Erlang / Gamma Distribution

Process goes through r sequential phases each of which has identical exponential distribution.

Then, overall time spent by process in all phases follows Erlang distribution

r- stage Erlang

$$f(t) = \frac{\lambda^r t^{r-1} e^{-\lambda t}}{(r-1)!}, \quad t>0, \lambda>0, \; r=1,2\ldots$$

$$F(t) = 1 - e^{-\lambda t} \sum_{k=0}^{r-1} \frac{(\lambda t)^k}{k!}$$

Consider a system that can withstand up less than r peak stresses. Stresses occur as a random Poisson process.

Exponential $(\lambda) \equiv$ Erlang $(\lambda, 1)$

Erlang / Gamma Distribution

If r in Erlang can take non-integral values, then distribution is called Gamma.

$$f(t) = \frac{\lambda^{\alpha} t^{\alpha-1} e^{-\lambda t}}{\Gamma(\alpha)}$$

where $\Gamma(\alpha) = \int_{0}^{\infty} x^{\alpha-1} e^{-x} dx$

(gamma function)

Hyperexponential Distribution

If process goes through only one of several alternate phases and each phase is exponential, then overall time follows hyperexponential distribution.

$$f(t) = \sum_{i=1}^{k} \alpha_i \lambda_i e^{-\lambda_i t}$$

$$\left(k \text{ phases}, \ \sum_{i=1}^{k} \alpha_i = 1\right)$$

$$F(t) = \sum_{i=1}^{k} \alpha_i \left(1 - e^{-\lambda_i t}\right)$$

$$h(t) = \frac{f(t)}{R(t)} = \frac{\sum \alpha_i \lambda_i e^{-\lambda_i t}}{\sum \alpha_i e^{-\lambda_i t}}$$

DFR with :

Weibull Distribution

Most widely used parametric family of failure distributions.

Proper choice of shape parameter can make it a IFR, DFR or CFR distribution.

$$f(t) = \lambda \alpha t^{\alpha-1} e^{-\lambda t^{\alpha}}$$

$$F(t) = 1 - e^{-\lambda t^{\alpha}}$$

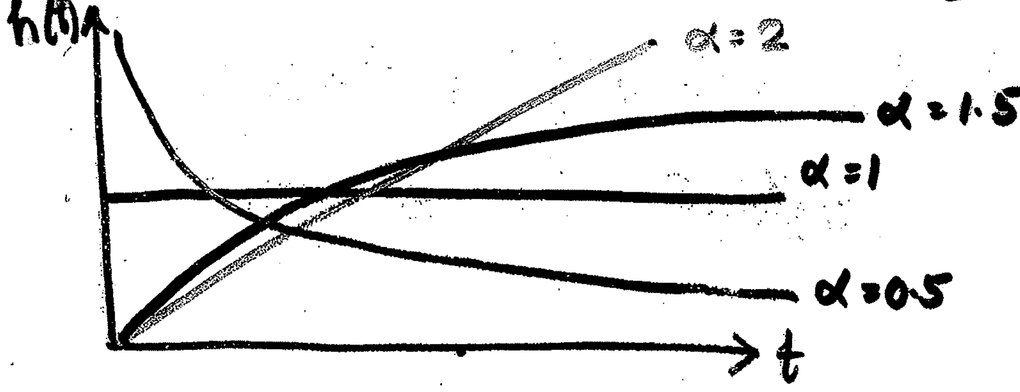
$$h(t) = \frac{\lambda \alpha t^{\alpha-1} e^{-\lambda t^{\alpha}}}{e^{-\lambda t^{\alpha}}}$$

PROBLEM #2

$$= \lambda \alpha t^{\alpha-1}$$

$$h(\alpha=2) = 2t$$

$$EXP(\lambda) \equiv WEI(\lambda, 1)$$



$\lambda = 1$

Normal or Gaussian Distribution

Central limit theorem gives that if you have n mutually independent random variables, then a sample of them will have a mean which is normally distributed as $n \to \infty$.

Examples: • Component lifetime in wear-out phase.

• Errors in measurements

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

$$X \sim N(\mu, \sigma^2)$$

Standard normal distribution $Z \sim N(0,1)$

$$Z = \frac{X-\mu}{\sigma}$$

$$F_X(x) = F_Z\left(\frac{x-\mu}{\sigma}\right)$$

PROBLEM #3