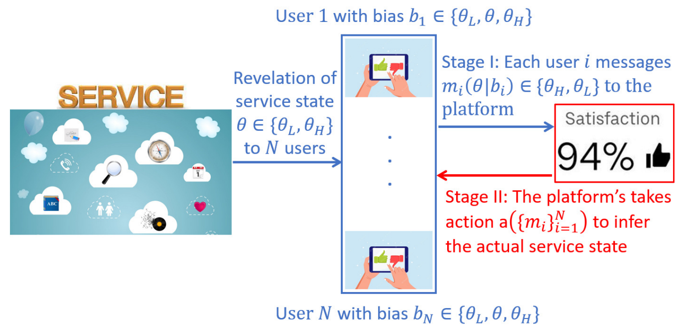

Fig. 1. System model of N users' messaging of service state (e.g., of hotels, restaurants and trips) to a crowdsourcing platform (e.g., TripAdvisor) who takes recommendation action $a(\{m_i\}_{i=1}^N)$ to infer the actual service state.

platform does not observe the actual state but knows the probability distribution of $Pr(\theta{=}\theta_H){=}p_H$ and $Pr(\theta{=}\theta_L){=}1{-}p_H$.

After observing service state θ, each user $i \in \{1, ..., N\}$ with bias type b_i sends the platform unverifiable message $m_i(\theta|b_i){\in}\{\theta_H, \theta_L\}$. His private bias b_i is realized from set $\{\theta_L, \theta, \theta_H\}$, telling negative bias/preference, no bias, or positive bias to mislead the platform from actual state θ. Here, a θ-biased user is equivalent to an unbiased user. The platform only knows probability distribution of possible biases as $Pr(b_i{=}\theta_L){=}Pr(b_i{=}\theta_H){=}q$ and $Pr(b_i{=}\theta){=}1{-}2q$ with $0{<}q{<}\frac{1}{2}$ for each user.[3] Denote set M to contain all the users' possible message combinations and set B to contain all the users' possible bias combinations. After receiving N users' messages $\{m_i\}_{i=1}^N{\in}M$, the platform takes recommendation action a about final rating to infer the actual service state. Note that the platform's action a is not limited to θ_H or θ_L, but can be any linear combination (e.g., 94% satisfaction in Figure 1).

Based on the above definitions, we next present users' and the platform's cost functions, respectively. Similar to the cheap-talk literature (e.g., [13], [14], [16]), user i's cost depends on his bias b_i to reflect self-preference for the platform's recommendation action $a(\{m_i\}_{i=1}^N)$ as follows:

$$u_{S_i}(a(\{m_i\}_{i=1}^N), b_i) = (a(\{m_i\}_{i=1}^N) - b_i)^2. \tag{1}$$

The quadratic loss function above tells a user's singly-preferred bias. As the platform's action strategy $a(\{m_i\}_{i=1}^N)$ is an inference from all the users' messages and observations, user i's expected cost depends on the service state and the other users' bias distribution. It is given by:

$$\bar{u}_{S_i}(b_i) = \sum_{\{m_i\}_{i=1}^N \in M} \sum_{j \in \{\theta_H, \theta_L\}} Pr(\theta = j) Pr(\{m_i\}_{i=1}^N | \theta = j) u_{S_i}(a(\{m_i\}_{i=1}^N), b_i). \tag{2}$$

Similar to the literature of cheap-talk (e.g., [13], [14], [16]), we consider that users participate rating in the long run and aim for minimizing their expected costs over service state realizations.

[1] We allow each user's observation to be noisy and will further study its impact in Section III.E.

[2] We can extend to multiple service states though it may not happen in the real world. For example, we can include another medium state in service state space $\{\theta_L, \theta_M, \theta_H\}$, where our PBE and PoA results still hold as the two-state scenario.

[3] For ease of exposition, we focus on the symmetric dual-bias distribution in this paper, and our main results such as PoA=2 can also be extended to the asymmetric case.

Similar to the cheap-talk literature (e.g., [13], [14], [16]), the platform's cost measures the square distance from its inference action a and realized state θ, and is given as follows:

$$u_R(a(\{m_i\}_{i=1}^N),\theta) = (a(\{m_i\}_{i=1}^N) - \theta)^2. \tag{3}$$

and its expected cost under a given inference strategy is thus

$$\bar{u}_R = \sum_{\{m_i\}_{i=1}^N \in M} \sum_{j\in\{\theta_H,\theta_L\}} Pr(\theta = j) Pr(\{m_i\}_{i=1}^N | \theta = j) u_R(a(\{m_i\}_{i=1}^N), \theta = j). \tag{4}$$

The platform's best strategy of inference action to minimize (4) is

$$a^*(\{m_i\}_{i=1}^N) = E[\theta|\{m_i\}_{i=1}^N] = \arg\min_a \sum_{j\in\{\theta_L,\theta_H\}} Pr\big(\theta = j|\{m_i\}_{i=1}^N\big) u_R(a,\theta = j), \tag{5}$$

We are ready to formulate a two-stage dynamic Bayesian game including N crowdsourcing users and the platform in Figure 1 as follows:

- Stage I: each user $i \in \{1, 2, \cdots, N\}$ with private bias $b_i \in \{\theta_L, \theta, \theta_H\}$ observes service state $\theta \in \{\theta_H, \theta_L\}$ and simultaneously messages $m_i(\theta|b_i) \in \{\theta_H, \theta_L\}$ to the crowdsourcing platform to minimize his expected cost in (2). The users have information advantages over the platform to know the service state and the bias types.
- Stage II: after observing users' messages $\{m_i\}_{i=1}^N$, the platform chooses an action strategy $a(\{m_i\}_{i=1}^N)$ to minimize its expected cost in (4).

Through Bayesian game theoretic analysis, we want to characterize the perfect Bayesian equilibrium (PBE) to best guide the platform's strategic learning against any biased messaging.

B. Definitions of Social Cost and Price of Anarchy (PoA)

We define social cost as the sum of N users' and the platform's costs as follows:

$$U(a,\theta,\{b_i\}_{i=1}^N) = \sum_{i=1}^N u_{S_i}(a,b_i) + u_R(a,\theta), \tag{6}$$

which can also incorporate different weights in the sum without affecting our following analysis. Note that the PBE (if exists) may not be unique. Define $\bar{U}_e^*$ as the expected social cost at a particular PBE e out of PBE set E of the Bayesian game as follows:

$$\bar{U}_e^*(N,p_H,q,\theta_H,\theta_L) = \sum_{i=1}^N \sum_{\{b_i\}_{i=1}^N\in B} Pr(\{b_i\}_{i=1}^N)\bar{u}^e_{S_i}(b_i) + \bar{u}^e_R, e\in E, \tag{7}$$

where both $\bar{u}^e_{S_i}$ and $\bar{u}^e_R$ consider any user i's messaging strategy $m_i^*(\theta|b_i)$ at the PBE.

At the social optimum, the social planner cares about minimizing expected social cost and is certain about the service state and users' biases. It decides the platform's inference as the average of all users' biases and the actual service state:

$$a^{**} = \arg\min_a U(a,\theta,\{b_i\}_{i=1}^N) = \frac{\theta + \sum_{i=1}^N b_i}{N+1}.$$

Define $\bar{U}^{**}$ as the minimum expected social cost, which is given by:

$$\bar{U}^{**}(N,p_H,q,\theta_H,\theta_L) = \sum_{\{b_i\}_{i=1}^N\in B} Pr(\{b_i\}_{i=1}^N) \sum_{j\in\{\theta_L,\theta_H\}} Pr(\theta=j) U(a^{**},\theta=j,\{b_i\}_{i=1}^N) = \frac{Nq(N+q-Nq)}{N+1}(\theta_H-\theta_L)^2, \tag{8}$$

where function U in (6) centrally decides each user i's messaging strategy $m_i^{**}(\theta|b_i)$ and the platform's inference action at the social optimum.

As in the game theory literature, price of anarchy (PoA) is defined to measure the maximum efficiency loss in the worst case. We then finally define PoA as the maximum ratio between expected social costs under the worst PBE and the social optimum. It is given by:

$$PoA := \max_{q,p_H,\theta_H,\theta_L} \frac{\max_{e\in E}\{\bar{U}_e^*\}}{\bar{U}^{**}}. \tag{9}$$

Note that PoA is greater than 1, and a large PoA implies large efficiency loss on expected social cost in our platform's strategic learning. We will examine it against different user number N.

III. PBE And PoA Analysis for One-user Case

In this section we first consider the case of one user, i.e., $N = 1$, and we skip the index in b_i and m_i. The Bayesian game theoretic analysis is still involved in this case, as the user's messaging has many ways to mislead the platform's inference from his observed state to his privately biased states. In the following we will find PBE first and then check its expected social cost gap from the social optimum via analyzing PoA.

A. User's Strategy Simplification

The following lemma helps us reduce the space to analyze the user's equilibrium strategy.

Lemma 1. *At the PBE of the dynamic Bayesian game, we have $m^*(\theta = \theta_L|b = \theta_L) = \theta_L$, $m^*(\theta|b = \theta) = \theta$, and $m^*(\theta = \theta_H|b = \theta_H) = \theta_H$.*

The unbiased user with realized $b = \theta$ type has the same cost function $u_{S_i}(a, b_i = \theta)$ in (1) as the platform's cost function $u_R(a, \theta)$ in (3). Thus, any deviation from truthfully messaging increases his cost and he will not cheat in messaging. Similarly, with high-state observation $\theta = \theta_H$, the θ_H-biased user has the same cost function as the platform, and will not cheat, either. This is also true for each θ_L-biased user facing $\theta = \theta_L$.

Thanks to Lemma 1, since the unbiased user is always honest, we focus on the two possible bias types for the user who only chooses one out of the four strategy candidates below:

- *User's honest strategy 1*: $m(\theta|b) = \theta$.
- *User's maximum dishonest strategy 2*: $m(\theta|b = \theta_L) = \theta_L$ and $m(\theta|b = \theta_H) = \theta_H$.
- *User's dishonest $b = \theta_L$ only strategy 3*: $m(\theta|b = \theta_L) = \theta_L$ and $m(\theta|b = \theta_H) = \theta$.
- *User's dishonest $b = \theta_H$ only strategy 4*: $m(\theta|b = \theta_L) = \theta$ and $m(\theta|b = \theta_H) = \theta_H$.

After receiving the user's message m, the platform updates its posterior state belief according to Bayes' theorem:

$$Pr(\theta = \theta_i|m = \theta_i) = \frac{Pr(m = \theta_i|\theta = \theta_i)Pr(\theta = \theta_i)}{\sum_{j\in\{H,L\}} Pr(m = \theta_i|\theta = \theta_j)Pr(\theta = \theta_j)}, i \in \{H, L\}, \tag{10}$$

where the truthful reporting probability is:

$$Pr(m = \theta_i|\theta = \theta_i) = \sum_{j\in\{\theta_L,\theta,\theta_H\}} Pr(m = \theta_i, b = j|\theta = \theta_i)$$
$$= \sum_{j\in\{\theta_L,\theta,\theta_H\}} Pr(m = \theta_i|\theta = \theta_i, b = j)Pr(b = j). \tag{11}$$

B. Platform's Best Response to Strategies 1-4

Next, we examine the platform's best response to the user's four possible strategies, respectively, and will use it to proactively determine the user's strategy choice later. First, assuming the user will adopt honest strategy 1, the platform expects:

$$Pr(m = \theta_i|\theta = \theta_i, b) = 1, \ i \in \{H, L\}. \tag{12}$$

Substituting (12) into (11) and (10), the platform's best-response action in (5) to strategy 1 is to predict

$$a_1^*(m) = m, \ m \in \{\theta_H, \theta_L\}. \tag{13}$$

Then the user's expected cost in (2) by using honest strategy 1 above is

$$\bar{u}_1(b) = \begin{cases} p_H(\theta_H - \theta_L)^2, & \text{if } b = \theta_L, \\ (1 - p_H)(\theta_H - \theta_L)^2, & \text{if } b = \theta_H. \end{cases} \tag{14}$$

Second, assuming the user will adopt maximum dishonest strategy 2, the platform expects

$$Pr(m = \theta_H|\theta, b = \theta_H) = Pr(m = \theta_L|\theta, b = \theta_L) = 1, \tag{15}$$

which is regardless of state realization θ. Substituting (15) into (11) and (10), the platform's best-response action in (5) to strategy 2 is to predict

$$a_2^*(m) = \begin{cases} \frac{p_H(1-q)\theta_H+(1-p_H)q\theta_L}{p_H(1-q)+(1-p_H)q}, & \text{if } m = \theta_H, \\ \frac{p_Hq\theta_H+(1-p_H)(1-q)\theta_L}{(1-p_H)(1-q)+p_Hq}, & \text{if } m = \theta_L. \end{cases} \tag{16}$$

Then the user's expected cost in (2) by using maximum dishonest strategy 2 above is

$$\bar{u}_2(b) = \begin{cases} \frac{p_H^2q^2}{((1-p_H)(1-q)+p_Hq)^2}(\theta_H - \theta_L)^2, & \text{if } b = \theta_L, \\ \frac{(1-p_H)^2q^2}{(p_H(1-q)+(1-p_H)q)^2}(\theta_H - \theta_L)^2, & \text{if } b = \theta_H. \end{cases} \tag{17}$$

