

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/235419085>

Combination of Evidence in Dempster-Shafer Theory

Book · January 2002

DOI: 10.2172/800792

CITATIONS

769

READS

2,666

2 authors, including:



Scott Ferson

University of Liverpool

193 PUBLICATIONS 8,727 CITATIONS

SEE PROFILE

Some of the authors of this publication are also working on these related projects:



Diagnostics without gold standards [View project](#)



Validation methods [View project](#)



SANDIA REPORT

SAND2002-0835
Unlimited Release
Printed April 2002

Combination of Evidence in Dempster-Shafer Theory

Karl Sentz and Scott Ferson

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract DE-AC04-94AL85000.

Approved for public release; further dissemination unlimited.



Sandia National Laboratories

Issued by Sandia National Laboratories, operated for the United States Department of Energy by Sandia Corporation.

NOTICE: This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government, nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, make any warranty, express or implied, or assume any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represent that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government, any agency thereof, or any of their contractors or subcontractors. The views and opinions expressed herein do not necessarily state or reflect those of the United States Government, any agency thereof, or any of their contractors.

Printed in the United States of America. This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from
U.S. Department of Energy
Office of Scientific and Technical Information
P.O. Box 62
Oak Ridge, TN 37831

Telephone: (865)576-8401
Facsimile: (865)576-5728
E-Mail: reports@adonis.osti.gov
Online ordering: <http://www.doe.gov/bridge>

Available to the public from
U.S. Department of Commerce
National Technical Information Service
5285 Port Royal Rd
Springfield, VA 22161

Telephone: (800)553-6847
Facsimile: (703)605-6900
E-Mail: orders@ntis.fedworld.gov
Online order: <http://www.ntis.gov/ordering.htm>



SAND 2002-0835
Unlimited Release
Printed April 2002

Combination of Evidence in Dempster-Shafer Theory

Kari Sentz
Ph.D. Student
Systems Science and Industrial Engineering Department
Thomas J. Watson School of Engineering and Applied Science
Binghamton University
P.O. Box 6000
Binghamton, NY 13902-6000

Scott Ferson
Applied Biomathematics
100 North Country Road
Setauket, NY 11733

Abstract

Dempster-Shafer theory offers an alternative to traditional probabilistic theory for the mathematical representation of uncertainty. The significant innovation of this framework is that it allows for the allocation of a probability mass to sets or intervals. Dempster-Shafer theory does not require an assumption regarding the probability of the individual constituents of the set or interval. This is a potentially valuable tool for the evaluation of risk and reliability in engineering applications when it is not possible to obtain a precise measurement from experiments, or when knowledge is obtained from expert elicitation. An important aspect of this theory is the combination of evidence obtained from multiple sources and the modeling of conflict between them. This report surveys a number of possible combination rules for Dempster-Shafer structures and provides examples of the implementation of these rules for discrete and interval-valued data.

ACKNOWLEDGEMENTS

The authors wish to thank Bill Oberkampf, Jon Helton, and Marty Pilch of Sandia National Laboratories for their many critical efforts in support of this project and the development of this report in particular. In addition, the initiative of Cliff Joslyn to organize the workshop on new methods in uncertainty quantification at Los Alamos National Laboratories (February, 2002) was extremely helpful to the final draft of this paper. Finally, we would like to thank George Klir of Binghamton University for his encouragement over the years.

TABLE OF CONTENTS

ABSTRACT	3
ACKNOWLEDGEMENTS	4
TABLE OF CONTENTS	5
LIST OF FIGURES	6
LIST OF TABLES	7
1.1: INTRODUCTION	8
1.2: TYPES OF EVIDENCE	10
2.1: DEMPSTER-SHAFER THEORY	13
2.2: RULES FOR THE COMBINATION OF EVIDENCE	15
2.2.1: THE DEMPSTER RULE OF COMBINATION	16
2.2.2: DISCOUNT+COMBINE METHOD	17
2.2.3: YAGER'S MODIFIED DEMPSTER'S RULE	18
2.2.4: INAGAKI'S UNIFIED COMBINATION RULE	20
2.2.5: ZHANG'S CENTER COMBINATION RULE	23
2.2.6: DUBOIS AND PRADE'S DISJUNCTIVE CONSENSUS RULE	24
2.2.7: MIXING OR AVERAGING	25
2.2.8: CONVOLUTIVE X-AVERAGING	26
2.2.9: OTHER RULES OF COMBINATION	26
3: DEMONSTRATION OF COMBINATION RULES	27
3.1: Data given by discrete values	27
3.1.1: Dempster's Rule	27
3.1.2: Yager's Rule	29
3.1.3: Inagaki's Rule	29
3.1.4: Zhang's Rule	30
3.1.5: Mixing	30
3.1.6: Dubois and Prade's Disjunctive Consensus Pooling	31
3.2: Data given by intervals	31
3.2.1: Dempster's Rule	34
3.2.2: Yager's Rule	35
3.2.3: Inagaki's Rule	36
3.2.4: Zhang's Rule	36
3.2.5: Mixing	39
3.2.6: Convolutional x-Averaging	40
3.2.7: Dubois and Prade's Disjunctive Consensus	43
3.2.8: Summary of Examples	45
4: CONCLUSIONS	46
REFERENCES	50
APPENDIX A	A-1

LIST OF FIGURES

Figure 1: Consonant evidence obtained from multiple sources.....	11
Figure 2: Consistent evidence obtained from multiple sensors.....	11
Figure 3: Arbitrary evidence obtained from multiple sensors.....	12
Figure 4: Disjoint evidence obtained from multiple sensors.....	12
Figure 5: The Possible Values of k in Inagaki's Unified Combination Rule	22
Figure 6: The value of $m(B)$ as a function of k in Inagaki's rule.....	30
Figure 7: The gcdf of A	32
Figure 8: The gcdf of B.....	32
Figure 9: The gcdf of C.....	33
Figure 10: The gcdf's of A, B, and C without any combination operation.....	33
Figure 11: The gcdf of the combination of A and B using Dempster's rule	34
Figure 12: The combination of A and B using Yager's rule	35
Figure 13: The gcdf of the combination of A and C using Yager's rule	35
Figure 14: The Inagaki combination of A and B for $k=0$	36
Figure 15: The Inagaki combination of A and B where $k = 1$	36
Figure 16: The Zhang combination of A and B.....	38
Figure 17: The mixture of A and B.....	39
Figure 18: The mixture of A and C.....	40
Figure 19: The gcdf of the combination of A and B using convolutive x-averaging	41
Figure 20: The Comparison of Combinations of A and B with Dempster's rule and Convolutive X-Averaging.....	41
Figure 21: The gcdf of the Combination of A and C using Convolutive x-Averaging	42
Figure 22: Comparison of Yager's rule and Convolutive x-averaging for A and C	43
Figure 23: The Disjunctive Consensus Pooling of A and B	44
Figure 24: The gcdf for the Disjunctive Consensus Pooling of A and C	44
Figure 25: Important Issues in the Combination of Evidence	48

LIST OF TABLES

Table 1: Dempster Combination of Expert 1 and Expert 2	28
Table 2: Unions obtained by Disjunctive Consensus Pooling.....	31
Table 3: The interval-based data for A and the basic probability assignments	31
Table 4: The interval-based data for B and the basic probability assignments	32
Table 5: The interval-based data for C and the basic probability assignments	33
Table 6: Combination of A and B with Dempster's Rule.....	34
Table 7: The combination of the marginals with Zhang's rule.....	37
Table 8: The length of the intervals and their intersections	37
Table 9: Calculation of the Measure of Intersection.....	37
Table 10: The product of m and $r(A,B)$	38
Table 11: The renormalized masses with Zhang's rule	38
Table 12: The mixture of A and B.....	39
Table 13: The mixture of A and C.....	40
Table 14: The Combination of A and B using Convolutional x-Averaging.....	40
Table 15: The Combination of A and C using Convolutional x-Averaging.....	42
Table 16: The Disjunctive Consensus Pooling of A and B	43
Table 17: Calculations for the Disjunctive Consensus Pooling of A and C	44
Table 18: The Combination of A and B Comparison Table.....	45
Table 19: The Combination of A and C Comparison Table.....	46
Table 20: Combination Rules and Their Algebraic Properties.....	47

