Fault-Tolerant Computer System Design
ECE 60872/CS 590

Topic 2: Discrete Distributions

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Outline

- Basic probability
- Conditional probability
- Independence of events
- Series-parallel system reliability evaluation
- Bayes' rule
- Bernoulli trials

Outline

- Discrete random variables
- Probability mass function
- Cumulative distribution function
- Distributions
  - Bernoulli
  - Binomial
  - Geometric
  - Poisson
  - Hypergeomteric
- *NOTE: This class material is substantially covered on the board. The topics are accompanied by sample problems discussed in class.*

Probability

- Basic probability definition
- Conditional probability of A given B
- Independence of events
  - Not transitive
  - Generalization to n events being independent
  - Example

Series-Parallel System

- Series system is one where the components are so interrelated that the whole system will fail if any of the components fails
- A series system of n components with probability of i^{th} component functioning correctly = p_i. Calculate reliability of system.
- What is the assumption here?
- Parallel system is one where the system will fail if all of the components fails
- What is the reliability of a parallel system with n components?
- Example of a series-parallel system and evaluate its reliability

Modeling Reliability: Reliability Block Diagram and Fault Tree

- Reliability Block Diagram (RBD)
  - Series-parallel system
  - Components are combined into blocks
  - The block may be series, parallel, or *k-out-of-n* configurations
  - *Example*
- Fault Tree
  - Graphical representation of the events that can cause system failure
  - Event can be a basic event or logical combination of lower-level events
  - Assume basic events are mutually independent
  - *Example*

Bayes' Rule

- If B1 and B2 are mutually exclusive (m.e.) and collectively exhaustive (c.e.) events then:
  $P(A) = P(A|B1) * P(B1) + P(A|B2) * P(B2)$
- Generalization to n m.e. and c.e. events gives Theorem of Total Probability
- Bayes' Rule
  $P(B|A) = P(A \cap B)/P(A) = P(B)P(A|B)/ \sum P(B_i)P(A|B_i)$
  Where B_i's are c.e. and m.e.

Bernoulli Trials

- Definition: Random experiment has two possible outcomes. N independent repetitions of the experiment are done.
- Examples
  - Jump taken or not taken in branch predictor
  - Transmitted bit received correctly or incorrectly in a faulty channel
- Probability of success = p(say), probability of failure = q (say)
- Probability of k successes in n trials = p(k) = ?
- Example problem: TMR reliability computation

Generalized Bernoulli trials

- There are k possible outcomes on each trial: b_1, b_2, …, b_k
- Probability of outcome b_i is p_i
- What is the probability there will be n_i occurrences of outcome b_i? Clarification: b_1 happens n_1 times, b_2 happens n_2 times, …, b_k happens n_k times.
- Example: At the end of CPU processing, a job is serviced by one of several I/O devices
- Example problem: Malfunctioning diodes

Discrete Random Variables

- Definition: A random variable X on sample space S is a function X: S $\rightarrow$ R that assigns a real number X(s) to each sample point s ε S
- Example: Consider three tosses of a coin with head=0, tail=1. Random variable examples are number of 1's, number of tosses before first 1.
- A discrete random variable is one which takes on values from the set of discrete numbers
- Inverse image of a random variable A_x

Probability Mass Function & Cumulative Distribution Function

- Probability mass function (pmf) gives probability that the value of the random variable X is equal to x. Denoted by $p_X(x)$
- Example: Three tosses of an unbiased coin. Random variable is number of 1's in the toss. What is the pmf?
- Cumulative distribution function (cdf)

  $F_X(t) = P(X \leq t)$
- $P(a < X \leq b) = ?$
- CDF for discrete random variables has jumps

Examples of PMF & CDF

- Bernoulli
- Binomial
  - Example problem: Defective shipments from two vendors
  - Laplace approximation of binomial for large n
- Geometric
  - Number of trials till first success in Bernoulli trials
  - Modified geometric distribution: number of failures till first success
  - Example: Time slices in a computer and program finishes execution with probability p in a given time slice. What is the number of time slices needed for the program to complete?
  - Memoryless property

Examples of PMF & CDF

- Poisson
  - Distribution gives probability of k arrivals in interval (0,t]
  - Rate of arrival constant λ
  - Single parameter $\alpha = \lambda t$. Denoted by $f(k; \alpha)$
  - Poisson approximation to Binomial for large n (>20), small p (<0.05)
  - Probability of k components failing in time λt
- Hypergeometric
  - Distribution gives probability of k defectives
  - m samples drawn from a total of n items with d defective
  - A drawn sample is not replaced
  - Denoted by $h(k; m,d,n)$
  - Binomial is generally a good approximation for hypergeometric when m << n