Third, assuming the user will adopt dishonest $b = \theta_L$ only strategy 3, the platform expects

$$Pr(m = \theta|\theta, b = \theta_H) = Pr(m = \theta_L|\theta, b = \theta_L) = 1. \tag{18}$$

Substituting (18) into (11) and (10), the platform's best-response action in (5) to strategy 3 is to predict

$$a_3^*(m) = \begin{cases} \theta_H, & \text{if } m = \theta_H, \\ \frac{p_Hq\theta_H+(1-p_H)\theta_L}{1-p_H+p_Hq}, & \text{if } m = \theta_L. \end{cases} \tag{19}$$

Then the user's expected cost in (2) by using dishonest $b = \theta_L$ only strategy 3 above is

$$\bar{u}_3(b) = \begin{cases} \frac{(p_Hq)^2}{(1-p_H+p_Hq)^2}(\theta_H - \theta_L)^2, & \text{if } b = \theta_L, \\ \frac{(1-p_H)^3}{(1-p_H+p_Hq)^2}(\theta_H - \theta_L)^2, & \text{if } b = \theta_H. \end{cases} \tag{20}$$

Finally, assuming the user will adopt dishonest $b = \theta_H$ only strategy 4, the platform expects

$$Pr(m = \theta_H|\theta, b = \theta_H) = Pr(m = \theta|\theta, b = \theta_L) = 1. \tag{21}$$

Substituting (21) into (11) and (10), the platform's best-response action in (5) to strategy 4 is to predict

$$a_4^*(m) = \begin{cases} \frac{p_H\theta_H+(1-p_H)q\theta_L}{p_H+(1-p_H)q}, & \text{if } m = \theta_H, \\ \theta_L, & \text{if } m = \theta_L. \end{cases} \tag{22}$$

Then the user's expected cost in (2) by using dishonest $b = \theta_H$ only strategy 4 above is

$$\bar{u}_4(b) = \begin{cases} \frac{p_H^3}{(p_H+(1-p_H)q)^2}(\theta_H - \theta_L)^2, & \text{if } b = \theta_L, \\ \frac{(1-p_H)^2q^2}{(p_H+(1-p_H)q)^2}(\theta_H - \theta_L)^2, & \text{if } b = \theta_H. \end{cases} \tag{23}$$

C. User's Final Messaging Strategy Choice and PBE

After comparing the user's expected costs in (14), (17), (20) and (23) under the platform's best-response actions for b=θ_H and b=θ_L, respectively, we are then able to determine his strategy for any bias type and finalize PBE of the whole game. Note that always honest strategy 1 cannot happen at the PBE, while maximum dishonest strategy 2 and partially dishonest strategies 3-4 may happen. To help introduce the user's strategy at PBE, we define two probability thresholds $p_{1,H}\in(0, 1)$ and $p_{1,L}\in(0, 1)$, where $p_{1,H}$=$1-p_{1,L}$ is the unique solution to

$$q^2(1 - p_{1,H} + p_{1,H}q)^2 - (1 - p_{1,H})(p_{1,H}(1 - q) + (1 - p_{1,H})q)^2 = 0.$$

We prove that the θ_L-biased user will misreport state $\theta = \theta_H$ to θ_L if $p_H > p_{1,L}$, and the θ_H-biased user will misreport state $\theta = \theta_L$ to θ_H if $p_H < p_{1,H}$.

Proposition 1. *In the one-user case, if the user has small bias probability (i.e., $q \leq \frac{\sqrt{1+2\sqrt{2}}-1}{2}$), we have $p_{1,L} \leq \frac{1}{2} \leq$*

TABLE I
PBE VERSUS HIGH-STATE PROBABILITY p_H IN SMALL BIAS PROBABILITY $q \leq \frac{\sqrt{1+2\sqrt{2}}-1}{2}$ IN THE ONE-USER CASE. SPECIFICALLY, $p_{1,L}$=$p_{1,H}$ =$\frac{1}{2}$ IF q=$\frac{\sqrt{1+2\sqrt{2}}-1}{2}$ AND THE MEDIUM p_H REGIME WITH MAXIMUM DISHONEST STRATEGY DIMINISHES.

p_H REGIME	PBE
SMALL $p_H \in [0, p_{1,L}]$	DISHONEST $b = \theta_H$ ONLY STRATEGY 4: $m^*(\theta \vert b{=}\theta_L){=}\theta$, $m^*(\theta \vert b{=}\theta_H){=}\theta_H$; AND THE PLATFORM'S INFERENCE $a^*(m)$ IN (22).
MEDIUM $p_H \in [p_{1,L}, p_{1,H}]$	MAXIMUM DISHONEST STRATEGY 2: $m^*(\theta \vert b{=}\theta_L){=}\theta_L$, $m^*(\theta \vert b{=}\theta_H){=}\theta_H$; AND THE PLATFORM'S INFERENCE $a^*(m)$ IN (16).
LARGE $p_H \in [p_{1,H}, 1]$	DISHONEST $b = \theta_L$ ONLY STRATEGY 3: $m^*(\theta \vert b{=}\theta_L){=}\theta_L$, $m^*(\theta \vert b{=}\theta_H){=}\theta$; AND THE PLATFORM'S INFERENCE $a^*(m)$ IN (19).

$p_{1,H}$. *PBE is given in closed-form in Table I with three high-state probability p_H regimes. If q=$\frac{\sqrt{1+2\sqrt{2}}-1}{2}$, $p_{1,L}$=$p_{1,H}$=$\frac{1}{2}$ and the medium p_H regime with maximum dishonest strategy diminishes in Table I.*

Proposition 1 implies that the biased user's cheap-talk does not happen at the PBE in most cases (see small and large p_H regimes in Table I). As shown in Table I, if $p_H > p_{1,H}$, the user type of bias θ_H truthfully messages to convince the platform of no cheap-talk, without losing much as state $\theta = \theta_L$ does not happen frequently to incur his cost. Similarly, if $p_H < p_{1,L}$, the user type of bias θ_L truthfully messages to convince the platform. For medium p_H, the cost in honest reporting is non-small and the user chooses maximum dishonest messaging. As bias probability q increases to $\frac{\sqrt{1+2\sqrt{2}}-1}{2}$, the user tries to be more honest to convince the platform of no cheap-talk, by skipping maximum dishonest strategy 2. Thus, the medium p_H regime of Table I diminishes and only strategies 3 and 4 may exhibit at PBE.

Proposition 2. *In the one-user case, if the user has large bias probability (i.e., $q \in (\frac{\sqrt{1+2\sqrt{2}}-1}{2}, \frac{1}{2})$), we have $0 \leq p_{1,H} < \frac{1}{2}$ and $\frac{1}{2} < p_{1,L} \leq 1$. PBE is given in closed-form in Table II with three high-state probability p_H regimes.*

Proposition 2 implies that the biased user's cheap-talk never happens at the PBE for larger bias probability $q \in (\frac{\sqrt{1+2\sqrt{2}}-1}{2}, \frac{1}{2})$. As bias probability q is no longer small to the platform, the biased user tries to be more honest to convince the platform, by skipping maximum dishonest strategy 2. As the two thresholds satisfy $p_{1,H} < \frac{1}{2}$ and $p_{1,L} > \frac{1}{2}$ here, we have two PBEs in the medium p_H regime of Table II.

D. PoA Analysis to Compare Expected Social Costs

Given PBE results in Propositions 1 and 2, we are ready to analyze the PoA in (9) by comparing PBE performance to the social optimum for any parameter including q.

TABLE II
PBE VERSUS HIGH-STATE PROBABILITY p_H IN LARGE REGIME OF $q \in (\frac{\sqrt{1+2\sqrt{2}}-1}{2}, \frac{1}{2})$ IN THE ONE-USER CASE.

p_H REGIME	PBE
SMALL $p_H \in [0, p_{1,H}]$	DISHONEST $b = \theta_H$ ONLY STRATEGY 4: $m^*(\theta \vert b{=}\theta_L){=}\theta$, $m^*(\theta \vert b{=}\theta_H){=}\theta_H$; AND THE PLATFORM'S INFERENCE $a^*(m)$ IN (22).
MEDIUM $p_H \in [p_{1,H}, p_{1,L}]$	DISHONEST $b = \theta_H$ ONLY STRATEGY 4: $m^*(\theta \vert b{=}\theta_L){=}\theta$, $m^*(\theta \vert b{=}\theta_H){=}\theta_H$; AND THE PLATFORM'S INFERENCE $a^*(m)$ IN (22).
	DISHONEST $b = \theta_L$ ONLY STRATEGY 3: $m^*(\theta \vert b{=}\theta_L){=}\theta_L$, $m^*(\theta \vert b{=}\theta_H){=}\theta$; AND THE PLATFORM'S INFERENCE $a^*(m)$ IN (19).
LARGE $p_H \in [p_{1,L}, 1]$	DISHONEST $b = \theta_L$ ONLY STRATEGY 3: $m^*(\theta \vert b{=}\theta_L){=}\theta_L$, $m^*(s \vert b{=}\theta_H){=}\theta$; AND THE PLATFORM'S INFERENCE $a^*(m)$ IN (19).

Proposition 3. *In the one-user case, we have $PoA = 2$ in the worst case, telling that the social cost can at most be doubled in the worst-case. This is obtained when the user exhibits maximum dishonest strategy 2 with $m^*(\theta|b = \theta_L) = \theta_L$ and $m^*(\theta|b = \theta_H) = \theta_H$, as shown in the medium p_H regime of Table I.*

The user does not use honest strategy 1 at all times, and the maximum efficiency loss is incurred when the user exhibits the maximum dishonest strategy 2 to confuse the platform.

E. Analysis on User's Imperfect Observation

Recall that in Section II we assume that each user perfectly observes the service state, which may not be true all the time. In this subsection we further model error probability $\epsilon \in (0, \frac{1}{2}]$ in the user's observation:

$$Pr(\theta' \neq j|\theta = j) = \epsilon \in (0, \frac{1}{2}], j \in \{\theta_L, \theta_H\},$$

where θ denotes the realized state and θ' is the user's observation. For example, a Waze user may not observe all the road condition (e.g., 'black-ice' segments) and obtain wrong navigation information [21]. Now the platform needs to further consider whether the user's observation (even if honest) is correct or not.

With similar analysis from Section III-A to III-D, we have the following.

Corollary 1. *Considering user's observation error probability $\epsilon \in (0, \frac{1}{2}]$, we still have $PoA = 2$. For the PBE, take the symmetric case of $p_H = \frac{1}{2}$ for example, at the PBE the biased user exhibits maximum dishonest strategy 2 if error probability ϵ satisfies*

$$\epsilon \leq \min\left\{\frac{-1 + 2q^2 + 4q^3 + 2q^4}{-2q - 4q^2 + 4q^3 + 4q^4}, \frac{1}{2}\right\}.$$

The PBE in Corollary 1 is same as that in the medium p_H regime of Table I. Corollary 1 implies that PoA and key PBE results can also be extended to the case of user's erroneous state observation as long as the error probability is small.

IV. PBE and PoA Analysis for Multi-user Case

In this section we extend to consider an arbitrary number of $N \geq 2$ users. Now each user needs to take the others' possible biases/messages into consideration when deciding his own messaging strategy, leading to more involved message combinations for the platform to strategically learn from. In the following we will find PBE first and then check its expected social cost gap from the social optimum via analyzing PoA.

Lemma 1 holds for N users here and each biased user i still chooses one out of strategies 2-4 in Section III-A.[4] Note that only unbiased users use honest strategy 1. Denote set $I_j = \{i | m_i = \theta_j, 0 \leq i \leq N\}$ to include all users for messaging θ_j with $j \in \{L, H\}$. By adding all possible bias/messaging combinations among N users to analysis in Section III-B, we can obtain the platform's best-response learning action in (5) and each user i's expected cost in (2) under maximum dishonest strategy 2 as follows:

$$\begin{aligned} & a_2^*(|I_H| = k) \\ =& \frac{p_H(1-q)^k q^{N-k}\theta_H + (1-p_H)q^k(1-q)^{N-k}\theta_L}{p_H(1-q)^k q^{N-k} + (1-p_H)q^k(1-q)^{N-k}}, \end{aligned} \tag{24}$$

$$\begin{aligned} \bar{u}_2(b_i = \theta_H) =& \sum_{k=1}^{N} C_{N-1}^{k-1}(\theta_H - \theta_L)^2 \\ & \left(p_H q^{N-k}(1-q)^{k-1} + (1-p_H)q^{k-1}(1-q)^{N-k}\right) \\ & \cdot \left(\frac{(1-p_H)q^k(1-q)^{N-k}}{p_H q^{N-k}(1-q)^k + (1-p_H)q^k(1-q)^{N-k}}\right)^2, \end{aligned} \tag{25}$$

$$\begin{aligned} \bar{u}_2(b_i = \theta_L) =& \sum_{k=0}^{N-1} C_{N-1}^{k}(\theta_H - \theta_L)^2 \\ & \left(p_H q^{N-k-1}(1-q)^k + (1-p_H)q^k(1-q)^{N-k-1}\right) \\ & \cdot \left(\frac{p_H q^{N-k}(1-q)^k}{p_H q^{N-k}(1-q)^k + (1-p_H)q^k(1-q)^{N-k}}\right)^2. \end{aligned} \tag{26}$$

These terms under dishonest b_i=θ_L only strategy 3 are given as follows:

$$a_3^*(|I_H| = k) = \begin{cases} \theta_H, & \text{if } k > 0, \\ \frac{p_H q^N \theta_H + (1-p_H)\theta_L}{p_H q^N + 1 - p_H}, & \text{if } k = 0, \end{cases} \tag{27}$$

$$\bar{u}_3(b_i = \theta_H) = (1-p_H)\left(\frac{1-p_H}{q^N p_H + 1 - p_H}\right)^2 (\theta_H - \theta_L)^2. \tag{28}$$

$$\begin{aligned} & \bar{u}_3(b_i = \theta_L) = (\theta_H - \theta_L)^2 \\ & \left(p_H(1-q^{N-1}) + (p_H q^{N-1} + 1 - p_H)\left(\frac{q^N p_H}{q^N p_H + 1 - p_H}\right)^2\right). \end{aligned} \tag{29}$$

[4]Here we consider users' symmetric messaging strategy under the same bias, i.e., $m_i(\theta|b_i)$=$m_j(\theta|b_j)$ if b_i=b_j, otherwise multiple PBEs may occur.