1.1: INTRODUCTION

Only very recently, the scientific and engineering community has begun to recognize the utility of defining multiple types of uncertainty. In part the greater depth of study into the scope of uncertainty is made possible by the significant advancements in computational power we now enjoy. As systems become computationally better equipped to handle complex analyses, we encounter the limitations of applying only one mathematical framework (traditional probability theory) used to represent the full scope of uncertainty. The dual nature of uncertainty is described with the following definitions from [Helton, 1997]:

Aleatory Uncertainty – the type of uncertainty which results from the fact that a system can behave in random ways
also known as: Stochastic uncertainty, Type A uncertainty, Irreducible uncertainty, Variability, Objective uncertainty

Epistemic Uncertainty- the type of uncertainty which results from the lack of knowledge about a system and is a property of the analysts performing the analysis.
also known as: Subjective uncertainty, Type B uncertainty, Reducible uncertainty, State of Knowledge uncertainty, Ignorance

Traditionally, probability theory has been used to characterize both types of uncertainty. It is well recognized that aleatory uncertainty is best dealt with using the frequentist approach associated with traditional probability theory. However, the recent criticisms of the probabilistic characterization of uncertainty claim that traditional probability theory is not capable of capturing epistemic uncertainty. The application of traditional probabilistic methods to epistemic or subjective uncertainty is often known as Bayesian probability. A probabilistic analysis requires that an analyst have information on the probability of all events. When this is not available, the uniform distribution function is often used, justified by Laplace's *Principle of Insufficient Reason*. [Savage, 1972] This can be interpreted that all simple events for which a probability distribution is not known in a given sample space are equally likely. Take for an example a system failure where there are three possible components that could have caused this type of failure. An expert in the reliability of one component assigns a probability of failure of that component with 0.3 (Component A). The expert knows nothing about the other two potential sources of failure (Components B and C). A traditional probabilistic analysis following the Principle of Insufficient Reason, could assign a probability of failure of 0.35 to each of the two remaining components (B and C). This would be a very precise statement about the probability of failure of these two components in the face of *complete* ignorance regarding these components on the part of the expert.

An additional assumption in classical probability is entailed by the axiom of additivity where all probabilities that satisfy specific properties must add to 1. This forces the conclusion that knowledge of an event necessarily entails knowledge of the complement of an event, i.e., knowledge of the probability of the likelihood of the occurrence of an event can be translated into the knowledge of the likelihood of that event not occurring. If an expert believes that a system may fail due to a particular

component with a likelihood of 0.3, does that necessarily mean that the expert believes that the system will *not* fail due to that component of 0.7? This articulates the challenge of modeling any uncertainty associated with an expert's subjective belief. Though the assumptions of additivity and the Principle of Insufficient Reason may be appropriate when modeling the random events associated with aleatoric uncertainty, these constraints are questionable when applied to an issue of knowledge or belief.

As a consequence of these concerns, applied mathematicians have investigated many more general representation of uncertainty to cope with particular situations involving epistemic uncertainty. Examples of these types of situations include:

1. When there is little information on which to evaluate a probability or
2. When that information is nonspecific, ambiguous, or conflicting.

Analysis of these situations can be required, for an example in risk assessment, though probability theory lacks the ability to handle such information. Where it is not possible to characterize uncertainty with a precise measure such as a precise probability, it is reasonable to consider a measure of probability as an interval or a set.

This characterization of a measure of probability as an interval or set has three important implications:

1. It is not necessary to elicit a precise measurement from an expert or an experiment if it is not realistic or feasible to do so.
2. The Principle of Insufficient Reason is not imposed. Statements can be made about the likelihood of multiple events together without having to resort to assumptions about the probabilities of the individual events under ignorance.
3. The axiom of additivity is not imposed. The measures do not have to add to 1. When they do, it corresponds to a traditional probabilistic representation. When the sum is less than 1, called the subadditive case, this implies an incompatibility between multiple sources of information, e.g. multiple sensors providing conflicting information. When the sum is greater than 1, the superadditive case, this implies a cooperative effect between multiple sources of information, e.g. multiple sensors providing the same information.

Because there is more than one kind of uncertainty and probability theory may not apply to every situation involving uncertainty, many theories of generalized uncertainty-based information have been developed. Currently, this discipline area is known as monotone measure theory or nonadditive measure theory but in older publications it is referred to as fuzzy measure theory. This latter designation is a misnomer as the majority of frameworks subsumed under this term are not fuzzy in the traditional use of the term as introduced by Zadeh. There are three major frameworks from which the problem of interval-based representation of uncertainty has been approached: imprecise probabilities (initial work by Walley, Fine; Kuznetsov); possibility theory (Zadeh; Dubois and Prade; Yager); and the Dempster-Shafer theory of evidence. (Dempster; Shafer; Yager; Smets).

This situation of multiple frameworks to characterize uncertainty poses an obvious problem to the analyst faced with epistemic uncertainty, namely, which method should be applied to a particular situation. While this is still a research question, this

decision is simplified somewhat by the level of development of the theories and their use in practical applications. This study uses Dempster-Shafer Theory as the framework for representing uncertainty and investigates the issue of combination of evidence in this theory. The motivation for selecting Dempster-Shafer theory can be characterized by the following reasons:

1. The relatively high degree of theoretical development among the non-traditional theories for characterizing uncertainty.
2. The relation of Dempster-Shafer theory to traditional probability theory and set theory.
3. The large number of examples of applications of Dempster-Shafer theory in engineering in the past ten years.
4. The versatility of the Dempster-Shafer theory to represent and combine different types of evidence obtained from multiple sources.

1.2: TYPES OF EVIDENCE

There are two critical and related issues concerning the combination of evidence obtained from multiple sources: one is the type of evidence involved and the other is how to handle conflicting evidence. We consider four types of evidence from multiple sources that impact the choice of how information is to be combined: consonant evidence, consistent evidence, arbitrary evidence, and disjoint evidence:

Consonant evidence can be represented as a nested structure of subsets where the elements of the smallest set are included in the next larger set... all of whose elements are included in the next larger set and so on. This can correspond to the situation where information is obtained over time that increasingly narrows or refines the size of the evidentiary set. Take a simple example from target identification. Suppose there are five sensors with varying degrees of resolution: Sensor 1; Sensor 2; Sensor 3; Sensor 4; Sensor 5.

Sensor 1 detects a target in vicinity A.

Sensor 2 detects two targets: one in vicinity A and one in vicinity B.

Sensor 3 detects three targets: one in vicinity A, one in vicinity B, one in vicinity C.

Sensor 4 detects four targets: one in vicinity A, one in vicinity B, one in vicinity C, one in vicinity D.

Sensor 5 detects five targets: one in vicinity A, one in vicinity B, one in vicinity C, one in vicinity D, one in vicinity E.

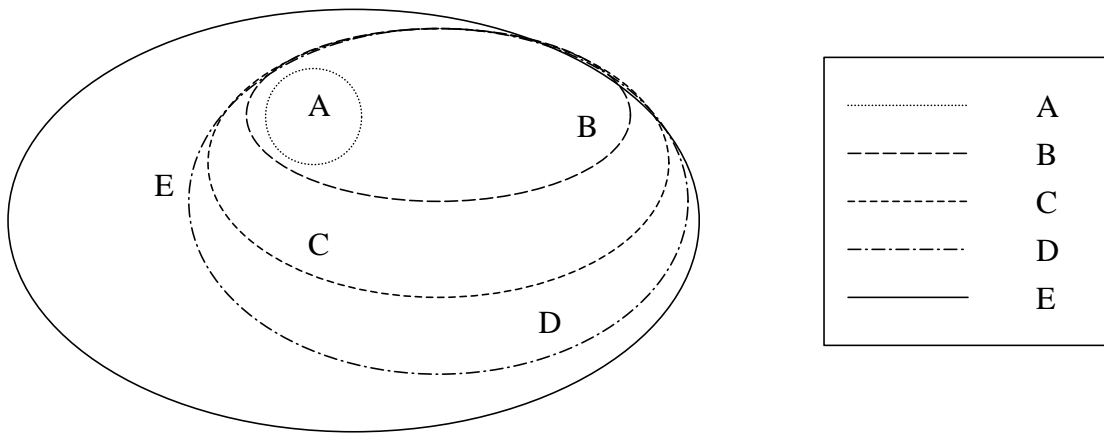


Figure 1: Consonant evidence obtained from multiple sources

Consistent evidence means that there is at least one element that is common to *all* subsets. From our target identification, this could look like:

Sensor 1 detects a target in vicinity A.

Sensor 2 detects two targets: one in vicinity A and one in vicinity B.

