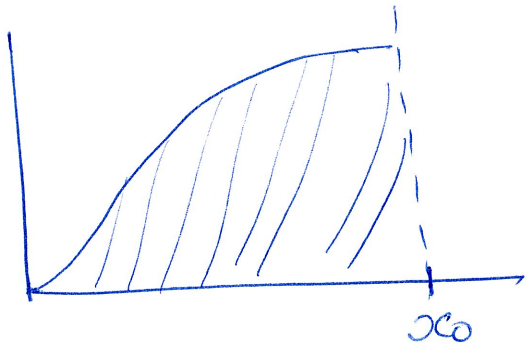
cdf

~~P(X ≤ x)~~

$F(x)$

X



$Pr(X \leq x_0)$

pdf $\quad f_X(x)$

Exponential distribution

$F(t) = 1 - e^{-\lambda t}$, $\Rightarrow$ Pr(Failing in $(0,t)$)

$\lambda \uparrow$ System dependability $\downarrow$

λ: failure rate

10^{-2} / hr

$R(t) = Pr$(No failure in $(0,t)$).

$\in [0,1]$ ✓ $\quad = 1 - F(t)$

$(0,1)$ ✗

Poisson distribution

\# events in $(0,t)$

Rate λ. Example: $10^{-2}/hr$

$$Pr(k \text{ events in } (0,t)) = e^{-\lambda t}\frac{(\lambda t)^k}{k!}.$$

$k \in (0,\infty)$

Events are failures.

$$R(t) = Pr(\emptyset \text{ event in } (0,t))$$
$$= e^{-\lambda t} \longrightarrow \text{Exponential distribution}$$

$\lambda \uparrow \quad R(t) \downarrow$

X : Time between failures.
Cont. variable

Y : # failures
Discrete variable

Memoryless property

Case 1

P_1

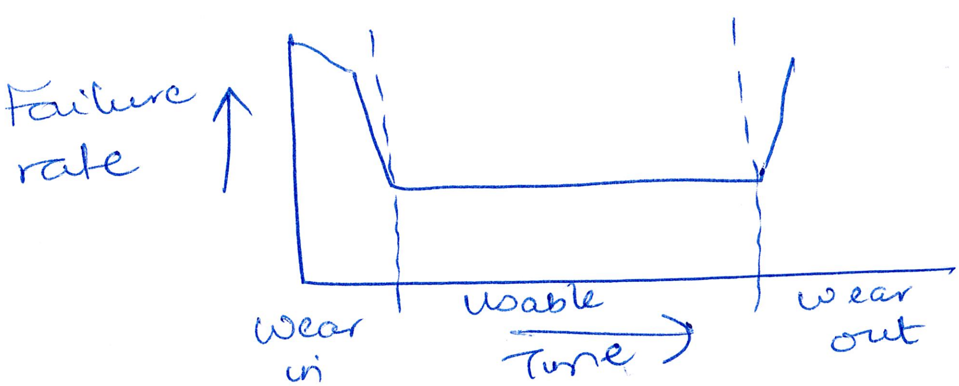
$Pr(\text{Failure in } (0, t_0))$

Case 2

P_2

$Pr(\text{Failure in } (T, T+t_0) \mid \text{System has survived till time } T)$.

Under memoryless property, these two values are the same.



Wear in: $P_1 > P_2$ usable lifetime: $P_1 = P_2$ wear out: $P_1 < P_2$

$$h(t) = \lambda_0 t.$$

$$R(t) = e^{-\int_0^t h(x)\,dx}$$

$$= \cancel{e^{-\int \lambda_0 \cancel{t}\, dx}}$$

$$= e^{-\int_0^t \lambda_0 x\, dx}$$

$$= e^{-\lambda_0 \frac{t^2}{2}}$$

Instantaneous failure rate for Exponential

$$h(t) = \frac{f(t)}{R(t)}$$

$$F(t) = 1 - e^{-\lambda t}$$

$$f(t) = \frac{dF(t)}{dt} = \lambda e^{-\lambda t}$$

$$R(t) = e^{-\lambda t}$$

$$\therefore h(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

$$\frac{\text{Useful/usable part of lifetime of system}}{\text{MTTF of system}}$$

$$\text{MTTF} = E(X) = \int_{min}^{max} x\, f(x)\, dx$$

Say $t \in (0, \infty)$

$$= \int_0^\infty x\, f(x)\, dx$$

$$= -\int x\, R'(x)\, dx$$

Take $f(x) = x$

$g'(x) = R'(x)$

$$= -x\, R(x) \Big|_0^\infty + \int_0^\infty R(x)\, dx$$

Integration by parts

$$\int f(x)\, g'(x)\, dx$$

$$= f(x)\, g(x) - \int g(x)\, f'(x)\, dx$$

$F(x) = 1 - R(x)$

$f(x) = F'(x) = -R'(x)$

$R(x) \to 0$ as $x \to \infty$

faster than $x \to \infty$

$$= \int_0^\infty R(x)\, dx$$