Some results from Combination

If a bag contains 6 white and 4 black balls, then

$$P\text{[drawing a white ball]} = \frac{6C_1}{10C_1}$$

$$P\text{[drawing 2 white balls]} = \frac{6C_2}{10C_2}$$

$$P\text{[drawing 3 white balls]} = \frac{6C_3}{10C_3}$$

$$P\text{[drawing 4 white balls]} = \frac{6C_4}{10C_4} \quad \text{and so on}$$

$$P\text{[drawing 2 white and 2 black balls]} = \frac{6C_2 \times 4C_2}{10C_4}$$

$$P\text{[drawing 1 white and 3 black balls]} = \frac{6C_1 \times 4C_3}{10C_4}$$

$$P\text{[drawing 3 white and 1 black balls]} = \frac{6C_3 \times 4C_1}{10C_4} \quad \text{and so on}$$

Problems based on Combination results

Ex. 14: What is the probability of getting 3 white balls in a draw of 3 balls from a box containing 5 white and 4 black balls ?

Ans: Favourable number of cases = $5C_3 = \frac{5\,.\,4\,.\,3}{3\,.\,2\,.\,1} = 10$

Total number of cases = $9C_3 = \frac{9\,.\,8\,.\,7}{3\,.\,2\,.\,1} = 84$

$\therefore$ The probability of getting 3 white balls = $\frac{5C_3}{9C_3} = \frac{10}{84} = \frac{5}{42}$

Ex. 15: A committee is to be constituted by selecting two people at random from a group consisting of 3 Economists and 4 Statisticians. Find the probability that the committee will consist of (1) 2 Economists (ii) 2 Statisticians (iii) 1 Economist and 1 Statistician.

Ans: (i) P [Selecting 2 Economists] $= \frac{3C_2}{7C_2} = \frac{3}{21} = \frac{1}{7}$

(ii) P [Selecting 2 Statisticians] $= \frac{4C_2}{7C_2} = \frac{6}{21} = \frac{2}{7}$

(iii) P [Selecting one Economists and one Statistician]

$= \frac{3C_1 \times 4C_1}{7C_2} = \frac{3 \times 4}{21} = \frac{4}{7}$

Ex. 16: A bag contains 7 white and 9 black balls. 3 balls are drawn together. What is the probability that (1) all are black (2) all are white (3) 1 white and 2 black (4) 2 white and 1 black

Ans: (1) P [all are black] $= \frac{9C_3}{16C_3} = \frac{84}{560} = \frac{3}{20}$

(2) P [all are white] $= \frac{7C_3}{16C_3} = \frac{35}{560} = \frac{1}{16}$

P [1 white and two black] $= \frac{7C_1 \times 9C_2}{16C_3} = \frac{252}{560} = \frac{9}{20}$

P [2 white and one black] $= \frac{7C_2 \times 9C_1}{16C_3} = \frac{189}{560} = \frac{27}{80}$

Ex. 17: The letters of the word 'STATISTICS' are written on 10 identical cards. If two cards are drawn at random, what is the probability that (i) one 'S' and one 'I' will occur (ii) Two 'T" will occur

Ans: Total letters 10; [S - 3, T - 3, A - 1, I - 2, C - 1]

(i) P (one 'S' and one 'I') $= \frac{3C_1 \times 2C_1}{10C_2} = \frac{6}{45} = \underline{\underline{\frac{2}{15}}}$

(ii) P (Two 'T') $= \frac{3C_2}{10C_2} = \frac{3}{45} = \underline{\underline{\frac{1}{15}}}$

Note: P (getting at least one) = 1 – P (getting none)

Ex. 18: There are 4 men and 3 women. Find the probability of selecting 3 of which (i) exactly two are women (ii) no woman (iii) at least one woman (iv) at least 2 women (v) at the most 2 women

Ans: There are 4 men and 3 women. 3 are to be selected

(1) P (Selecting exactly 2 women) = P (Selecting 2 women and 1 man)

$= \frac{3C_2 \times 4C_1}{7C_3} = \frac{3 \times 4}{35} = \frac{12}{35}$

(2) P (Selecting no woman) = P (Selecting 3 men) = $\frac{4C_3}{7C_3} = \frac{4}{35}$

(3) P (at least one woman) = 1 – P (no woman) = $1 - \frac{4}{35} = \frac{31}{35}$

(4) P (at least 2 women) = P [2 women or 3 women]

= P [(2 women and 1 man) or (3 women)] = $\frac{(3C_2 \times 4C_1) + 3C_3}{7C_3} = \frac{13}{35}$

(5) P (selecting at the most 2 women) = P [2 women or 1 woman or no woman]
= P [(2 women and 1 man) or (1 woman and two men) or (3 men)]

$$= \frac{(3C_2 \times 4C_1) + (3C_1 \times 4C_2) + 4C_3}{7C_3} = \frac{12 + 18 + 4}{35} = \frac{34}{35}$$