Sensor 3 detects two targets: one in vicinity A, one in vicinity C.

Sensor 4 detects three targets: one in vicinity A, one in vicinity B, one in vicinity D.

Sensor 5 detects four targets: one in vicinity A, one in vicinity B, one in vicinity C, one in vicinity E.

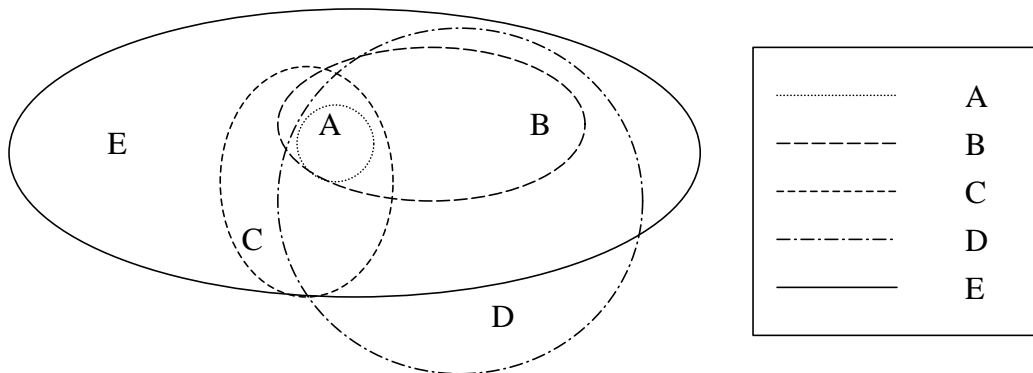


Figure 2: Consistent evidence obtained from multiple sensors

Arbitrary evidence corresponds to the situation where there is no element common to *all* subsets, though some subsets may have elements in common. One possible configuration in our target identification example:

Sensor 1 detects a target in vicinity A.

Sensor 2 detects two targets: one in vicinity A and one in vicinity B.

Sensor 3 detects two targets: one in vicinity A, one in vicinity C.
 Sensor 4 detects two targets: one in vicinity C, one in vicinity D.
 Sensor 5 detects two targets: one in vicinity C, one in vicinity E.

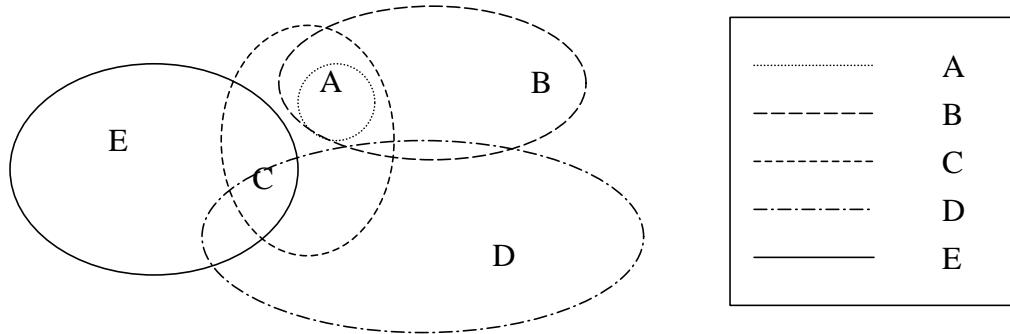


Figure 3: Arbitrary evidence obtained from multiple sensors

Disjoint evidence implies that any two subsets have no elements in common with any other subset.

Sensor 1 detects a target in vicinity A.
 Sensor 2 detects a target in vicinity B.
 Sensor 3 detects a target in vicinity C.
 Sensor 4 detects a target in vicinity D.
 Sensor 5 detects a target in vicinity E.

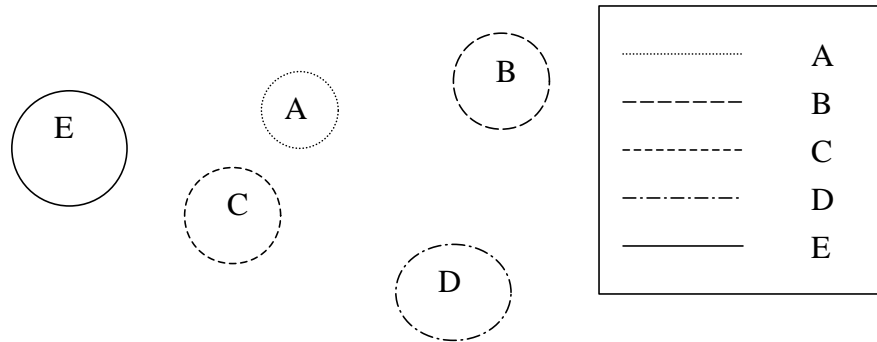


Figure 4: Disjoint evidence obtained from multiple sensors

Each of these possible configurations of evidence from multiple sources has different implications on the level of conflict associated with the situation. Clearly in the case of disjoint evidence, all of the sources supply conflicting evidence. With arbitrary evidence, there is some agreement between some sources but there is no consensus among sources on any one element. Consistent evidence implies an agreement on at least one evidential set or element. Consonant evidence represents the situation where each set is supported by the next larger set and implies an agreement on the smallest evidential set; however, there is conflict between the additional evidence that the larger set represents in relation to the smaller set. Traditional probability theory cannot handle

consonant, consistent, or arbitrary evidence without resorting to further assumptions of the probability distributions within a set, nor can probability theory express the level of conflict between these evidential sets. Dempster-Shafer theory is a framework that can handle these various evidentiary types by combining a notion of probability with the traditional conception of sets. In addition, in Dempster Shafer theory, there are many ways which conflict can be incorporated when combining multiple sources of information.

2.1: DEMPSTER-SHAFFER THEORY

Dempster-Shafer Theory (DST) is a mathematical theory of evidence. The seminal work on the subject is [Shafer, 1976], which is an expansion of [Dempster, 1967]. In a finite discrete space, Dempster-Shafer theory can be interpreted as a generalization of probability theory where probabilities are assigned to *sets* as opposed to mutually exclusive singletons. In traditional probability theory, evidence is associated with only one possible event. In DST, evidence can be associated with multiple possible events, e.g., sets of events. As a result, evidence in DST can be meaningful at a higher level of abstraction without having to resort to assumptions about the events within the evidential set. Where the evidence is sufficient enough to permit the assignment of probabilities to single events, the Dempster-Shafer model collapses to the traditional probabilistic formulation. One of the most important features of Dempster-Shafer theory is that the model is designed to cope with varying levels of precision regarding the information and no further assumptions are needed to represent the information. It also allows for the direct representation of uncertainty of system responses where an imprecise input can be characterized by a set or an interval and the resulting output is a set or an interval.

There are three important functions in Dempster-Shafer theory: the *basic probability assignment* function (bpa or m), the *Belief* function (Bel), and the *Plausibility* function (Pl).

The basic probability assignment (bpa) is a primitive of evidence theory. Generally speaking, the term “basic probability assignment” does *not* refer to probability in the classical sense. The bpa, represented by m , defines a mapping of the power set to the interval between 0 and 1, where the bpa of the null set is 0 and the summation of the bpa’s of all the subsets of the power set is 1. The value of the bpa for a given set A (represented as $m(A)$), expresses the proportion of all relevant and available evidence that supports the claim that a particular element of X (the universal set) belongs to the set A but to no particular subset of A [Klir, 1998]. The value of $m(A)$ pertains only to the set A and makes no additional claims about any subsets of A . Any further evidence on the subsets of A would be represented by another bpa, i.e. $B \subset A$, $m(B)$ would be the bpa for the subset B . Formally, this description of m can be represented with the following three equations:

$$m: P(X) \rightarrow [0,1] \quad (1)$$

$$m(\emptyset) = 0 \quad (2)$$

$$\sum_{A \in \mathcal{P}(X)} m(A) = 1 \quad (3)$$

where $\mathcal{P}(X)$ represents the power set of X , \emptyset is the null set, and A is a set in the power set ($A \in \mathcal{P}(X)$). [Klir, 1998]

Some researchers have found it useful to interpret the basic probability assignment as a classical probability, such as [Chokr and Kreinovich, 1994], and the framework of Dempster-Shafer theory can support this interpretation. The theoretical implications of this interpretation are well developed in [Kramosil, 2001]. This is a very important and useful interpretation of Dempster-Shafer theory but it does *not* demonstrate the full scope of the representational power of the basic probability assignment. As such, the bpa *cannot* be equated with a classical probability in general.