TABLE III
PBE versus high-state probability p_H in the two-user case.

p_H regime	PBE
Small $p_H \in [0, p_{2,L}]$	Dishonest $b_i = \theta_H$ only strategy 4: $m_i^*(\theta \mid b_i = \theta_L) = \theta$, $m_i^*(\theta \mid b_i = \theta_H) = \theta_H$.
Medium $p_H \in [p_{2,L}, p_{2,H}]$	Maximum dishonest strategy 2: $m_i^*(\theta \mid b_i = \theta_L) = \theta_L$, $m_i^*(\theta \mid b_i = \theta_H) = \theta_H$.
Large $p_H \in [p_{2,H}, 1]$	Dishonest $b_i = \theta_L$ only strategy 3: $m_i^*(\theta \mid b_i = \theta_L) = \theta_L$, $m_i^*(\theta \mid b_i = \theta_H) = \theta$.

Finally, they under dishonest b_i=θ_H only strategy 4 are given as follows:

$$a_4^*(|I_H| = k) = \begin{cases} \theta_L, & \text{if } k < N, \\ \frac{p_H \theta_H + (1-p_H)q^N \theta_L}{p_H + (1-p_H)q^N}, & \text{if } k = N. \end{cases} \tag{30}$$

$$\begin{aligned} & \bar{u}_4(b_i = \theta_H) = (\theta_H - \theta_L)^2 \Bigg((p_H + (1-p_H)q^{N-1}) \\ & \left(\frac{(1-p_H)q^N}{p_H + (1-p_H)q^N}\right)^2 + (1-p_H)(1-q^{N-1}) \Bigg), \end{aligned} \tag{31}$$

$$\bar{u}_4(b_i = \theta_L) = p_H \left(\frac{p_H}{p_H + (1-p_H)q^N}\right)^2 (\theta_H - \theta_L)^2. \tag{32}$$

A. PBE and PoA Analysis for $N = 2$ Users

To inspire a more general analysis, we first look at N=2 users. To help determine N=2 users' strategy at PBE, we define two probability thresholds $p_{2,H} \in (\frac{1}{2}, 1)$ and $p_{2,L} \in (0, \frac{1}{2})$, where $p_{2,H}$=$1 - p_{2,L}$ is the unique solution in the range (0, 1) to

$$\begin{aligned} & \left(p_{2,H}(1-q) + (1-p_{2,H})q\right)q^4(1 - p_{2,H} + p_{2,H}q^2)^2 \\ & -(1-p_{2,H})\left((1-q)^2 p_{2,H} + q^2(1-p_{2,H})\right)^2 \\ & +\left(p_{2,H}q + (1-p_{2,H})(1-q)\right)\left((1-q)^2 p_{2,H} + q^2(1-p_{2,H})\right)^2 \\ & \cdot(1 - p_{2,H} + p_{2,H}q^2)^2 = 0. \end{aligned} \tag{33}$$

Similar to Section III-C, each θ_L-biased user here will misreport state $\theta = \theta_H$ to θ_L if $p_H > p_{2,L}$, and each user of bias θ_H will misreport state $\theta = \theta_L$ to θ_H if $p_H < p_{2,H}$. However, here the PBE with one more user becomes less complicated due to $p_{2,L} < p_{2,H}$ for any bias probability q.

Proposition 4. *In the two-user case, $p_{2,L} < \frac{1}{2} < p_{2,H}$ always holds. Users' strategies at the unique PBE are given in closed-form in Table III with three high-state probability p_H regimes. The platform's closed-form equilibrium learning action depends on high-state probability p_H in the following.*

- *If $p_H < p_{2,L}$, we have*

$$a^*(m_1, m_2) = \begin{cases} \frac{p_H \theta_H + (1-p_H)q^2 \theta_L}{p_H + (1-p_H)q^2}, & \textit{if } m_1 = m_2 = \theta_H, \\ \theta_L, & \textit{otherwise.} \end{cases}$$

- *If $p_H \in [p_{2,L}, p_{2,H}]$, we have*

$$a^*(m_1, m_2) = \begin{cases} \frac{(1-q)^2 p_H \theta_H + q^2(1-p_H)\theta_L}{(1-q)^2 p_H + q^2(1-p_H)}, & \text{if } m_1 = m_2 = \theta_H, \\ p_H\theta_H + (1-p_H)\theta_L, & \text{if } m_i = \theta_H, m_{-i} = \theta_L, \\ \frac{q^2 p_H \theta_H + (1-q)^2(1-p_H)\theta_L}{q^2 p_H + (1-q)^2(1-p_H)}, & \text{if } m_1 = m_2 = \theta_L. \end{cases}$$

- *If $p_H > p_{2,H}$, we have*

$$a^*(m_1, m_2) = \begin{cases} \frac{p_H\theta_H + (1-p_H)q^2\theta_L}{p_H + (1-p_H)q^2}, & \text{if } m_1 = m_2 = \theta_L, \\ \theta_H, & \text{otherwise.} \end{cases}$$

Proposition 4 implies that biased users' cheap-talks do not happen at the PBE in most cases (see small and large p_H regimes in Table III) . Table III with two-users is similar to Table I with one-user for small bias probability q. However, for non-small q, here the PBE structure changes substantially and we no longer observe similar results in Table II with one-user. In the medium p_H regime of Table III, as the other user to message may be equally likely to have opposite bias, a user with different bias from the state ($b_i \neq \theta$) has greater cost to message truthfully and convince the platform. Thus, users choose maximum dishonest strategy 2 as cheap-talk.

Perhaps surprisingly, the following corollary shows that the platform does not always incur smaller expected cost from inviting one more random user of hidden bias.

Corollary 2. *The platform's expected cost decreases with one more random user if high-state probability $p_H \in [0, p_{2,L}] \cup [p_{2,H}, 1]$ and bias probability $q \leq \frac{\sqrt{1+2\sqrt{2}}-1}{2}$, but increases if $p_H \in (p_{2,L}, p_{2,H})$ and $q > \frac{\sqrt{1+2\sqrt{2}}-1}{2}$.*

According to Propositions 2 and 4, cheap-talk does not happen at the PBE in the large regime of bias probability $q > \frac{\sqrt{1+2\sqrt{2}}-1}{2}$ in the one-user case, but may still occur in the two-user case. Thus, the platform might learn nothing from two biased users' cheap-talk, and its expected cost increases with one more user.

Given PBE results in Proposition 4, we are ready to analyze the PoA in (9) by comparing PBE performance to the social optimum.

Proposition 5. *In the two-user case, we have $PoA = 2$ in the worst case, telling that the social cost can at most be doubled in the worst-case. This is obtained when $p_H \in [p_{2,L}, p_{2,H}]$ for each user i's maximum dishonest strategy 2 with $m_i^*(\theta|b_i = \theta_L) = \theta_L$ and $m_i^*(\theta|b_i = \theta_H) = \theta_H$, as shown in the medium p_H regime of Table III.*

Similar to the one-user case, each user does not use honest strategy 1 anyway, and the maximum efficiency loss is incurred when each uses the maximum dishonest strategy 2 to confuse the platform.

B. PBE and PoA Analysis for Arbitrary N Users

For arbitrary N users, it includes many combinations of users' biases and the analysis is more involved. Nonetheless, we manage to provide asymptotic analysis as $N \to \infty$ and numerically study the case of finite N.

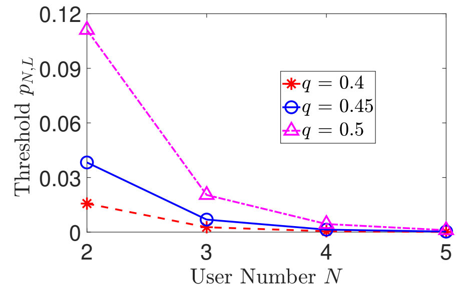

Fig. 2. Threshold $p_{N,L}$ versus user number N and bias probability q, and its special case $p_{N=2,L} = 1 - p_{N=2,H}$ and $p_{N=2,H}$ is given in (33). If $p_H > p_{N,L}$, a user of bias θ_L misreports state $\theta = \theta_H$ to bias θ_L.

Lemma 2. *Given user number $N \to \infty$, at the PBE each biased user i may arbitrarily choose any strategy 1-4. The platform can always learn the actual service state, and its equilibrium action is*

$$a^*(|I_H| = k) = \begin{cases} \theta_L, & \text{if } \frac{k}{N} < \frac{1}{2}, \\ \theta_H, & \text{if } \frac{k}{N} > \frac{1}{2}, \end{cases}$$

where $I_H = \{i | m_i = \theta_H, 1 \leq i \leq N\}$ is the set of users for messaging θ_H.

As $N \to \infty$, positively and negatively biased users' reviews well negate each other and the platform always learns the actual service state from the majority including unbiased user(s). Note that the infinite number of users eliminates effect of biased users' cheap-talk, and the platform does not suffer from greater cost as in Corollary 2. Given the PBE results in Lemma 2, we examine the platform's expected cost and the system PoA below.

Proposition 6. *As $N \to \infty$, at the PBE the platform's expected cost in* (4) *becomes zero. In the worst case of social cost, we have $PoA < 2$.*

At the PBE the platform always recognizes the actual state and learns correctly, and unbiased users thus have zero cost, improving the PoA=2 (under N=1 and 2) to PoA<2 here.

Now we numerically study the PBE under finite N. We similarly generalize the two thresholds $p_{2,L}$ and $p_{2,H}$ in (33) to $p_{N,L}$ and $p_{N,H}$, respectively. Note that the PBE here is similar to Table III with three p_H regimes, where a user of bias θ_L will misreport state $\theta = \theta_H$ to bias θ_L if $p_H > p_{N,L}$, and he of bias θ_H will misreport state $\theta = \theta_L$ to bias θ_H if $p_H < p_{N,H}$. As N increases from 2, $p_{N,L}$ decreases in Figure 2 and $p_{N,H}$ increases in Figure 3, telling that the medium regime of Table III enlarges and the maximum dishonest messaging strategy prevails. Note that the PBE for large N may be still different from Lemma 2 with arbitrary user strategy under $N \to \infty$.

As bias probability q increases and the platform is more suspicious on users' messages, Figures 2 and 3 show that $p_{N,L}$

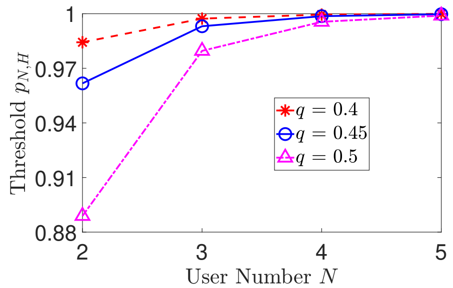

Fig. 3. Threshold $p_{N,H}$ versus user number N and bias probability q, and its special case $p_{N=2,H}$ is given in (33). If $p_H < p_{N,H}$, a user of bias θ_H misreports state $\theta = \theta_L$ to bias θ_H.

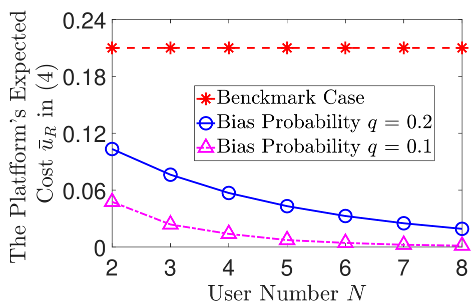

Fig. 4. The platform's expected cost $\bar{u}_R^*$ in (4) versus user number N and bias probability q, respectively. In the benchmark case, the platform simply treats biased reviews as cheap-talk and abandons (e.g., [8]–[11]). Here we normalize $(\theta_H - \theta_L)^2 = 1$ and set $p_H = 0.3$.

increases with q and $p_{N,H}$ decreases with q. The maximum dishonest strategy regime in the medium p_H regime of Table III reduces. This is because users still want to convince the platform of no cheap-talks and hope to use honest messaging to influence the platform's learning action.

Figure 4 numerically shows that at the PBE of our dynamic Bayesian game, the platform's expected cost $\bar{u}_R^*$ decreases with N and approaches zero. However, as shown in the benchmark case, if the platform simply treats biased reviews as cheap-talk and abandons as the literature of crowdsourcing systems (e.g., [8]–[11]), its expected cost is independent of N and never diminishes as shown in Figure 4.