From the basic probability assignment, the upper and lower bounds of an interval can be defined. This interval contains the precise probability of a set of interest (in the classical sense) and is bounded by two nonadditive continuous measures called Belief and Plausibility. The lower bound *Belief* for a set A is defined as the sum of all the basic probability assignments of the proper subsets (B) of the set of interest (A) ($B \subseteq A$). The upper bound, *Plausibility*, is the sum of all the basic probability assignments of the sets (B) that intersect the set of interest (A) ($B \cap A \neq \emptyset$). Formally, for all sets A that are elements of the power set ($A \in \mathcal{P}(X)$), [Klir, 1998]

$$Bel(A) = \sum_{B|B \subseteq A} m(B) \quad (4)$$

$$Pl(A) = \sum_{B|B \cap A \neq \emptyset} m(B) \quad (5)$$

The two measures, *Belief* and *Plausibility* are nonadditive. This can be interpreted as is not required for the sum of all the Belief measures to be 1 and similarly for the sum of the Plausibility measures.

It is possible to obtain the basic probability assignment from the *Belief* measure with the following inverse function:

$$m(A) = \sum_{B|B \subseteq A} (-1)^{|A-B|} Bel(B) \quad (6)$$

where $|A-B|$ is the difference of the cardinality of the two sets.

In addition to deriving these measures from the basic probability assignment (m), these two measures can be derived from each other. For example, *Plausibility* can be derived from *Belief* in the following way:

$$Pl(A) = 1 - Bel(\bar{A}) \quad (7)$$

where \bar{A} is the classical complement of A . This definition of Plausibility in terms of Belief comes from the fact that all basic assignments must sum to 1.

$$Bel(\bar{A}) = \sum_{B \mid B \subseteq \bar{A}} m(B) = \sum_{B \mid B \cap A = \emptyset} m(B) \quad (8)$$

$$\sum_{B \mid B \cap A \neq \emptyset} m(B) = 1 - \sum_{B \mid B \cap A = \emptyset} m(B) \quad (9)$$

From the definitions of Belief and Plausibility, it follows that $Pl(A) = 1 - Bel(\bar{A})$. As a consequence of Equations 6 and 7, given any one of these measures ($m(A)$, $Bel(A)$, $Pl(A)$) it is possible to derive the values of the other two measures.

The precise probability of an event (in the classical sense) lies within the lower and upper bounds of *Belief* and *Plausibility*, respectively.

$$Bel(A) = P(A) = Pl(A) \quad (10)$$

The probability is uniquely determined if $Bel(A) = Pl(A)$. In this case, which corresponds to classical probability, all the probabilities, $P(A)$ are uniquely determined for all subsets A of the universal set X [Yager, 1987, p.97]. Otherwise, $Bel(A)$ and $Pl(A)$ may be viewed as lower and upper bounds on probabilities, respectively, where the actual probability is contained in the interval described by the bounds. Upper and lower probabilities derived by the other frameworks in generalized information theory can *not* be directly interpreted as *Belief* and *Plausibility* functions. [Dubois and Prade, 1992, p.216]

2.2: RULES FOR THE COMBINATION OF EVIDENCE

The purpose of aggregation of information is to meaningfully summarize and simplify a corpus of data whether the data is coming from a single source or multiple sources. Familiar examples of aggregation techniques include arithmetic averages, geometric averages, harmonic averages, maximum values, and minimum values [Ayuub, 2001]. Combination rules are the special types of aggregation methods for data obtained from *multiple* sources. These multiple sources provide different assessments for the same frame of discernment and Dempster-Shafer theory is based on the assumption that these sources are *independent*. The requirement for establishing the independence of sources is an important philosophical question.

From a set theoretic standpoint, these rules can potentially occupy a continuum between conjunction (AND-based on set intersection) and disjunction (OR-based on set union) [Dubois and Prade, 1992]. In the situation where *all* sources are considered reliable, a conjunctive operation is appropriate (A and B and $C \dots$). In the case where there is one reliable source among many, we can justify the use of a disjunctive combination operation (A or B or $C \dots$). However, many combination operations lie between these two extremes (A and B or C , A and C or B , etc.). Dubois and Prade [Dubois, Prade, 1992] describe these three types of combinations as *conjunctive pooling* ($A \cap B$, if $A \cap B \neq \emptyset$), *disjunctive pooling* ($A \cup B$), and *tradeoff* (There are many ways a tradeoff between $A \cap B$ and $A \cup B$ can be achieved).

There are multiple operators available in each category of pooling by which a corpus of data can be combined. One means of comparison of combination rules is by

comparing the algebraic properties they satisfy. With the tradeoff type of combination operations, less information is assumed than in a Bayesian approach and the precision of the result may suffer as a consequence. On the other hand, a precise answer obtained via the Bayesian approach does not express any uncertainty associated with it and may have hidden assumptions of additivity or Principle of Insufficient Reason. [Dubois and Prade, 1992]

In keeping with this general notion of a continuum of combination operations, there are multiple possible ways in which evidence can be combined in Dempster-Shafer theory. The original combination rule of multiple basic probability assignments known as the Dempster rule is a generalization of Bayes' rule. [Dempster, 1967] This rule strongly emphasizes the agreement between multiple sources and ignores *all* the conflicting evidence through a normalization factor. This can be considered a strict AND-operation. The use of the Dempster rule has come under serious criticism when significant conflict in the information is encountered. [Zadeh, 1986; Yager, 1987] Consequently, other researchers have developed modified Dempster rules that attempt to represent the degree of conflict in the final result. This issue of conflict and the allocation of the bpa mass associated with it is the critical distinction between all of the Dempster-type rules. To employ any of these combination rules in an application, it is essential to understand how conflict should be treated in that particular application context.

In addition to the Dempster rule of combination, we will discuss four modified Dempster rules: Yager's rule; Inagaki's unified combination rule; Zhang's center combination rule; and Dubois and Prade's disjunctive pooling rule. Three types of averages will be considered: discount and combine; convolutive averaging; and mixing. All of the combination rules will be considered relative to four algebraic properties: commutativity, $A * B = B * A$; idempotence, $A * A = A$; continuity, $A * B \approx A' * B$, where $A' \approx A$ (A' is very close to A); and associativity, $A * (B * C) = (A * B) * C$; where $*$ denotes the combination operation. The motivation for these properties is discussed at length in [Ferson and Kreinovich, 2002].

2.2.1: THE DEMPSTER RULE OF COMBINATION

The Dempster rule of combination is critical to the original conception of Dempster-Shafer theory. The measures of *Belief* and *Plausibility* are derived from the combined basic assignments. Dempster's rule combines multiple belief functions through their basic probability assignments (m). These belief functions are defined on the same frame of discernment, but are based on *independent* arguments or bodies of evidence. The issue of independence is a critical factor when combining evidence and is an important research subject in Dempster-Shafer theory. The Dempster rule of combination is purely a conjunctive operation (AND). The combination rule results in a belief function based on conjunctive pooled evidence [Shafer, 1986, p.132].

Specifically, the combination (called the joint m_{12}) is calculated from the aggregation of two bpa's m_1 and m_2 in the following manner:

$$m_{12}(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - K} \quad \text{when } A \neq \emptyset \quad (11)$$

$$m_{12}(\emptyset) = 0 \quad (12)$$

$$\text{where } K = \sum_{B \cap C = \emptyset} m_1(B)m_2(C) \quad (13)$$

K represents basic probability mass associated with conflict. This is determined by the summing the products of the bpa's of all sets where the intersection is null. This rule is commutative, associative, but not idempotent or continuous.

The denominator in Dempster's rule, $1-K$, is a normalization factor. This has the effect of *completely* ignoring conflict and attributing any probability mass associated with conflict to the null set [Yager, 1987]. Consequently, this operation will yield counterintuitive results in the face of significant conflict in certain contexts. The problem with conflicting evidence and Dempster's rule was originally pointed out by Lotfi Zadeh in his review of Shafer's book, *A Mathematical Theory of Evidence* [Zadeh, 1984]. Zadeh provides a compelling example of erroneous results. Suppose that a patient is seen by two physicians regarding the patient's neurological symptoms. The first doctor believes that the patient has either meningitis with a probability of 0.99 or a brain tumor, with a probability of 0.01. The second physician believes the patient actually suffers from a concussion with a probability of 0.99 but admits the possibility of a brain tumor with a probability of 0.01. Using the values to calculate the m (brain tumor) with Dempster's rule, we find that $m(\text{brain tumor}) = Bel(\text{brain tumor}) = 1$. Clearly, this rule of combination yields a result that implies complete support for a diagnosis that both physicians considered to be very unlikely. [Zadeh, 1984, p.82]

In light of this simple but dramatic example of the counterintuitive results of normalization factor in Dempster's rule, a number of methods and combination operations that have been developed to address this problem posed by strongly conflicting evidence. We will discuss many of these alternatives in the following sections as well as the importance of conflict and context in the rule selection. We will find that in addition to the level or degree of conflict is important in determining the propriety of using Dempster's rule, the *relevance* of conflict also plays a critical role.

2.2.2: DISCOUNT+COMBINE METHOD

This *tradeoff* method was initially discussed in [Shafer, 1976] and deals with conflict just in the manner that the name implies. Specifically, when an analyst is faced with conflicting evidence, he/she can discount the sources first, and then combine the resulting functions with Dempster's rule (or an alternative rule) using a discounting function. This discounting function must account for the absolute reliability of the sources. Absolute reliability implies that the analyst is qualified to make distinctions between the reliability of experts, sensors, or other sources of information and can express this distinction between sources mathematically. [Dubois and Prade, 1992]

Shafer applies the discounting function to each specified *Belief*. Let $1-\alpha_i$ be the degree of reliability attributable to a particular belief function, A (Shafer calls this a degree of trust), where $0 \leq \alpha_i \leq 1$ and i is an index used to specify the particular

discounting function associated with a particular belief measure. $Bel^{\alpha_i}(A)$ then represents the discounted belief function defined by:

$$Bel^{\alpha_i}(A) = (1 - \alpha_i) Bel(A) \quad (14)$$

Shafer then averages all the belief functions associated with set A ($Bel^{\alpha_1}(A)$, $Bel^{\alpha_2}(A)$, ..., $Bel^{\alpha_n}(A)$) to obtain an average of n Bel, denoted by \overline{Bel} .

$$\overline{Bel}(A) = \frac{1}{n} (Bel^{\alpha_1}(A) + \dots + Bel^{\alpha_n}(A)) \quad (15)$$

for all subsets A of the universal set X .

Consequently, the discount and combine method uses an averaging function as the method of combination. This is to be used when all the belief functions to be combined are highly conflicting and the discounting rate is not too small. This can also be used to eliminate the influence of any strongly conflicting single belief function provided that the remaining belief functions do not conflict too much with each other and the discount rate is not too small or too large. Alternatively, for this case one could also eliminate the strongly conflicting belief altogether if that is reasonable. [Shafer, 1976]

2.2.3: YAGER'S MODIFIED DEMPSTER'S RULE

The most prominent of the alternative combination rules is a class of unbiased operators developed by Ron Yager. [Yager, 1987a] Yager points out that an important feature of combination rules is the ability to update an already combined structure when new information becomes available. This is frequently referred to as updating and the algebraic property that facilitates this is associativity. Dempster's rule is an example of an associative combination operation and the order of the information does not impact the resulting fused structure. [Yager, 1987b]

Yager points out that in many cases a non-associative operator is necessary for combination. A familiar example of this is the arithmetic average. The arithmetic average is not itself associative, i.e., one cannot update the information by averaging an average of a given body of data and a new data point to yield a meaningful result. However, the arithmetic average can be updated by adding the new data point to the sum of the pre-existing data points and dividing by the total number of data points. This is the concept of a *quasi-associative* operator that Yager introduced in [Yager, 1987b]. Quasi-associativity means that the operator can be broken down into associative suboperations. Through the notion of quasi-associative operator, Yager develops a general framework to look at combination rules where associative operators are a proper subset.

To address the issue of conflict, Yager starts with an important distinction between the basic probability mass assignment (m) and what he refers to as the *ground probability mass assignment* (designated by q). The major differences between the basic probability assignment and the ground probability assignment are in the normalization factor and the mass attributed to the universal set. The combined ground probability assignment is defined in equation 16.

$$q(A) = \sum_{B \cap C = A} m_1(B) m_2(C) \quad (16)$$

where A is the intersection of subsets B and C (both in the power set $P(X)$), and $q(A)$ denotes the ground probability assignment associated with A . Note that there is no normalization factor. This rule is known as Yager's combination rule or sometimes the Modified Dempster's Rule.

Though the Yager rule of combination is not associative, the combined structure $q(A)$ can be used to include any number of pieces of evidence. Assume m_1, m_2, \dots, m_n are the basic probability assignments for n belief structures. Let F_i represent the set of focal elements associated with the i^{th} belief structure (m_i) which are subsets of the universal set X . A_i represents an element of the focal set. Then the combination of n basic probability assignment structures is defined by [Yager, 1987a]:

$$q(A) = \sum_{\cap_{i=1}^n A_i = A} m_1(A_1) m_2(A_2) \dots m_n(A_n) \quad (17)$$

Through the quasiassociativity that Yager describes, the combined structure $q(A)$ can be updated based on new evidence. This is performed by combining the ground probability assignment associated with the new evidence and the ground probability assignment of the already existing combination through the above formulas (Equation 16) and then converting the ground probability assignments to basic probability assignments described below. (Equations 19-21)

As previously mentioned, one obvious distinction between combination with the basic and the ground probability assignment functions is the absence of the normalization factor $(1-K)$. In Yager's formulation, he circumvents normalization by allowing the ground probability mass assignment of the null set to be greater than 0, i.e.

$$q(\emptyset) \geq 0 \quad (18)$$

$q(\emptyset)$ is calculated in exactly in the same manner as Dempster's K (conflict) in Equation 13. Then Yager adds the value of the conflict represented by $q(\emptyset)$ to the ground probability assignment of the universal set, $q(X)$, to yield the conversion of the ground probabilities to the basic probability assignment of the universal set $m^Y(X)$:

$$m^Y(X) = q(X) + q(\emptyset) \quad (19)$$

Consequently, instead of normalizing out the conflict, as we find in the case of the Dempster rule, Yager ultimately attributes conflict to the universal set X through the conversion of the ground probability assignment to the basic probability assignments. The interpretation of the mass of the universal set (X) is the degree of *ignorance*. Dempster's rule has the effect of changing the evidence through the normalization and the allocation of conflicting mass to the null set. Yager's rule can be considered as an epistemologically honest interpretation of the evidence as it does not change the evidence by normalizing out the conflict. In Yager's rule, the mass associated with conflict is attributed to the universal set and thus enlarges this degree of ignorance. [Yager, 1987a]

Upon inspection of the two combination formulas it is clear that Yager's rule of combination yields the same result as Dempster's rule when conflict is equal to zero, ($K = 0$ or $q(\emptyset) = 0$). [Yager, 1987a] The basic algebraic properties that this rule satisfies is commutativity and quasiassociativity, but not idempotence or continuity.

The ground probability assignment functions (q) for the null set, \emptyset , and an arbitrary set A , are converted to the basic probability assignment function associated with this Yager's rule (m^Y) by [Yager 1987a]:

$$m^Y(\emptyset) = 0 \quad (20)$$

$$m^Y(A) = q(A) \quad (21)$$

The basic probability assignments associated with Yager's rule (m^Y) are not the same as with Dempster's rule (m). Yager provides the relation between the ground assignments and Dempster's rule [Yager 1987a]:

$$m(\emptyset) = 0 \quad (22)$$

$$m(X) = \frac{q(X)}{1 - q(\emptyset)} \quad (23)$$

$$m(A) = \frac{q(A)}{1 - q(\emptyset)} \quad (24)$$

for $A \neq \emptyset, X$

To summarize, these are the important attributes of Yager's rule of combination:

1. The introduction of the general notion of quasi-associative operators and the expansion of the theoretical basis for the combination and updating of evidence where the associative operators are a proper subset of the quasi-associative operators.
2. The introduction of the ground probability assignment functions (q) and their relation to the basic probability assignments (m^Y) associated with Yager's rule and the basic probability assignments (m) associated with Dempster's rule.
3. The rule does not filter or change the evidence through normalization.
4. The allocation of conflict to the universal set (X) instead of to the null set (\emptyset). Thus mass associated with conflict is interpreted as the degree of ignorance.