V. Conclusion

In this paper, we study how to save crowdsourcing from cheap-talk and strategically learn the actual service state from biased users' reviews. We formulate the problem as a dynamic Bayesian game, including the unknown users' messaging and the platform's follow-up inference. Through involved analysis, we provide closed-form expressions for the PBE. Our PBE shows that the platform's strategic learning can successfully prevent biased users from cheap-talk in most cases, where a user even with extreme bias still honestly messages to convince the platform of listening to his review. We prove that the PoA is 2. As the user number becomes large, our platform always learns the actual service. Perhaps surprisingly, the platform's expected cost may be worse off after adding one more user.

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Appendix A
Proofs of Propositions 1 and 2

Proof. We will first show strategy 1 is not an equilibrium for the user with bias θ_H nor θ_L. With (14) and (23), we have

$$\begin{aligned}&\bar{u}_1(b=\theta_H)-\bar{u}_4(b=\theta_H)\\=&(1-p_H)\left(1-\frac{(1-p_H)q^2}{(p_H+(1-p_H)q)^2}\right)(\theta_H-\theta_L)^2\geq 0\end{aligned}$$

due to $1-p_H\in[0,1]$ and $q^2/(p_H+(1-p_H)q)^2\in[0,1]$. The user with bias θ_H thus deviates from strategy 1. With (14) and (20), we have

$$\begin{aligned}&\bar{u}_1(b=\theta_L)-\bar{u}_3(b=\theta_L)\\=&p_H\left(1-\frac{p_Hq^2}{(1-p_H+p_Hq)^2}\right)(\theta_H-\theta_L)^2\geq 0\end{aligned}$$

due to $p_H\in[0,1]$ and $q^2/(1-p_H+p_Hq)^2\in[0,1]$. The user with bias θ_L thus deviates from strategy 1. Therefore, user's honest strategy 1 is never an equilibrium.

We will then find condition for the user with bias θ_H prefers to always send high-state message than truthfully message, i.e., when $\bar{u}_2(b=\theta_H)$ in (17) is less than $\bar{u}_3(b=\theta_H)$ in (23), which is equal to

$$\begin{aligned}&(1-p_H)^2q^2(1-p_H+p_Hq)^2\\&-(1-p_H)^3(p_H(1-q)+(1-p_H)q)^2\leq 0.\end{aligned}$$

Denote

$$\begin{aligned}f_0(p_H):=&(1-p_H)^2q^2(1-p_H+p_Hq)^2\\&-(1-p_H)^3(p_H(1-q)+(1-p_H)q)^2.\end{aligned}$$

After checking derivatives of $f_0(p_H)$, we obtain that $f_0(p_H)\leq 0$ if $p_H\in[0,p_{1,H}]$ and $f_0(p_H)\geq 0$ if $p_H\in(p_{1,H},1]$, where $q\in(0,\frac{1}{2})$ and $p_{1,H}\in(0,1)$ satisfying $f(p_{1,H})=0$. If $q=\frac{1}{2}$, we have $f_0(p_H)\geq 0$. Therefore, the desired condition is $p_H\in[0,p_{1,H}]$ if $q\in(0,\frac{1}{2})$ and $p_H\in\emptyset$ if $q=\frac{1}{2}$. We further have

$$\begin{aligned}&f_0(p_H=\frac{1}{2})\\&=(\frac{1}{2})^2q^2(\frac{1}{2}+(1-\frac{1}{2})q)^2-(\frac{1}{2})^3((1-\frac{1}{2})(1-q)+\frac{1}{2}q)^2.\end{aligned}$$

By checking derivatives of $f_0(p_H=\frac{1}{2})$ on q , we have $f_0(p_H=\frac{1}{2})$ increases with q and $f_0(p_H=\frac{1}{2})<0$ for $q<\frac{\sqrt{1+2\sqrt{2}}-1}{2}$, $f_0(p_H=\frac{1}{2})=0$ for $q=\frac{\sqrt{1+2\sqrt{2}}-1}{2}$ and $f_0(p_H=\frac{1}{2})>0$ for $q>\frac{\sqrt{1+2\sqrt{2}}-1}{2}$. We thus have $p_{1,H}>\frac{1}{2}$ if $q<\frac{\sqrt{1+2\sqrt{2}}-1}{2}$, $p_{1,H}=\frac{1}{2}$ if $q=\frac{\sqrt{1+2\sqrt{2}}-1}{2}$ and $p_{1,H}<\frac{1}{2}$ if $q>\frac{\sqrt{1+2\sqrt{2}}-1}{2}$.

We will then find condition for the user with bias θ_L prefers to always send low-state message than truthfully message, i.e., $\bar{u}_2(b=\theta_L)$ in (17) is less than $\bar{u}_4(b=\theta_L)$ in (23), which is equal to

$$p_H^2q^2(p_H+(1-p_H)q)^2-p_H^3((1-p_H)(1-q)+p_Hq)^2\leq 0.$$

Denote

$$\begin{aligned}&g_0(p_H)\\&:=p_H^2q^2(p_H+(1-p_H)q)^2-p_H^3((1-p_H)(1-q)+p_Hq)^2.\end{aligned}$$

After checking derivatives of $g_0(p_H)$ on p_H, we obtain that $g_0(p_H)\geq 0$ if $p_H\in[0,p_{1,L}]$ and $g_0(p_H)\leq 0$ if $p_H\in(p_{1,L},1]$, where $q\in[0,\frac{1}{2})$ and $p_{1,L}\in(0,1)$ satisfying $g(p_{1,L})=0$. If $q=\frac{1}{2}$, we have $g_0(p_H)\geq 0$. Therefore, the desired condition is $p_H\in[p_{1,L},1]$ if $q\in(0,\frac{1}{2})$ and $p_H\in\emptyset$ if $q=\frac{1}{2}$. We further have

$$\begin{aligned}&g_0(p_H=\frac{1}{2})\\&=(\frac{1}{2})^2q^2(\frac{1}{2}+(1-\frac{1}{2})q)^2-(\frac{1}{2})^3((1-\frac{1}{2})(1-q)+\frac{1}{2}q)^2.\end{aligned}$$

By checking derivatives of $g_0(p_H=\frac{1}{2})$ on q, we have $g_0(p_H=\frac{1}{2})$ increases with q and $g_0(p_H=\frac{1}{2})<0$ for $q<\frac{\sqrt{1+2\sqrt{2}}-1}{2}$, $g_0(p_H=\frac{1}{2})=0$ for $q=\frac{\sqrt{1+2\sqrt{2}}-1}{2}$ and $g_0(p_H=\frac{1}{2})>0$ for $q>\frac{\sqrt{1+2\sqrt{2}}-1}{2}$. We thus have $p_{1,L}<\frac{1}{2}$ if $q<\frac{\sqrt{1+2\sqrt{2}}-1}{2}$, $p_{1,L}=\frac{1}{2}$ if $q=\frac{\sqrt{1+2\sqrt{2}}-1}{2}$ and $p_{1,L}>\frac{1}{2}$ if $q>\frac{\sqrt{1+2\sqrt{2}}-1}{2}$.

After summarizing the above results, we have PBE as shown in the proposition. □

Appendix B
Proof of Proposition 3

Proof. According to the PBE in Propositions 1 and 2, we derive the user's expected cost according to (17), (20), and (23), where only strategies 2-4 can be chosen without honest strategy 1. Then we further include the platform's expected cost and write down expected social cost in (7) as follows:

$$\bar{U}_2^*=q(\theta_H-\theta_L)^2, \tag{34}$$

$$\bar{U}_3^*=\frac{q(1+2p_H^2q-p_H(1+q))}{1+p_H(q-1)}(\theta_H-\theta_L)^2, \tag{35}$$

$$\bar{U}_4^*=\frac{q(p_H+q+p_Hq(2p_H-3))}{p_H+q-p_Hq}(\theta_H-\theta_L)^2. \tag{36}$$

According to (8), we have expected social cost under social optimum with $N=1$ as follows:

$$\bar{U}_1^{**}=\frac{q}{2}(\theta_H-\theta_L)^2.$$

To analyze PoA in the worst case, we need to compare (34)-(36) to find the maximum. Note

$$\begin{aligned}\bar{U}_2^*-\bar{U}_3^*&=\frac{2p_Hq^2(1-p_H)}{1-p_H(1-q)}(\theta_H-\theta_L)^2>0,\\ \bar{U}_2^*-\bar{U}_4^*&=\frac{2p_Hq^2(1-p_H)}{p_H(1-q)+q}(\theta_H-\theta_L)^2>0,\end{aligned}$$

which tells that $\bar{U}_2^*$ is the maximum. Therefore, we have

$$PoA=\max_{p_H,q,\theta_H,\theta_L}\frac{\bar{U}_2^*}{\bar{U}_1^{**}}=\max_{p_H,q,\theta_H,\theta_L}\frac{q(\theta_H-\theta_L)^2}{\frac{q}{2}(\theta_H-\theta_L)^2}=2. \quad\square$$

APPENDIX C
PROOF OF COROLLARY 1

A. PBE with One user's Imperfect State Observation and Proof of PBE

Due to existence of observation error, the user's expected cost in (2) becomes:

$$\bar{u}_S(b) = \sum_{m\in\{\theta_H,\theta_L\}} \sum_{j\in\{\theta_H,\theta_L\}} \sum_{j'\in\{\theta_H,\theta_L\}} Pr(\theta = j)Pr(\theta' = j'|\theta = j)Pr(m|\theta' = j')u_S(a(m), b). \tag{37}$$

The platform's expected cost in (4) becomes:

$$\bar{u}_R = \sum_{m\in\{\theta_H,\theta_L\}} \sum_{j\in\{\theta_H,\theta_L\}} \sum_{j'\in\{\theta_H,\theta_L\}} Pr(\theta = j)Pr(\theta' = j'|\theta = j)Pr(m|\theta' = j')u_R(a(m), \theta). \tag{38}$$

After receiving the user's message m, the platform updates her posterior state belief according to Bayes' theorem as shown in (10), but the truthful reporting probability changes to:

$$Pr(m = \theta_i|\theta = \theta_i) = \sum_{j\in\{H,L\}} Pr(m = \theta_i, \theta' = \theta_j|\theta = \theta_i) = \sum_{j\in\{H,L\}} Pr(m = \theta_i|\theta' = \theta_j)Pr(\theta' = \theta_j|\theta = \theta_i), \tag{39}$$

and

$$Pr(m = \theta_i|\theta' = \theta_j) = \sum_{l\in\{\theta_L,\theta,\theta_H\}} Pr(m = \theta_i, b = l|\theta' = \theta_j) = \sum_{l\in\{\theta_L,\theta,\theta_H\}} Pr(m = \theta_i|\theta' = \theta_j, b = l)Pr(b = l). \tag{40}$$

Analysis on equilibrium here follows same steps as in Section III-A. We first give some definitions on boundary and denote $q = q_0 \in (0, \frac{1}{2})$ is the unique solution to

$$-1 + q_0 + 4q_0^2 + 2q_0^3 = 0,$$

and $\epsilon = \bar{\epsilon} \in (0, \frac{1}{2})$ is the unique solution to

$$(1-\bar{\epsilon}+\bar{\epsilon}q)^2+\bar{\epsilon}^2(1+q)^2-2(\bar{\epsilon}(1-q)+(1-\bar{\epsilon})q)^2(1+q)^2 = 0,$$

where $q \in (q_0, \frac{\sqrt{1+2\sqrt{2}}-1}{2})$.

Proposition 7. *For the one-user case with small bias probability $q < q_0$ and high-state probability $p_H = \frac{1}{2}$, we have unique PBE that the biased user chooses maximum dishonest strategy 2: $m^*(\theta'|b = \theta_L) = \theta_L$ and $m^*(\theta'|b = \theta_H) = \theta_H$. The platform's equilibrium action $a^*(m) =$*

$$\begin{cases} \big((1-\epsilon)(1-q)+\epsilon q\big)\theta_H + \big((1-\epsilon)q+\epsilon(1-q)\big)\theta_L, & \textit{if } m = \theta_H, \\ \big((1-\epsilon)q+\epsilon(1-q)\big)\theta_H + \big((1-\epsilon)(1-q)+\epsilon q\big)\theta_L, & \textit{if } m = \theta_L. \end{cases} \tag{41}$$

Proposition 8. *For the one-user case with medium bias probability $q \in [q_0, \frac{\sqrt{1+2\sqrt{2}}-1}{2})$ and high-state probability $p_H = \frac{1}{2}$, PBE depends on user's observation error ϵ in the following.*

- *If $\epsilon \le \bar{\epsilon}$, the biased user chooses maximum dishonest strategy 2: $m^*(\theta'|b = \theta_L) = \theta_L$ and $m^*(\theta'|b = \theta_H) = \theta_H$. The platform's equilibrium action is same as* (41).
- *If $\epsilon > \bar{\epsilon}$, we have two PBEs. The first PBE is that the biased user chooses dishonest $b = \theta_H$ only strategy ($m^*(\theta'|b = \theta_L) = \theta$ and $m^*(\theta'|b = \theta_H) = \theta_H$), and the platform's equilibrium action is*

$$a^*(m) = \begin{cases} (1-\epsilon)\theta_H + \epsilon\theta_L, & \textit{if } m = \theta_H, \\ \frac{((1-\epsilon)q+\epsilon)\theta_H+(1-\epsilon+\epsilon q)\theta_L}{1+q}, & \textit{if } m = \theta_L. \end{cases} \tag{42}$$

The second PBE is that the biased user chooses dishonest $b = \theta_L$ only strategy ($m^*(\theta|b = \theta_L) = \theta_L$ and $m^*(\theta|b = \theta_H) = \theta$), and the platform's equilibrium action is

$$a^*(m) = \begin{cases} \frac{(1-\epsilon+\epsilon q)\theta_H+((1-\epsilon)q+\epsilon)\theta_L}{1+q}, & \textit{if } m = \theta_H, \\ \epsilon\theta_H + (1-\epsilon)\theta_L, & \textit{if } m = \theta_L. \end{cases} \tag{43}$$

Proposition 9. *For the one-user case with large bias probability $q \ge \frac{\sqrt{1+2\sqrt{2}}-1}{2}$ and high-state probability $p_H = \frac{1}{2}$, we have two PBEs. The first PBE is that the biased user chooses dishonest $b = \theta_H$ only strategy ($m^*(\theta'|b = \theta_L) = \theta$ and $m^*(\theta'|b = \theta_H) = \theta_H$), and the platform's equilibrium action is same as* (42)*. The second PBE is that the biased user chooses dishonest $b = \theta_L$ only strategy ($m^*(\theta|b = \theta_L) = \theta_L$ and $m^*(\theta|b = \theta_H) = \theta$), and the platform's equilibrium action is same as* (43).