2.2.4: INAGAKI'S UNIFIED COMBINATION RULE

This combination rule was introduced by Toshiyuki Inagaki. [Inagaki, 1991] Inagaki takes advantage of the ground probability assignment function (q) that Yager defined in [Yager, 1987a] to define a continuous parametrized class of combination operations which subsumes both Dempster's rule and Yager's rule. Specifically, Inagaki argues that every combination rule can be expressed as:

$$m(C) = q(C) + f(C)q(\emptyset) \quad (25)$$

where $C \neq \emptyset$

$$\sum_{C \subset X, C \neq \emptyset} f(C) = 1 \quad (26)$$

$$f(C) \geq 0 \quad (27)$$

From Equation 25 the function, f , can be interpreted as a scaling function for $q(\emptyset)$, where the conflict (represented by the parameter k) is defined by:

$$k = \frac{f(C)}{q(C)} \quad \text{for any } C \neq X, \emptyset \quad (28)$$

Inagaki restricts consideration to the class of combination rules that satisfy the following property:

$$\frac{m(C)}{m(D)} = \frac{q(C)}{q(D)} \quad (29)$$

for any nonempty sets C and D which are distinct from X or \emptyset . By maintaining the ratio between m and q consistently, this equation implies that there is no “meta-knowledge” of the credibility or reliability of sources/experts. If an analyst applied a weighting factor to the evidence based on some extra knowledge about the credibility of the sources, in general, this would change the ratio and the equality would not hold. As a result of this restriction and its implication, Inagaki’s rule applies *only* to the situations where there is no information regarding the credibility or reliability of the sources. [Inagaki, 1991]

From the general expression (Equation 25) and the restriction (Equation 26) and the definition of k (Equation 28), Inagaki derives his unified combination rule denoted by m_k^U .

$$m_k^U(C) = [1 + kq(\emptyset)] q(C), \text{ where } C \neq X, \emptyset \quad (30)$$

$$m_k^U(X) = [1 + kq(\emptyset)] q(X) + [1 + kq(\emptyset) - k] q(\emptyset) \quad (31)$$

$$0 \leq k \leq \frac{1}{1 - q(\emptyset) - q(X)} \quad (32)$$

The parameter k is used for normalization. The determination of k is an important step in the implementation of this rule, however, a developed well-justified procedure for determining k is lacking in the literature reviewed for this report. Tanaka and Klir refer to the determination of k either through experimental data, simulation, or the expectations of an expert in the context of a specific application. In addition, they provide an example for the determination of k and the resulting affect on m for monitoring systems [Tanaka

and Klir, 1999]. In [Inagaki, 1991], Inagaki poses the optimization problem for the selection of k to be an open and critical research question. Despite this, Inagaki discusses the rules in the context of an application where he demonstrates the values of *Belief* and *Plausibility* as a function of k and the implications on the choice of a safety control policy.

The value of k directly affects the value of the combined basic probability assignments and will collapse to either Dempster's rule or Yager's rule under certain circumstances. When $k = 0$, the unified combination rule coincides with Yager's rule.

When $k = \frac{1}{1 - q(\emptyset)}$, the rule corresponds to Dempster's rule. The parameter k gives

rise to an entire parametrized class of possible combination rules that interpolate or extrapolate Dempster's rule. [Inagaki, 1991] This is schematically represented in the Figure 5 from [Inagaki, 1991]:

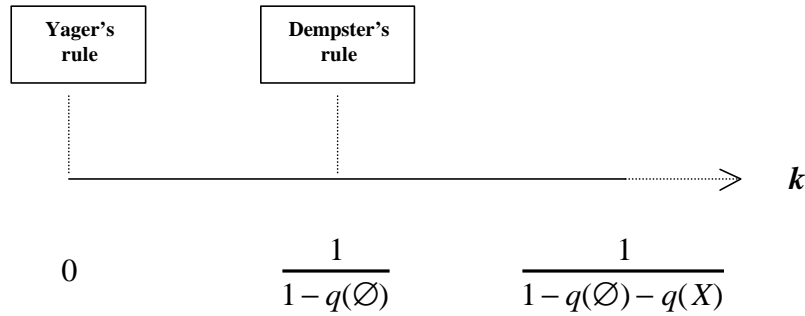


Figure 5: The Possible Values of k in Inagaki's Unified Combination Rule

The only combination rule of this parametrized class that is associative is the one that corresponds to Dempster's rule. Every combination rule represented by the unified combination rule is commutative though not idempotent or continuous. Inagaki considers the effect of non-associativity in applications to be an open research question. [Inagaki, 1991]

As is pointed out by Tanaka and Klir [Tanaka and Klir, 1999], the most extreme rule (referred to as “the extra rule” and denoted by the parameter k_{ext}) availed by this formulation is when k is equal to the upper bound:

$$m_{k_{ext}}^U(C) = \frac{1 - q(X)}{1 - q(X) - q(\emptyset)} q(C) \quad (33)$$

for $C \neq X$,

$$m_{k_{ext}}^U(X) = q(X) \quad (34)$$

As can be seen in Equation 33, the value of $q(C)$ is scaled by the factor, $\frac{1 - q(X)}{1 - q(X) - q(\emptyset)}$ to yield the corresponding basic probability function $m_{k_{ext}}^U$. The interpretation of the

extreme rule of Inagaki's class is that both conflict (represented by $q(\emptyset)$) and the degree of ignorance (represented by the probability mass associated with the universal set, $q(X)$) are used to scale the resulting combination. This acts as a filter for the evidence.

Inagaki studied the ordering relations of the three rules: Dempster's rule, Yager's rule, and this "extra rule" and the propriety of their application in fault-warning safety control policy. [Inagaki, 1991] Tanaka and Klir point out that the selection of the parameter k essentially determines how to cope with conflicting information. Yager's rule ($k=0$) assigns conflict to the universal set and does not change the evidence. Dempster's rule ($k=1/[1-q(\emptyset)]$) tremendously filters the evidence by ignoring all conflict. Inagaki's extreme rule ($k=1/[1-q(\emptyset)-q(X)]$) also filters the evidence by scaling both conflict and ignorance, but the degree of influence of the scaling is determined by the relative values of $q(X)$ and $q(\emptyset)$. k has the effect of scaling the importance of conflict as it is represented in the resulting combination. The greater the value of k , the greater the change to the evidence. As noted earlier, a well-justified procedure for the selection of k is as essential step toward implementing this rule in an application.

The important contributions of Inagaki's Unified rule of combination can be summarized as follows:

1. The use of Yager's ground functions to develop a parametrized class of combination rules that subsumes both Dempster's rule and Yager's rule.
2. Inagaki compares and orders three combination rules: Dempster's rule, Yager's rule, and the Inagaki extra rule, in terms of the value of m in the context of an application.

2.2.5: ZHANG'S CENTER COMBINATION RULE

Lianwen Zhang [Zhang, 1994] also provides an alternative combination rule to Dempster's rule. In addition, he offers a two frame interpretation of Dempster-Shafer theory: Suppose there are two frames of discernment, S and T . These could be the opinions of two experts. Between these frames is a compatibility relation, C , which is a subset of the Cartesian product $S \times T$. We are concerned with the truth in T but the only available probability P is about the truth in S . Because of this compatibility relation it follows that information about S provides some information of T . This information is summarized as a Belief function for any subset of A of T . The belief function for A can be written as:

$$Bel(A) = P\{s | s \in S \text{ and } \exists t \in A \text{ s.t. } (s, t) \in C\} \quad (35)$$

The value of this two frame interpretation of Dempster-Shafer Theory is recognizing the contribution of DST as a new technique for propagating probabilities through logical links, i.e. one can obtain information about one frame of discernment from its logical relation to another frame. Specifically, if the only information available between the elements of S and T (denoted by s and t , respectively) is through the logical constraint (i.e., their compatibility relation C), traditional Bayesian theory has difficulty providing for a meaningful inference regarding s and t . Dempster-Shafer theory can

represent the relationship, C , between s and t by a subset of the joint frame $S \times T$. [Zhang, 1994]

With respect to the rule of combination, Zhang points out that Dempster's rule fails to consider how focal elements intersect. [Zhang, 1994] To define an alternative rule of combination, he introduces a measure of the intersection of two sets A and B assuming finite sets. This is defined as the ratio of the cardinality of the intersection of two sets divided by the product of the cardinality of the individual sets. Zhang denotes this relation with $r(A,B)$:

$$r(A, B) = \frac{|A \cap B|}{|A| |B|} = \frac{|C|}{|A| |B|} \quad (36)$$

where $A \cap B = C$. The resulting combination rule scales the products of the basic probability assignments of the intersecting sets ($A \cap B = C$) by using a measure of intersection, $r(A,B)$ defined in Equation 36. This is repeated for every intersecting pair that yields C . The scaled products of the masses for all pairs whose intersection equals C are summed and multiplied by a factor k . In this case, k is a renormalization factor that is independent of C , m_1 , and m_2 . This renormalization factor provides that the sum of the basic assignments to add to 1.

$$m(C) = k \sum_{A \cap B = C} \left[\frac{|C|}{|A| |B|} m_1(A) m_2(B) \right] \quad (37)$$

The case where $|C| = |A||B|$, this rule will correspond to the Dempster rule.

It is important to note that the measure of intersection of two sets ($r(A,B)$) can be defined in other ways, for example by dividing the cardinality of intersection of A and B by the cardinality of the union of sets A and B . This would have the effect of a different scaling on the product of the m 's that could be compensated for in the sum of all the basic probability assignments by the renormalization factor k . Many combination rules could be devised in the spirit of Zhang's center combination rule by defining a reasonable measure of intersection. This particular rule is commutative but not idempotent, continuous, or associative.

The important contributions of Zhang's work:

1. The two frame interpretation of Dempster-Shafer theory
2. The introduction of a measure of intersection of two sets ($r(A,B)$) based on cardinality.
3. The center combination rule based on a measure of intersection of two sets that could be modified by any other reasonable measure of intersection.

2.2.6: DUBOIS AND PRADE'S DISJUNCTIVE CONSENSUS RULE

Dubois and Prade take a set-theoretic view of a body of evidence to form their disjunctive consensus rule in [Dubois, Prade, 1986; Dubois, Prade, 1992]. They define the union of the basic probability assignments $m_1 \cup m_2$ (denoted by $m_{\cup}(C)$) by extending the set-theoretic union:

$$m_{\cup}(C) = \sum_{A \cup B = C} m_1(A) m_2(B) \quad (38)$$

For all A of the power set X . The union does not generate any conflict and does not reject any of the information asserted by the sources. As such, no normalization procedure is required. The drawback of this method is that it may yield a more imprecise result than desirable.

The union can be more easily performed via the belief measure: Let $Bel_1 \cup Bel_2$ be the belief measure associated with $m_1 \cup m_2$. Then for every subset A of the universal set X ,

$$Bel_1(A) \cup Bel_2(A) = Bel_1(A) Bel_2(A) \quad (39)$$

The disjunctive pooling operation is commutative, associative, but not idempotent.

2.2.7: MIXING OR AVERAGING

Mixing (or p-averaging or averaging) is a generalization of averaging for probability distributions. [Ferson and Kreinovich, 2002] This describes the frequency of different values within an interval of possible values in the continuous case or in the discrete case, the possible simple events. The formula for the "mixing" combination rule is just

$$m_{1...n}(A) = \frac{1}{n} \sum_{i=1}^n w_i m_i(A) \quad (40)$$

where m_i 's are the bpa's for the belief structures being aggregated and the w_i 's are weights assigned according to the reliability of the sources. This is very similar to the discount and combine rule proposed by Shafer in that they are both averaging operations, but they differ in which structures are being pooled. In the case of mixing, it is the basic probability assignment, m ; in the case of discount and combine, it is Bel .

Mixing generalizes the averaging operation that is usually used for probability distributions. In particular, suppose that the input Dempster-Shafer structures are probability distributions, that is, suppose that both structures consist of an element in which each basic probability mass is associated with a single point. If one applies the mixing operation to these inputs, the result will be a Dempster-Shafer structure all of whose masses are also at single points. These masses and points are such that the Dempster-Shafer structure is equivalent to the probability distribution that would have been obtained by mixing the probability distributions, that is, by simply averaging the probabilities for every point. None of the other Dempster-Shafer aggregation rules would give this same answer. Insofar as averaging of probability distributions via mixing is regarded as a natural method of aggregating probability distributions, it might also be considered as a reasonable approach to employ with Dempster-Shafer structures, and that is why it is considered here. Like mixing of probability distributions, mixing in Dempster-Shafer theory is idempotent and commutative. It's not associative but it is quasi-associative.

2.2.8: CONVOLUTIVE X-AVERAGING

Convoluteive x-averaging (or c-averaging) is a generalization of the average for scalar numbers. [Ferson and Kreinovich, 2002] This is given by the formula:

$$m_{12}(A) = \sum_{\frac{B+C}{2}=A} m_1(B)m_2(C) \quad (41)$$

Like the mixing average, this can be formulated to include any number of bpa's, n , in the following equation:

$$m_{1,...,n}(A) = \sum_{\frac{A_1+...+A_n}{n}=A} \prod_{i=1}^n m_i(A_i) \quad (42)$$

Suppose that the input Dempster-Shafer structures are scalar numbers, that is, suppose that both structures consist of a single element where all mass is at a single point. If one applies the convoluteive average operation to these inputs, the result will be a Dempster-Shafer structure all of whose mass is at a single point, the same point one gets by simply averaging the two scalar numbers. None of the other Dempster-Shafer aggregation rules would give this answer. Insofar as "averaging" is regarded as a natural method of aggregating disparate pieces of information, it might also be considered as a reasonable approach to employ with Dempster-Shafer structures, and that is why it is considered here.

Like averaging of scalar numbers, the convoluteive average is commutative. Also like scalar averaging, the convoluteive average is not associative, although it is quasi-associative. Unlike scalar averaging, however, it is not idempotent.

2.2.9: OTHER RULES OF COMBINATION

There are still other rules of combination available for Dempster-Shafer theory that will not be considered here. The remaining rules and the motivation for their exclusion are summarized as follows:

Smets' rule: Some authors refer to this as a distinct rule, however, this is essentially the Dempster rule applied in Smets' Transferable Belief Model. Smet's model entails a slightly different conception and formulation of Dempster-Shafer theory, though it essentially distills down to the same ideas. [Smets, 2000]

Qualitative Combination Rule: This rule was proposed by Yao and Wong in their paper [Yao and Wong, 1994]. This rule requires the definition of a binary relation expressing the preference of one proposition or source, over another. Then a distance function is defined between two belief relations. All the distances over all the pairs of the relation are summed to obtain an overall distance. The resulting combination rule combines the relations in such a way as to minimize the overall distance. This type of formulation of

DST, as its name implies is qualitative, whereas in engineering analyses, we expect to be dealing with quantitative data. Consequently, it is beyond the scope of this study.

Yen's rule: This rule is based on an extension of Dempster-Shafer theory by randomizing the compatibility relations and using Zadeh's relational model of Dempster-Shafer theory. As this extension of DST is not the focus of the current paper and the rule is similar to Zhang's rule, a discussion of Yen's rule is not included. [Yen, 1989]

Envelope, Imposition, and Horizontal x-Averaging: These are three methods of combination that can be applied to belief structures that have been converted to "generalized cumulative distribution functions" or p-boxes. The resultant combination can be reinterpreted as a belief structure but with a complicated relationship with the original inputs. A discussion of these methods in the context of p-boxes can be found in [Ferson and Kreinovich, 2002].

3: DEMONSTRATION OF COMBINATION RULES

In this section, we demonstrate the differences between the various combination rules for discrete and interval-type data. In Section 3.1, the data will be given by discrete values and in Section 3.2 the data will be given by intervals.

3.1: Data given by discrete values

Suppose two experts are consulted regarding a system failure. The failure could be caused by Component A, Component B or Component C. The first expert believes that the failure is due to Component A with a probability of 0.99 or Component B with a probability of 0.01 (denoted by $m_1(A)$ and $m_1(B)$, respectively). The second expert believes that the failure is due to Component C with a probability of 0.99 or Component B with a probability of 0.01 (denoted by $m_2(C)$ and $m_2(B)$, respectively). The distributions can be represented by the following:

Expert 1:

$m_1(A) = 0.99$ (failure due to Component A)

$m_1(B) = 0.01$ (failure due to Component B)

Expert 2:

$m_2(B) = 0.01$ (failure due to Component B)

$m_2(C) = 0.99$ (failure due to Component C)

3.1.1: Dempster's Rule

The combination of the masses associated with the experts is summarized in Table 1.