Proof. Lemma 1 holds here and the biased user still chooses one out of strategies 2-4 in Section III-A. Note that only unbiased user uses honest strategy 1.

Next, we examine the platform's best response to the user's four possible strategies, respectively, and will examine the user's strategy choice later. First, assuming the user will adopt honest strategy 1, the platform expects:

$$Pr(m = \theta'|\theta', b) = 1. \tag{44}$$

Substituting (44) into (39) and (40), the platform's best-response action in (5) to strategy 1 is to predict

$$a_1^*(m) = \begin{cases} \frac{p_H(1-\epsilon)\theta_H+(1-p_H)\epsilon\theta_L}{p_H(1-\epsilon)+(1-p_H)\epsilon}, & \text{if } m = \theta_H, \\ \frac{p_H\epsilon\theta_H+(1-p_H)(1-\epsilon)\theta_L}{p_H\epsilon+(1-p_H)(1-\epsilon)}, & \text{if } m = \theta_L. \end{cases}$$

Then the user's expected cost in (37) by using honest strategy 1 above is

$$\bar{u}_1(b) = \begin{cases} \left(\frac{(p_H)^2(1-\epsilon)^2}{p_H(1-\epsilon)+(1-p_H)\epsilon} + \frac{(p_H)^2\epsilon^2}{p_H\epsilon+(1-p_H)(1-\epsilon)}\right)(\theta_H - \theta_L)^2, & \text{if } b = \theta_L, \\ \left(\frac{(1-p_H)^2\epsilon^2}{p_H(1-\epsilon)+(1-p_H)\epsilon} + \frac{(1-p_H)^2(1-\epsilon)^2}{p_H\epsilon+(1-p_H)(1-\epsilon)}\right)(\theta_H - \theta_L)^2, & \text{if } b = \theta_H. \end{cases} \tag{45}$$

Similar to Section III-B, we can obtain the platform's best-response learning action in (5) and each user i's expected cost in (37) under strategy 2 as follows:

$$a_2^*(m=\theta_H)= \frac{p_H\big((1-\epsilon)(1-q)+(\epsilon)q\big)\theta_H}{p_H\big((1-\epsilon)(1-q)+\epsilon q\big)+(1-p_H)\big((1-\epsilon)q+\epsilon(1-q)\big)} + \frac{(1-p_H)\big((1-\epsilon)q+\epsilon(1-q)\big)\theta_L}{p_H\big((1-\epsilon)(1-q)+\epsilon q\big)+(1-p_H)\big((1-\epsilon)q+\epsilon(1-q)\big)},$$

$$a_2^*(m=\theta_L)= \frac{p_H\big((1-\epsilon)q+\epsilon(1-q)\big)\theta_H}{p_H\big((1-\epsilon)q+\epsilon(1-q)\big)+(1-p_H)\big((1-\epsilon)(1-q)+\epsilon q\big)} + \frac{(1-p_H)\big((1-\epsilon)(1-q)+\epsilon q\big)\theta_L}{p_H\big((1-\epsilon)q+\epsilon(1-q)\big)+(1-p_H)\big((1-\epsilon)(1-q)+\epsilon q\big)}.$$

$$\bar{u}_2(b)=\begin{cases}\left(\frac{p_H\big((1-\epsilon)q+\epsilon(1-q)\big)(\theta_H-\theta_L)}{p_H\big((1-\epsilon)q+\epsilon(1-q)\big)+(1-p_H)\big((1-\epsilon)(1-q)+\epsilon q\big)}\right)^2, & \text{if } b=\theta_L,\\ \left(\frac{(1-p_H)\big((1-\epsilon)q+\epsilon(1-q)\big)(\theta_H-\theta_L)}{p_H\big((1-\epsilon)(1-q)+\epsilon q\big)+(1-p_H)\big((1-\epsilon)q+\epsilon(1-q)\big)}\right)^2, & \text{if } b=\theta_H,\end{cases} \tag{46}$$

they under strategy 3 are as follows:

$$a_3^*(m=\theta_H)= \frac{p_H(1-q)(1-\epsilon)\theta_H+(1-p_H)\epsilon(1-q)\theta_L}{p_H(1-q)(1-\epsilon)+(1-p_H)\epsilon(1-q)},$$

$$a_3^*(m=\theta_L)= \frac{p_H((1-\epsilon)q+\epsilon)\theta_H+(1-p_H)((1-\epsilon)+\epsilon q)\theta_L}{p_H((1-\epsilon)q+\epsilon)+(1-p_H)((1-\epsilon)+\epsilon q)}.$$

$$\bar{u}_3(b=\theta_L)= \left(\frac{p_H((1-\epsilon)q+\epsilon)(\theta_H-\theta_L)}{p_H((1-\epsilon)q+\epsilon)+(1-p_H)((1-\epsilon)+\epsilon q)}\right)^2, \tag{47}$$

$$\bar{u}_3(b=\theta_H)= (p_H(1-\epsilon)+(1-p_H)\epsilon)\left(\frac{(1-p_H)(\epsilon)(\theta_H-\theta_L)}{p_H(1-\epsilon)+(1-p_H)\epsilon}\right)^2 + (p_H\epsilon+(1-p_H)(1-\epsilon)) \cdot\left(\frac{(1-p_H)((1-\epsilon)+\epsilon q)(\theta_H-\theta_L)}{p_H((1-\epsilon)q+\epsilon)+(1-p_H)((1-\epsilon)+\epsilon q)}\right)^2, \tag{48}$$

they under strategy 4 are as follows:

$$a_4^*(m=\theta_H)= \frac{p_H((1-\epsilon)+\epsilon q)\theta_H+(1-p_H)((1-\epsilon)q+\epsilon)\theta_L}{p_H((1-\epsilon)+\epsilon q)+(1-p_H)((1-\epsilon)q+\epsilon)},$$

$$a_4^*(m=\theta_L)=\frac{p_H\epsilon\theta_H+(1-p_H)(1-\epsilon)\theta_L}{p_H\epsilon+(1-p_H)(1-\epsilon)}.$$

$$\bar{u}_4(b=\theta_L)= (p_H\epsilon+(1-p_H)(1-\epsilon))\left(\frac{p_H\epsilon(\theta_H-\theta_L)}{p_H\epsilon+(1-p_H)(1-\epsilon)}\right)^2 + (p_H(1-\epsilon)+(1-p_H)\epsilon) \cdot\left(\frac{p_H((1-\epsilon)+\epsilon q)(\theta_H-\theta_L)}{p_H((1-\epsilon)+\epsilon q)+(1-p_H)(\epsilon+(1-\epsilon)q)}\right)^2, \tag{49}$$

$$\bar{u}_4(b=\theta_H)= \left(\frac{(1-p_H)((1-\epsilon)q+\epsilon)(\theta_H-\theta_L)}{p_H((1-\epsilon)+\epsilon q)+(1-p_H)((1-\epsilon)q+\epsilon)}\right)^2. \tag{50}$$

If $p_H=\frac{1}{2}$, user 1 with biases $b=\theta_H$ and $b=\theta_L$ are symmetric. we have user 1 with bias θ_H or θ_L has expected cost under 4 different strategies as follows:

$$\begin{aligned}&\bar{u}_1(b=\theta_H)=\bar{u}_1(b=\theta_L)\\ =&\frac{1}{2}(\epsilon^2+(1-\epsilon)^2)(\theta_H-\theta_L)^2,\\ &\bar{u}_2(b=\theta_H)=\bar{u}_2(b=\theta_L)\\ =&(\epsilon(1-q)+(1-\epsilon)q)^2(\theta_H-\theta_L)^2,\\ &\bar{u}_3(b=\theta_H)=\bar{u}_3(b=\theta_L)\\ =&\frac{1}{2}\left(\epsilon^2+\left(\frac{1-\epsilon+\epsilon q}{1+q}\right)^2\right)(\theta_H-\theta_L)^2,\\ &\bar{u}_4(b=\theta_H)=\bar{u}_4(b=\theta_L)\\ =&\left(\frac{(1-\epsilon)q+\epsilon}{1+q}\right)^2(\theta_H-\theta_L)^2.\end{aligned}$$

To show honest strategy 1 is not an equilibrium, we have

$$\begin{aligned}&\bar{u}_1(b=\theta_H)-\bar{u}_4(b=\theta_H)\\ =&\left(\frac{1}{2}(\epsilon^2+(1-\epsilon)^2)-\left(\frac{(1-\epsilon)q+\epsilon}{1+q}\right)^2\right)(\theta_H-\theta_L)^2\\ \geq&\left(\frac{1}{2}(\epsilon^2+(1-\epsilon)^2)-\left(\frac{1}{2}\right)^2\right)(\theta_H-\theta_L)^2\\ \geq&0,\end{aligned}$$

where we have the first inequality due to $(\frac{(1-\epsilon)q+\epsilon}{1+q})^2$ increases with q for $\epsilon\leq\frac{1}{2}$ and we thus take $q=1$, and the second due to $\epsilon\leq\frac{1}{2}$. Due to symmetry, we have

$$\bar{u}_1(b=\theta_L)-\bar{u}_3(b=\theta_L)\geq 0.$$

We then want to check

$$\begin{aligned}&\bar{u}_3(b=\theta_H)-\bar{u}_2(b=\theta_H)\\ =&\left(\frac{1}{2}\left(\epsilon^2+\left(\frac{1-\epsilon+\epsilon q}{1+q}\right)^2\right)-(\epsilon(1-q)+(1-\epsilon)q)^2\right)(\theta_H-\theta_L)^2\\ =&\left(\frac{(1-\epsilon+\epsilon q)^2+\epsilon^2(1+q)^2-2(\epsilon(1-q)+(1-\epsilon)q)^2(1+q)^2}{2(1+q)^2}\right)(\theta_H-\theta_L)^2.\end{aligned}$$

Denote the above numerator as

$$g(\epsilon) =(1-\epsilon+\epsilon q)^2+\epsilon^2(1+q)^2 \\ -2(\epsilon(1-q)+(1-\epsilon)q)^2(1+q)^2.$$

We take second derivative on ϵ over $g(\epsilon)$ as follows:

$$g''(\epsilon)=8q(1+2q-2q^2-2q^3)>0, q\leq \frac{1}{2},$$

which implies $g'(\epsilon)$ increases with $\epsilon \leq \frac{1}{2}$. Since

$$g'(0)=-2-2q+12q^3+8q^4<0, q\leq\frac{1}{2},$$
$$g'(\frac{1}{2})=2(-1+q+4q^2+2q^3)\begin{cases}<0, & \text{if } q<q_0,\\ \geq 0, & \text{if } q\geq q_0.\end{cases}$$

where $q_0<\frac{1}{2}$ is the unique solution to

$$-1+q_0+4q_0^2+2q_0^3=0,$$

we then have $g'(\epsilon)<0$ if $q<q_0$, and $g'(\epsilon)<0$ then $g'(\epsilon)\geq 0$ if $q\geq q_0$, implying $g(\epsilon)$ decreases with $\epsilon\leq\frac{1}{2}$ if $q<q_0$ and first decreases then increases otherwise. Since

$$g(0)=1-2q^2-4q^3-2q^4\begin{cases}>0, & \text{if } q<\frac{\sqrt{1+2\sqrt{2}}-1}{2},\\ \leq 0, & \text{if } q\geq\frac{\sqrt{1+2\sqrt{2}}-1}{2},\end{cases}$$
$$g(\frac{1}{2})=0,$$

we then have $g(\epsilon)\geq 0$ if $q\leq q_0$, $g(\epsilon)\geq 0$ then $g(\epsilon)<0$ if $q\in(q_0,\frac{\sqrt{1+2\sqrt{2}}-1}{2})$, and $g(\epsilon)<0$ if $q\geq\frac{\sqrt{1+2\sqrt{2}}-1}{2}$. Finally, we have

$$\bar{u}_3(b=\theta_H)-\bar{u}_2(b=\theta_H)=\bar{u}_4(b=\theta_L)-\bar{u}_2(b=\theta_L)$$
$$=\begin{cases}\geq 0, & \text{if } q\leq q_0 \text{ or } q\in(q_0,\frac{\sqrt{1+2\sqrt{2}}-1}{2}) \text{ and } \epsilon\leq\bar{\epsilon},\\ <0, & \text{otherwise..}\end{cases}$$

We then have equilibrium as shown in the proposition. □

B. Proof of PoA of Corollary 1

Proof. According to analysis in Propositions 7-9, we have expected social cost at PBE in (7) under user's strategies 2 and 4 as shown in (51)-(53).

We have expected social cost under social optimum with one user as shown in (54), where the platform's action to predict is

$$a^{**}(\theta_{j'})=\arg\min_a \sum_{j\in\{L,H\}} Pr(\theta=\theta_j|\theta'=\theta_{j'}) \\ \cdot\Big((a-\theta_j)^2+(a-b)^2\Big).$$

Denote r_i as the maximum ratio between $\bar{U}^*_{i'}$ and $\bar{U}^{**}_{1'}$ as follows:

$$r_i=\max_{q,p_H,\epsilon,\theta_H,\theta_L}\frac{\bar{U}^*_{i'}}{\bar{U}^{**}_{1'}},\ i\in\{2,3,4\}.$$

By checking derivatives of $r_2(\epsilon)$ on ϵ, we have $r_2(\epsilon)$ first decreases then increases with $\epsilon\in[0,\frac{1}{2}]$, which implies

$$r_2(\epsilon)\leq\max\{r_2(0),r_2(\frac{1}{2})\}=2,$$

which is achievable at $\epsilon=0$, $q\leq\frac{\sqrt{1+2\sqrt{2}}-1}{2}$ and $p_H\in[p_{1,L},p_{1,H}]$.

By following same methods to show $r_2=2$, we have

$$r_3\leq 2,\ r_4\leq 2.$$

Finally, PoA is

$$PoA=\max\{r_2,r_3,r_4\}=2.$$

□

Appendix D
Proof of Proposition 4

Proof. We will then show user i's strategy 1 is not an equilibrium for each user with bias θ_L or θ_H. With $N=2$, we have

$$l_0(q):=\frac{\bar{u}_1(b_i=\theta_H)-\bar{u}_4(b_i=\theta_H)}{(\theta_H-\theta_L)^2} \\ =\Bigg((1-p_H)q-(p_H+(1-p_H)q)\Big(\frac{(1-p_H)q^2}{p_H+(1-p_H)q^2}\Big)^2\Bigg).$$

By checking derivatives of $l_0(q)$ on q, we have $l_0(q)$ increases with q and thus $l_0(q)\geq l_0(0)=0$. Each user with bias θ_H thus deviates from strategy 1. We also have

$$l_1(q):=\frac{\bar{u}_1(b_i=\theta_L)-\bar{u}_3(b_i=\theta_L)}{(\theta_H-\theta_L)^2} \\ =p_Hq-(p_Hq+1-p_H)\Big(\frac{q^2p_H}{q^2p_H+(1-p_H)}\Big)^2.$$

By checking derivatives of $l_1(q)$ on q, we have $l_1(q)$ increases with q and thus $l_1(q)\geq l_1(0)=0$. Each user with bias θ_L thus deviates from strategy 1. Therefore, user i's honest strategy 1 is never an equilibrium.

We will then find condition for user i with bias θ_H prefers to always send high-state message than truthfully message, i.e., $\bar{u}_2(b_i=\theta_H)$ in (25) is greater than $\bar{u}_3(b_i=\theta_H)$ in (28) with $N=2$, which is equal to

$$\frac{1}{((1-q)^2p_H+q^2(1-p_H))^2(1-p_H+p_Hq^2)^2} \\ \cdot\Bigg((p_H(1-q)+(1-p_H)q)q^4(1-p_H)^2(1-p_H+p_Hq^2)^2 \\ -(1-p_H)^3((1-q)^2p_H+q^2(1-p_H))^2 \\ +(p_Hq+(1-p_H)(1-q))(1-p_H)^2 \\ \cdot((1-q)^2p_H+q^2(1-p_H))^2(1-p_H+p_Hq^2)^2\Bigg)\leq 0.$$

$$\bar{U}^{*}_{2'} = (\theta_H-\theta_L)^2 \Bigg(\frac{p_H^2(-(2q-1))\left(2q^2(1-2\epsilon)^2+q\left(-8\epsilon^2+6\epsilon-1\right)+2(\epsilon-1)\epsilon\right)+p_H(2q-1)\left(2q^2(1-2\epsilon)^2+q\left(-8\epsilon^2+6\epsilon-1\right)+2(\epsilon-1)\epsilon\right)}{(p_H(2q-1)(2\epsilon-1)-2q\epsilon+q+\epsilon-1)(p_H(2q-1)(2\epsilon-1)-2q\epsilon+q+\epsilon)} + \frac{q\left(-q^2(1-2\epsilon)^2+q(1-2\epsilon)^2-(\epsilon-1)\epsilon\right)}{(p_H(2q-1)(2\epsilon-1)-2q\epsilon+q+\epsilon-1)(p_H(2q-1)(2\epsilon-1)-2q\epsilon+q+\epsilon)}\Bigg), \tag{51}$$

$$\bar{U}^{*}_{3'} = -(\theta_H-\theta_L)^2 \Bigg(\frac{p_H^2\left(q^2\left(-4\epsilon^2-2\epsilon+1\right)+q\left(14\epsilon^2-10\epsilon+1\right)-2(\epsilon-1)\epsilon\right)+2p_H^3q(2\epsilon-1)(q-\epsilon)+p_H\left(4q^2\epsilon^2+q\left(-10\epsilon^2+8\epsilon-1\right)+2(\epsilon-1)\epsilon\right)}{(p_H(2\epsilon-1)-\epsilon)(p_H(q-1)(2\epsilon-1)-q\epsilon+\epsilon-1)} + \frac{q\epsilon(-q\epsilon+\epsilon-1)}{(p_H(2\epsilon-1)-\epsilon)(p_H(q-1)(2\epsilon-1)-q\epsilon+\epsilon-1)}\Bigg), \tag{52}$$

$$\bar{U}^{*}_{4'} = -(\theta_H-\theta_L)^2 \Bigg(\frac{p_H^2\left(q^2\left(-4\epsilon^2+10\epsilon-5\right)+q\left(2\epsilon^2-4\epsilon+1\right)-2(\epsilon-1)\epsilon\right)-2p_H^3q(2\epsilon-1)(q-\epsilon)+p_H\left(4q^2(\epsilon-1)^2+q\left(-6\epsilon^2+6\epsilon-1\right)+2\epsilon(\epsilon-1)\right)}{(p_H(2\epsilon-1)-\epsilon+1)(p_H(q-1)(2\epsilon-1)+q(-\epsilon)+q+\epsilon)} - \frac{q(\epsilon-1)(q(\epsilon-1)-\epsilon)}{(p_H(2\epsilon-1)-\epsilon+1)(p_H(q-1)(2\epsilon-1)+q(-\epsilon)+q+\epsilon)}\Bigg). \tag{53}$$

$$\begin{aligned}\bar{U}^{**}_{1'} &= \sum_{j\in\{L,H\}} Pr(\theta=\theta_j)\sum_{j'\in\{L,H\}} Pr(\theta'=\theta_{j'}|\theta=\theta_j)\Big((a^{**}(\theta_{j'})-\theta_j)^2+(a^{**}(\theta_{j'})-b)^2\Big)\\ &= \Bigg(\frac{p_H^2\left(q\left(10\epsilon^2-10\epsilon+1\right)-4(\epsilon-1)\epsilon\right)-p_Hq\left(10\epsilon^2-10\epsilon+1\right)+4p_H(\epsilon-1)\epsilon+q(\epsilon-1)\epsilon}{2(p_H(2\epsilon-1)-\epsilon)(p_H(2\epsilon-1)-\epsilon+1)}\Bigg)(\theta_H-\theta_L)^2.\end{aligned} \tag{54}$$

Denote the items in the big brackets as

$$\begin{aligned}&f(p_H)\\ &:= (p_H(1-q)+(1-p_H)q)q^4(1-p_H)^2(1-p_H+p_Hq^2)^2\\ &-(1-p_H)^3((1-q)^2p_H+q^2(1-p_H))^2\\ &+(p_Hq+(1-p_H)(1-q))(1-p_H)^2\\ &((1-q)^2p_H+q^2(1-p_H))^2(1-p_H+p_Hq^2)^2.\end{aligned}$$

After checking derivatives of $f(p_H)$ on p_H, we obtain that $f(p_H)\le 0$ if $p_H\in[0,p_{2,H}]$ and $f(p_H)\ge 0$ if $p_H\in(p_{2,H},1]$, where $q\in[0,\frac{1}{2}]$ and $p_{2,H}\in(0,1)$ satisfying $f(p_{2,H})=0$. therefore, the desired condition is $p_H\in[0,p_{2,H}]$. Since we have

$$\begin{aligned}&f(p_H=\frac{1}{2})=\frac{1}{32}\\ &(-3+12q-22q^2+16q^3+9q^4-20q^5+24q^6-8q^7+8q^8)\\ &<0, q\le\frac{1}{2},\end{aligned}$$

together with $f(p_H)\le 0$ for $p_H\in[0,p_{2,H}]$, we obtain $p_{2,H}>\frac{1}{2}$.

We will finally find condition for each user with bias θ_L prefers to always share low-state message than truthfully share, i.e., $\bar{u}_2(b_i=\theta_L)$ in (26) is greater than $\bar{u}_4(b_i=\theta_L)$ in (32) with $N=2$, which is equal to

$$\begin{aligned}&\frac{1}{(q^2p_H+(1-q)^2(1-p_H))^2(q^2(1-p_H)+p_H)^2}\\ &\cdot\Bigg((p_Hq+(1-p_H)(1-q))q^4p_H^2(q^2(1-p_H)+p_H)^2\\ &-p_H^3(q^2p_H+(1-q))^2+(p_H(1-q)\\ &\quad+(1-p_H)q)p_H^2(q^2p_H+(1-p_H))^2(1-p_H))^2)\Bigg)\le 0.\end{aligned}$$

Denote the items in the big brackets as

$$\begin{aligned}&g(p_H):=\\ &(p_Hq+(1-p_H)(1-q))q^4p_H^2(q^2(1-p_H)+p_H)^2\\ &-p_H^3(q^2p_H+(1-q)^2(1-p_H))^2\\ &+(p_H(1-q)+(1-p_H)q)p_H^2\\ &\quad\cdot(q^2p_H+(1-q)^2(1-p_H))^2(q^2(1-p_H)+p_H)^2.\end{aligned}$$

After checking derivatives of $g(p_H)$ on p_H, we obtain that $g(p_H)\ge 0$ if $p_H\in[0,p_{2,L}]$ and $g(p_H)\le 0$ if $p_H\in(p_{2,L},1]$, where $q\in[0,\frac{1}{2}]$ and $p_{2,L}\in(0,1)$ satisfying $g(p_{2,L})=0$. Therefore, the desired condition is $p_H\in[p_{2,L},1]$. Since we have

$$\begin{aligned}&g(p_H=\frac{1}{2})=\frac{1}{32}\\ &(-3+12q-22q^2+16q^3+9q^4-20q^5+24q^6-8q^7+8q^8)\\ &<0, q\le\frac{1}{2},\end{aligned}$$

together with $g(p_H)\le 0$ for $p_H\in(p_{2,L},1]$, we obtain $p_{2,L}<\frac{1}{2}$. Since $p_{2,H}>\frac{1}{2}$, we have

$$p_{2,L}<\frac{1}{2}<p_{2,H}.$$

After summarizing all the obtained results, we have PBE under different cases as shown in the proposition. $\square$

APPENDIX E
PROOF OF COROLLARY 2

Proof. We will first show the platform's expected cost decreases with one more user than one if $p_H \in [0, p_{2,L}] \cup [p_{2,H}, 1]$ and $q \leq \frac{\sqrt{1+2\sqrt{2}}-1}{2}$ by showing that she obtains more under honest $b = \theta_H$ and $b = \theta_L$ only PBE with two users than any PBE with one, respectively. According to Propositions 1 and 2, the platform's expected cost under different PBEs are as follows:

$$\begin{aligned}
&\bar{u}_R(N=1, \text{Maximum Dishonest}) = (\theta_H - \theta_L)^2 \\
&\frac{p_H(1-p_H)q(1-q)}{(p_H(1-q)+(1-p_H)q)(p_H q + (1-p_H)(1-q))}, \\
&\bar{u}_R(N=1, \text{Dishonest } b=\theta_H \text{ Only}) \\
&= \frac{p_H(1-p_H)q}{p_H + (1-p_H)q}(\theta_H - \theta_L)^2, \\
&\bar{u}_R(N=1, \text{Dishonest } b=\theta_L \text{ Only}) \\
&= \frac{p_H(1-p_H)q}{1-p_H+p_H q}(\theta_H - \theta_L)^2.
\end{aligned}$$

According to Proposition 4, the platform's expected cost under different PBEs are as follows:

$$\begin{aligned}
&\bar{u}_R(N=2, \text{Maximum Dishonest}) = (\theta_H - \theta_L)^2 \Bigg(\\
&\frac{p_H(1-p_H)(1-q)^2 q^2 \big(q^2 + (1-q)^2\big)}{(p_H(1-q)^2 + (1-p_H)q^2)(p_H q^2 + (1-p_H)(1-q)^2)} \\
&- 2p_H(1-p_H)(1-q)q \Bigg), p_H \in [p_{2,L}, p_{2,H}], \\
&\bar{u}_R(N=2, \text{Dishonest } b_i=\theta_H \text{ Only}) \\
&= \frac{p_H(1-p_H)q^2}{p_H + (1-p_H)q^2}(\theta_H - \theta_L)^2, p_H \leq p_{2,L}, \\
&\bar{u}_R(N=2, \text{Dishonest } b_i=\theta_L \text{ Only}) \\
&= \frac{p_H(1-p_H)q^2}{1-p_H+p_H q^2}(\theta_H - \theta_L)^2, p_H \geq p_{2,H}.
\end{aligned}$$

We will first show for all $p_H \in [0,1]$ and $q \in [0, \frac{1}{2}]$, we always have

$$\begin{aligned}
&\bar{u}_R(N=2, \text{Dishonest } b_i=\theta_H \text{ Only}) \\
\leq &\bar{u}_R(N=1, \text{Maximum Dishonest}).
\end{aligned}$$

Denote

$$\begin{aligned}
f(p_H) :=& \frac{\bar{u}_R(N=2, \text{Dishonest } b_i=\theta_H \text{ Only})}{(\theta_H - \theta_L)^2} \\
&- \frac{\bar{u}_R(N=1, \text{Maximum Dishonest})}{(\theta_H - \theta_L)^2}.
\end{aligned}$$

First derivative of $f(p_H)$ on p_H is as follows:

$$\begin{aligned}
&f'(p_H) = p_H q^2 \Bigg(\\
&\frac{p_H^5(1-2q)^4(-1+q^2) - 2p_H^4(1-2q)^4(-1+2q^2)}{(p_H+q-2p_Hq)^2(p_H+q^2-p_Hq^2)^2(1-q+p_H(-1+2q))^2} \\
&\frac{-2q^2(1-3q+5q^2-6q^3+3q^4)}{(p_H+q-2p_Hq)^2(p_H+q^2-p_Hq^2)^2(1-q+p_H(-1+2q))^2} \\
&\frac{+p_H^3(1-2q)^2(-1+6q-26q^3+26q^4)}{(p_H+q-2p_Hq)^2(p_H+q^2-p_Hq^2)^2(1-q+p_H(-1+2q))^2} \\
&\frac{+p_H^2(2-6q+4q^2+30q^3-120q^4+172q^5-86q^6)}{(p_H+q-2p_Hq)^2(p_H+q^2-p_Hq^2)^2(1-q+p_H(-1+2q))^2} \\
&\frac{+p_H(-1+2q+5q^2-24q^3+55q^4-72q^5+36q^6)}{(p_H+q-2p_Hq)^2(p_H+q^2-p_Hq^2)^2(1-q+p_H(-1+2q))^2} \Bigg).
\end{aligned}$$

Since denominator of $f'(p_H)$ is always positive, we focus on its numerator and define it as

$$\begin{aligned}
&f_1(p_H) := p_H q^2 \big(p_H^5(1-2q)^4(-1+q^2) \\
&- 2p_H^4(1-2q)^4(-1+2q^2) - 2q^2(1-3q+5q^2-6q^3+3q^4) \\
&+ p_H^3(1-2q)^2(-1+6q-26q^3+26q^4) \\
&+ p_H^2(2-6q+4q^2+30q^3-120q^4+172q^5-86q^6) \\
&+ p_H(-1+2q+5q^2-24q^3+55q^4-72q^5+36q^6)\big).
\end{aligned}$$

After checking derivatives of $f_1(p_H)$ on p_H, we have $f_1(p_H)$ first decreases then increase with $p_H \in [0,1]$. Since

$$\begin{aligned}
&f_1(0) = 0,\ q \in \left[0, \frac{1}{2}\right], \\
&f_1(1) = (1-q)^3 q^2 (1+q) \geq 0,\ q \in \left[0, \frac{1}{2}\right],
\end{aligned}$$

we then have $f_1(p_H) \leq 0$ then $f_1(p_H) \geq 0$ as p_H increases from 0 to 1, which means $f'(p_H) \leq 0$ then $f'(p_H) \geq 0$ for $p_H \in [0,1]$. We finally have $f(p_H)$ first decreases then increases with $p_H \in [0,1]$. Finally,

$$f(p_H) \leq f(0) = f(1) = 0.$$

According to definition of $f(p_H)$, we obtain

$$\begin{aligned}
&\bar{u}_R(N=2, \text{Dishonest } b_i=\theta_H \text{ Only}) \\
\leq &\bar{u}_R(N=1, \text{Maximum Dishonest}).
\end{aligned}$$

To show

$$\begin{aligned}
&\bar{u}_R(N=2, \text{Dishonest } b_i=\theta_H \text{ Only}) \\
\leq &\bar{u}_R(N=1, \text{Dishonest } b=\theta_H \text{ Only}),
\end{aligned}$$

we have

$$\begin{aligned}
&\bar{u}_R(N=2, \text{Dishonest } b_i=\theta_H \text{ Only}) \\
-&\bar{u}_R(N=1, \text{Dishonest } b=\theta_H \text{ Only}) \\
=&-\frac{(1-p_H)p_H^2(1-q)q(\theta_H-\theta_L)^2}{(p_H(1-q)+q)(q^2+p_H(1-q^2))} \leq 0
\end{aligned}$$

for any $p_H \in [0,1], q \in [0, \frac{1}{2}]$. Regarding $p_H \leq p_{2,L}$, with one user the PBE cannot be Dishonest $b_i = \theta_L$ Only, so we have shown that the platform incurs smaller cost under Dishonest

$b_i = \theta_H$ Only PBE with two users than any PBE with one for $q \leq \frac{\sqrt{1+2\sqrt{2}}-1}{2}$. We can use the same methods to show that the platform incurs smaller cost under Dishonest $b_i = \theta_L$ Only PBE with two users than any PBE with one for $p_H \geq p_{2,H}$ and $q \leq \frac{\sqrt{1+2\sqrt{2}}-1}{2}$.

We next need to show the platform incurs greater cost under Maximum Dishonest PBE with two users than any PBE with one for $q > \frac{\sqrt{1+2\sqrt{2}}-1}{2}$. By applying same methods as above, we have for $p_H \in (p_{2,L}, p_{2,H})$ and $q > \frac{\sqrt{1+2\sqrt{2}}-1}{2}$,

$$\begin{aligned}&\bar{u}_R(N=2, \text{Maximum Dishonest})\\ >&\bar{u}_R(N=1, \text{Dishonest } b=\theta_H \text{ Only}),\end{aligned}$$

and

$$\begin{aligned}&\bar{u}_R(N=2, \text{Maximum Dishonest})\\ >&\bar{u}_R(N=1, \text{Dishonest } b=\theta_L \text{ Only}).\end{aligned}$$

Finally, we have the platform incurs greater cost under Maximum Dishonest PBE with two users than PBE with one for $q > \frac{\sqrt{1+2\sqrt{2}}-1}{2}$ and $p_H \in (p_{2,L}, p_{2,H})$. □

APPENDIX F
PROOF OF PROPOSITION 5

Proof. According to analysis in Proposition 4, we have expected social cost at PBE in (7) as follows:

$$\begin{aligned}\bar{U}_2^* =& (\theta_H-\theta_L)^2\\ &\Big(\frac{q(-4p_H^3(1-2q)^2(-1+q)+2p_H^4(1-2q)^2(-1+q))}{((p_H-2p_Hq+q^2)((-1+q)^2+p_H(-1+2q)))}\\ &+\frac{q(2(-1+q)^2q^2-p_H(-2+9q-10q^2+q^3))}{((p_H-2p_Hq+q^2)((-1+q)^2+p_H(-1+2q)))}\\ &+\frac{q(p_H^2(-4+19q-26q^2+9q^3))}{((p_H-2p_Hq+q^2)((-1+q)^2+p_H(-1+2q)))}\Big),\end{aligned} \tag{55}$$

$$\bar{U}_3^* = \frac{q(2+p_H^2q(1+4q)-p_H(2+q+2q^2))}{1+p_H(-1+q^2)}(\theta_H-\theta_L)^2, \tag{56}$$

$$\bar{U}_4^* = \frac{q(2q^2+p_H^2q(1+4q)-p_H(-2+q+6q^2)}{q^2+p_H(1-q^2)}(\theta_H-\theta_L)^2, \tag{57}$$

where $\bar{U}_j^*$ is under user i's strategy j, $j=2,3,4$. According to (8), we have expected social cost under social optimum with two users as follows:

$$\bar{U}_2^{**} = \frac{2(2q-q^2)}{3}(\theta_H-\theta_L)^2.$$

To show PoA=2, we will show each $\bar{U}_j^*/\bar{U}_2^{**} \leq 2$ according to (55)-(57).

We first define worst-case ratio r_3 between $\bar{U}_4^*$ in (57) and $\bar{U}^{**}$ as follows:

$$\begin{aligned}r_3 =& \max_{p_H,q}\frac{\bar{U}_4^*}{\bar{U}_2^{**}}\\ =& \max_{p_H,q}\frac{6q^2+3p_H^2q(1+4q)-3p_H(-2+q+6q^2)}{2(-2+q)(-q^2+p_H(-1+q^2))}.\end{aligned}$$

Define

$$h(p_H) := \frac{6q^2+3p_H^2q(1+4q)-3p_H(-2+q+6q^2)}{2(-2+q)(-q^2+p_H(-1+q^2))}.$$

First derivative of $h(p_H)$ on p_H is

$$h'(p_H) = \frac{3q(1+4q)(-q^2+2p_Hq^2+p_H^2(1-q^2))}{2(2-q)(p_H+q^2-p_Hq^2)^2}.$$

Since denominator of $h'(p_H)$ is positive, we further need to check its numerator and denote it as follows:

$$h_1(p_H) := 3q(1+4q)(-q^2+2p_Hq^2+p_H^2(1-q^2)).$$

By taking second derivative of $h_1(p_H)$ on p_H, we have

$$h_1''(p_H) = -6q(1+4q)(-1+q^2) > 0,\ q \in \big(0, \frac{1}{2}\big],$$

which means $h_1'(p_H)$ increases with $p_H \in [0,1]$. By taking first derivative of $h_1(p_H)$ on p_H, we have

$$h_1'(p_H) = -3q(1+4q)(-2q^2+2p_H(-1+q^2)).$$

Furthermore, we have $h_1'(0) > 0$ and $h_1'(1) > 0$, so $h_1'(p_H) > 0$ for $p_H \in [0,1]$ and $h_1(q)$ increases with p_H. Since $h_1(0) < 0$ and $h_1(1) > 0$, we have $h_1(p_H) > 0$ then $h_1(p_H) < 0$ as p_H increases from 0 to 1, which means $h'(p_H) > 0$ then $h'(p_H) < 0$. We finally have $h(p_H)$ first decreases then increases with $p_H \in [0,1]$, implying

$$h(p_H) \leq h(0) = h(1) = \frac{3}{2-q},$$

where $p_H = 0$ is achievable with with dishonest $b_i = \theta_H$ only strategy 4. Then ratio r_3 becomes

$$r_3 = \max_q \frac{3}{2-q} = \frac{3}{2-\frac{1}{2}} = 2,$$

where $q = \frac{1}{2}$ is achievable with dishonest θ_H only strategy 4.

Denote worst-case ratios r_1 and r_2 between $\bar{U}_j^*$ in (55)-(56), $j=2,3$, and $\bar{U}_2^{**}$, respectively, as follows:

$$\begin{aligned}r_1 &= \max_{p_H,q}\frac{\bar{U}_2^*}{\bar{U}_2^{**}},\\ r_2 &= \max_{p_H,q}\frac{\bar{U}_3^*}{\bar{U}_2^{**}}.\end{aligned}$$

By using same methods to show $r_3 = 2$, we obtain that $r_1 < 2$ and $r_2 = 2$. Then we have PoA as follows:

$$PoA = \max_{i=1,2,3}\{r_i\} = 2. \qquad \square$$

APPENDIX G
PROOF OF LEMMA 2

Proof. We will show the platform's action and each user i's expected cost strategy by strategy. Strategy 1 is clear as $N \rightarrow$

∞. With strategy 2, we have the platform's action in (24) as $N \to \infty$ as follows:

$$\begin{aligned}
&\lim_{N\to\infty} a_2^*(|I_H| = k)\\
=&\lim_{N\to\infty} \frac{p_H(1-q)^k q^{N-k}\theta_H + (1-p_H)q^k(1-q)^{N-k}\theta_L}{p_H(1-q)^k q^{N-k} + (1-p_H)q^k(1-q)^{N-k}}\\
=&\lim_{N\to\infty} \frac{\theta_H}{1+\frac{1-p_H}{p_H}\left(\frac{1-q}{q}\right)^{N-2k}} + \lim_{N\to\infty} \frac{\theta_L}{1+\frac{p_H}{1-p_H}\left(\frac{q}{1-q}\right)^{N-2k}}\\
=&\begin{cases} \theta_L, & \text{if } \frac{k}{N} < \frac{1}{2},\\ p_H\theta_H + (1-p_H)\theta_L, & \text{if } \frac{k}{N} = \frac{1}{2},\\ \theta_H, & \text{if } \frac{k}{N} > \frac{1}{2} \end{cases}
\end{aligned}$$

due to $q \in (0, \frac{1}{2})$. We then have θ_H-biased user's expected cost in (25) as follows:

$$\begin{aligned}
&\lim_{N\to\infty} \bar{u}_2(b_i = \theta_H)\\
=&\lim_{N\to\infty} \sum_{k=1}^{N} C_{N-1}^{k-1}(\theta_H - \theta_L)^2\\
&\left(p_H q^{N-k}(1-q)^{k-1} + (1-p_H)q^{k-1}(1-q)^{N-k}\right)\\
&\cdot\left(\frac{(1-p_H)q^k(1-q)^{N-k}}{p_H q^{N-k}(1-q)^k + (1-p_H)q^k(1-q)^{N-k}}\right)^2
\end{aligned}$$

Since

$$\begin{aligned}
&\lim_{N\to\infty}\left(\frac{(1-p_H)q^k(1-q)^{N-k}}{p_H q^{N-k}(1-q)^k + (1-p_H)q^k(1-q)^{N-k}}\right)^2\\
=&\lim_{N\to\infty}\left(\frac{1}{\frac{p_H}{1-p_H}(\frac{q}{1-q})^{N-2k}+1}\right)^2\\
=&\begin{cases} 1, & \text{if } \frac{k}{N} < \frac{1}{2},\\ (1-p_H)^2, & \text{if } \frac{k}{N} = \frac{1}{2},\\ 0, & \text{if } \frac{k}{N} > \frac{1}{2}, \end{cases}
\end{aligned}$$

we then have

$$\begin{aligned}
&\lim_{N\to\infty} \bar{u}_2(b_i = \theta_H)\\
=&(\theta_H - \theta_L)^2\Bigg(\lim_{N\to\infty} \sum_{k=1}^{\frac{N}{2}-1}\\
&C_{N-1}^{k-1}\left(p_H q^{N-k}(1-q)^{k-1} + (1-p_H)q^{k-1}(1-q)^{N-k}\right)\\
&+C_{N-1}^{\frac{N}{2}-1}\left(p_H q^{\frac{N}{2}}(1-q)^{\frac{N}{2}-1} + (1-p_H)q^{\frac{N}{2}-1}(1-q)^{\frac{N}{2}}\right)\\
&(1-p_H)^2\Bigg).
\end{aligned}$$

As $N \to \infty$, we can approximate binomial distribution $B(N, 1-q)$ as normal distribution $\mathcal{N}(N(1-q), Nq(1-q))$, we then have

$$\begin{aligned}
&\lim_{N\to\infty} \sum_{k=1}^{\frac{N}{2}} C_{N-1}^{k-1} p_H q^{N-k}(1-q)^{k-1}\\
=&\lim_{N\to\infty} \frac{1}{2} + \frac{1}{2}erf\left(\frac{\frac{N}{2} - 1 - N(1-q)}{\sqrt{2Nq(1-q)}}\right)\\
=&0
\end{aligned} \tag{58}$$

due to $q < \frac{1}{2}$. To show

$$\lim_{N\to\infty} C_{N-1}^{k-1} p_H q^{N-k}(1-q)^{k-1} = 0, k \leq \frac{N}{2}, \tag{59}$$

we have

$$\begin{aligned}
&\lim_{N\to\infty} C_{N-1}^{k-1} p_H q^{N-k}(1-q)^{k-1}\\
=&\lim_{N\to\infty} \frac{(N-1)\cdots(N-k+1)p_H}{(k-1)!(\frac{1-q}{q})^{\frac{N+1}{2}-k}(\frac{1}{q(1-q)})^{\frac{N-1}{2}}}\\
=&\lim_{N\to\infty} \frac{\frac{\partial^{k-1}((N-1)\cdots(N-k+1)p_H)}{\partial N^{k-1}}}{\frac{\partial^{k-1}((k-1)!(\frac{1-q}{q})^{\frac{N+1}{2}-k}(\frac{1}{q(1-q)})^{\frac{N-1}{2}})}{\partial N^{k-1}}}\\
=&\lim_{N\to\infty} \frac{(k-1)!}{(k-1)!(\frac{1-q}{q})^{\frac{N+1}{2}-k}(\frac{1}{q(1-q)})^{\frac{N-1}{2}}}\\
&\cdot\frac{p_H}{(\frac{1}{2}\ln\frac{1-q}{q})^{k-1} + (\frac{1}{2}\ln\frac{1}{q(1-q)}))^{k-1}}\\
=&0
\end{aligned}$$

due to $k \leq N/2$ and $q \in (0, \frac{1}{2})$. Similarly, we have

$$\lim_{N\to\infty} C_{N-1}^{\frac{N}{2}-1}(1-p_H)q^{\frac{N}{2}-1}(1-q)^{\frac{N}{2}} = 0. \tag{60}$$

With (58)-(60), we then have

$$\begin{aligned}
&\lim_{N\to\infty} \bar{u}_2(b_i = \theta_H)\\
=&(1-p_H)\lim_{N\to\infty} \sum_{k=1}^{\frac{N}{2}-1} C_{N-1}^{k-1} q^{k-1}(1-q)^{N-k}(\theta_H - \theta_L)^2\\
=&(1-p_H)(\theta_H - \theta_L)^2 \lim_{N\to\infty} \frac{1}{2} + \frac{1}{2}erf\left(\frac{\frac{N}{2} - 2 - Nq}{\sqrt{2Nq(1-q)}}\right)\\
=&(1-p_H)(\theta_H - \theta_L)^2,
\end{aligned}$$

where we approximate binomial distribution $B(N, q)$ as normal distribution $\mathcal{N}(Nq, Nq(1-q))$.

We have θ_L-biased user's expected cost in (29) as follows:

$$\begin{aligned}
&\lim_{N\to\infty} \bar{u}_2(b_i = \theta_L)\\
=&\lim_{N\to\infty} \sum_{k=0}^{N-1} C_{N-1}^{k}(\theta_H - \theta_L)^2\\
&\left(p_H q^{N-k-1}(1-q)^k + (1-p_H)q^k(1-q)^{N-k-1}\right)\\
&\cdot\left(\frac{p_H q^{N-k}(1-q)^k}{p_H q^{N-k}(1-q)^k + (1-p_H)q^k(1-q)^{N-k}}\right)^2.
\end{aligned}$$

Since

$$\lim_{N\to\infty}\left(\frac{p_H q^{N-k}(1-q)^k}{p_H q^{N-k}(1-q)^k+(1-p_H)q^k(1-q)^{N-k}}\right)^2$$
$$=\lim_{N\to\infty}\left(\frac{1}{\frac{1-p_H}{p_H}(\frac{1-q}{q})^{N-2k}+1}\right)^2$$
$$=\begin{cases}0, & \text{if } \frac{k}{N}<\frac{1}{2},\\ (p_H)^2, & \text{if } \frac{k}{N}=\frac{1}{2},\\ 1, & \text{if } \frac{k}{N}>\frac{1}{2},\end{cases}$$

we then have

$$\lim_{N\to\infty}\bar{u}_2(b_i=\theta_L)$$
$$=\lim_{N\to\infty}\sum_{k=\frac{N}{2}+1}^{N-1}C_{N-1}^k p_H q^{N-k-1}(1-q)^k(\theta_H-\theta_L)^2$$
$$+\lim_{N\to\infty}\sum_{k=\frac{N}{2}+1}^{N-1}C_{N-1}^k(1-p_H)q^k(1-q)^{N-k-1}(\theta_H-\theta_L)^2$$
$$+C_{N-1}^{\frac{N}{2}}\left((1-p_H)q^{\frac{N}{2}}(1-q)^{\frac{N}{2}-1}+p_H q^{\frac{N}{2}-1}(1-q)^{\frac{N}{2}}\right)p_H^2$$
$$(\theta_H-\theta_L)^2$$
$$=\lim_{N\to\infty}\sum_{k=1}^{\frac{N}{2}-1}C_{N-1}^{k-1}p_H q^{k-1}(1-q)^{N-k}(\theta_H-\theta_L)^2$$
$$+\lim_{N\to\infty}\sum_{k=1}^{\frac{N}{2}-1}C_{N-1}^{k-1}(1-p_H)q^{N-k}(1-q)^{k-1}(\theta_H-\theta_L)^2$$
$$+C_{N-1}^{\frac{N}{2}}\left((1-p_H)q^{\frac{N}{2}}(1-q)^{\frac{N}{2}-1}+p_H q^{\frac{N}{2}-1}(1-q)^{\frac{N}{2}}\right)p_H^2$$
$$(\theta_H-\theta_L)^2.$$

According to (58), (59) and (60), we further simplify the above equation as

$$\lim_{N\to\infty}\bar{u}_2(b_i=\theta_L)$$
$$=p_H\lim_{N\to\infty}\sum_{k=1}^{\frac{N}{2}-1}C_{N-1}^{k-1}q^{k-1}(1-q)^{N-k}(\theta_H-\theta_L)^2$$
$$=p_H(\theta_H-\theta_L)^2\lim_{N\to\infty}\frac{1}{2}+\frac{1}{2}erf\left(\frac{\frac{N}{2}-2-Nq}{\sqrt{2Nq(1-q)}}\right)$$
$$=p_H(\theta_H-\theta_L)^2.$$

With strategy 3, we have the platform's action in (27) as $N\to\infty$ as follows:

$$\lim_{N\to\infty}a_3^*(|I_H|=k)=\begin{cases}\theta_H, & \text{if } k\neq 0,\\ \theta_L, & \text{if } k=0.\end{cases}$$

We then have θ_H-biased user's expected cost in (28) as follows:

$$\lim_{N\to\infty}\bar{u}_3(b_i=\theta_H)$$
$$=\lim_{N\to\infty}(1-p_H)\left(\frac{(1-p_H)}{q^N p_H+(1-p_H)}\right)^2(\theta_H-\theta_L)^2$$
$$=(1-p_H)(\theta_H-\theta_L)^2,$$

due to $q<1$, and θ_L-biased user's expected cost in (29) as follows:

$$\lim_{N\to\infty}\bar{u}_3(b_i=\theta_L)$$
$$=\lim_{N\to\infty}(\theta_H-\theta_L)^2\Bigg(p_H(1-q^{N-1})+$$
$$(p_H q^{N-1}+1-p_H)\left(\frac{q^N p_H}{q^N p_H+(1-p_H)}\right)^2\Bigg)$$
$$=p_H(\theta_H-\theta_L)^2,$$

due to $q<\frac{1}{2}$.

With strategy 4, we have the platform's action in (30) as $N\to\infty$ as follows:

$$\lim_{N\to\infty}a_4^*(|I_H|=k)=\begin{cases}\theta_L, & \text{if } k<N,\\ \theta_H, & \text{if } k=N.\end{cases}$$

We then have θ_H-biased user's expected cost in (31) as follows:

$$\lim_{N\to\infty}\bar{u}_4(b_i=\theta_H)$$
$$=\lim_{N\to\infty}\Bigg((p_H+(1-p_H)q^{N-1})\left(\frac{(1-p_H)q^N}{p_H+(1-p_H)q^N}\right)^2$$
$$+(1-p_H)(1-q^{N-1})\Bigg)(\theta_H-\theta_L)^2$$
$$=(1-p_H)(\theta_H-\theta_L)^2,$$

due to $q<\frac{1}{2}$, and θ_L-biased user's expected cost in (32) as follows:

$$\lim_{N\to\infty}\bar{u}_4(b_i=\theta_L)$$
$$=\lim_{N\to\infty}\left(p_H\left(\frac{p_H}{p_H+(1-p_H)q^N}\right)^2(\theta_H-\theta_L)^2\right.$$
$$=p_H(\theta_H-\theta_L)^2,$$

due to $q<1$.

We then have

$$\lim_{N\to\infty}\bar{u}_1(b_i)=\lim_{N\to\infty}\bar{u}_2(b_i)$$
$$=\lim_{N\to\infty}\bar{u}_3(b_i)=\lim_{N\to\infty}\bar{u}_4(b_i),\ b_i\in\{\theta_L,\theta_H\}.$$

After summarizing the above results, we have PBE shown in the proposition. □

Appendix H
Proof of Proposition 6

Proof. As $N \to \infty$, we have already showed in the proof of Lemma 2 that

$$\lim_{N\to\infty} \bar{u}^*_{S_i}(b_i = \theta_H) = (1 - p_H)(\theta_H - \theta_L)^2$$

and

$$\lim_{N\to\infty} \bar{u}^*_{S_i}(b_i = \theta_L) = p_H(\theta_H - \theta_L)^2.$$

Since the platform can always recognize the actual state, we have

$$\lim_{N\to\infty} \bar{u}^*_{S_i}(b_i = \theta) = \lim_{N\to\infty} \bar{u}^*_R = 0.$$

Together with (7) and (8), we have PoA under $N \to \infty$ as follows:

$$\begin{aligned} PoA &= \lim_{N\to\infty} \max_{p_H, q, \theta_H, \theta_L} \frac{\bar{U}^*}{\bar{U}^{**}} \\ &= \lim_{N\to\infty} \max_q \frac{Nq}{\frac{Nq(N+q-Nq)}{N+1}} \\ &= \lim_{N\to\infty} \max_q \frac{N+1}{N+q-Nq} \\ &= \max_q \frac{1}{1-q} < 2, \ q < \frac{1}{2}. \end{aligned}$$

□