			Expert 1			
			A	B	C	Failure Cause
			0.99	0.01	0	m_1
Expert 2	Failure Cause	m_2				
	A	0	$m_1(A) m_2(A) = 0$	$m_1(B) m_2(A) = 0$	$m_1(C) m_2(A) = 0$	
	B	0.01	$m_1(A) m_2(B) = 0.0099$	$m_1(B) m_2(B) = 0.0001$	$m_1(C) m_2(B) = 0$	
	C	0.99	$m_1(A) m_2(C) = 0.9801$	$m_1(B) m_2(C) = 0.0099$	$m_1(C) m_2(C) = 0$	

Table 1: Dempster Combination of Expert 1 and Expert 2

Using Equations 11-13:

1. To calculate the combined basic probability assignment for a particular cell, simply multiply the masses from the associated column and row.
2. Where the intersection is nonempty, the masses for a particular set from each source are multiplied, e.g., $m_{12}(B) = (0.01)(0.01) = 0.0001$.
3. Where the intersection is empty, this represents conflicting evidence and should be calculated as well. For the empty intersection of the two sets A and C associate with Expert 1 and 2, respectively, there is a mass associated with it. $m_1(A) m_2(C) = (0.99)(0.99) = 0.9801$.
4. Then sum the masses for all sets and the conflict.
5. The only nonzero value is for the combination of B , $m_{12}(B) = 0.0001$. In this example there is only one intersection that yields B , but in a more complicated example it is possible to find more intersections to yield B .
6. For K , there are three cells that contribute to conflict represented by empty intersections. Using Equation 13, $K = (0.99)(0.01) + (0.99)(0.01) + (0.99)(0.99) = 0.9999$
7. Using Equation 11, calculate the joint, $m_1(B) m_2(B) = (0.01)(0.01) / [1 - 0.9999] = 1$

Though there is highly conflicting evidence, the basic probability assignment for the failure of Component B is 1, which corresponds to a $Bel(B) = 1$. This is the result of normalizing the masses to exclude those associated with conflict. This points to the inconsistency when Dempster's rule is used in the circumstances of significant relevant conflict that was pointed out by Zadeh.

3.1.2: Yager's Rule

For this simple problem, Yager's rule will yield the almost the same matrix as with Dempster's rule. However, there are some important exceptions in the nomenclature and eventually the allocation of conflict:

1. Instead of basic probability assignments (m), Yager calls these ground probability assignments (q)
2. Instead of using K to represent the conflict, Yager uses the $q(\emptyset)$ which is calculated in the exact same way as K . (Equation 13)

Using Equation 16, the combination is calculated:

$$q_{12}(B) = m_{12}(B) = (.01)(.01) = .0001$$

Here the combination is not normalized by the factor $(1-K)$. When Yager converts the ground probability assignments (q) to the basic probability assignments (m), the mass for a particular joint remains the same and the mass associated with conflict is attributed to the universal set X that represents the degree of ignorance (or lack of agreement). So in this case the $m(X)$ is 0.9999. To convert the basic probability assignment to the lower bound *Bel*, the $Bel(B)$ is equal to the $m(B)$ ($Bel(B) = .0001$), as this is the only set that satisfies the criteria for *Belief* ($B \subseteq B$). This approach results in a significant reduction of the value for *Belief* and a large expansion of *Plausibility*. Note that the value of *Belief* is substantially smaller than either the experts' estimates would yield individually and in such a case, this may be counterintuitive.

3.1.3: Inagaki's Rule

Once again the matrix is calculated in the same manner as in case of the Dempster rule. Inagaki uses the ground probability functions similar to Yager. Ultimately, the value of $m_{12}(B)$ obtained by Inagaki's rule depends on the value of k which is now a parameter. It is suggested by the literature that the value of k should be determined experimentally or by expert expectation though an exact procedure is lacking. Figure 6 demonstrates the behavior of the Inagaki combination as a function of the value of k for this problem.

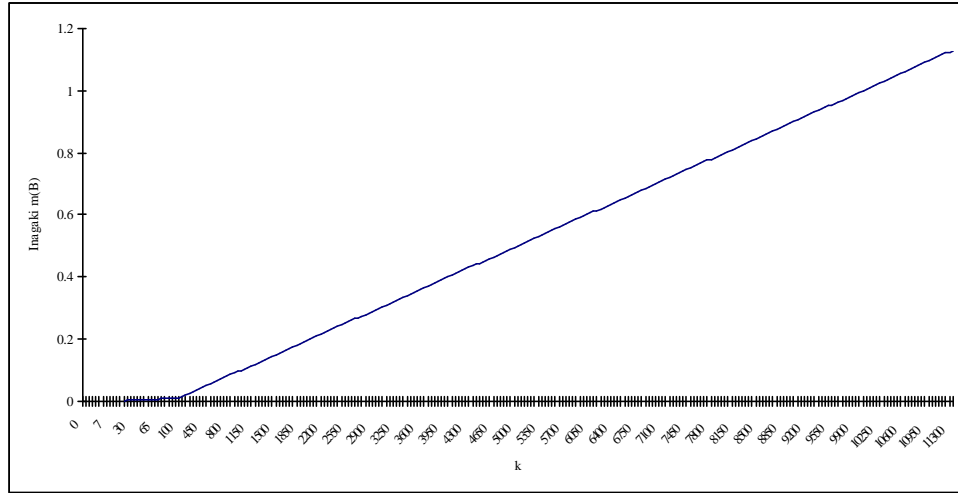


Figure 6: The value of $m_{12}(B)$ as a function of k in Inagaki's rule

When $k = 0$, Inagaki's combination will obtain the same result as Yager's ($m_{12}(B) = .0001$). When $k = \frac{1}{1 - q(\emptyset)} = \frac{1}{1 - 0.9999} = 10000$, Inagaki's rule corresponds to Dempster's rule ($m_{12}(B) = 1$). Because there is no mass associated with the universal set $q(X)$, in this case, Inagaki's extra rule is the same as Dempster's rule. Although, the calculation can be extended beyond Dempster's rule, any value for the combination greater than 1 does not make sense because sums of all masses must be equal to 1. Corresponding to the increasing value of k , is the increase in the filtering of the evidence.

3.1.4: Zhang's Rule

Recall from Equations 36 and 37 for Zhang's rule, in addition to calculating the product of the masses like in Table 1, we must also calculate the measure of intersection based on the cardinality of the sets. The cardinality of each of the sets A, B, and C is 1. In this case we find that the only nonzero intersection of the sets is set B obtained from the evaluation of B by both Experts 1 and 2. Since $|B| = |B||B|$, we find that the Zhang combination corresponds to the Dempster combination. This points to two problems with Zhang's measure of intersection:

1. The equivalence with Dempster's rule when the cardinality is 1 for all relevant sets or when the $|C| = |A||B|$ in the circumstance of conflicting evidence. (This should not pose a problem if there is no significant conflict.)
2. If the cardinality of B was greater than 1, even completely overlapping sets will be scaled.

3.1.5: Mixing

The formulation for mixing in this case corresponds to the sum of $m_1(B)(1/2)$ and $m_2(B)(1/2)$. From Equation 40:

$$m_{12}(A) = (1/2)(0.99) = 0.445$$

$$m_{12}(B) = (1/2)(0.01) + (1/2)(0.01) = 0.01$$

$$m_{12}(C) = (1/2)(0.99) = 0.445$$

3.1.6: Dubois and Prade's Disjunctive Consensus Pooling

The unions of multiple sets based on the calculations from Table 1 that can be summarized in Table 2.

Union	m_E	Linguistic Interpretation
$A \cup A$	0	Failure of Component A
$A \cup B$	0.0099	Failure of Component A or B
$A \cup C$	0.9801	Failure of Component A or C
$B \cup B$	0.0001	Failure of Component B
$B \cup C$	0.0099	Failure of Component B or C
$C \cup C$	0	Failure of Component C
$A \cup B \cup C$	1	Failure of Component A or B or C

Table 2: Unions obtained by Disjunctive Consensus Pooling

3.2: Data given by intervals

Using the operations discussed above, now we will consider the aggregation of three sources of information where the information is given as intervals. Interval-based data is common to problems involving parametric uncertainty for physical parameters like conductivity, diffusivity, or viscosity. Suppose there is an experiment that provides multiple intervals for an uncertain parameter from three sources A, B, and C that must be combined. The intervals associated with sources A, B, and C are summarized in the Tables 3, 4, and 5, respectively. Figures 7, 8, and 9 depict the intervals and the basic probability assignments graphically with a “generalized cumulative distribution function” (gcdf). This is the probabilistic concept of cumulative distribution function generalized to Dempster-Shafer structures where the focal elements (intervals) are represented on the x-axis and the cumulative basic probability assignments on the y-axis. A discussion of the generalization of some of the ideas from the theory of random variable to the Dempster-Shafer environment is discussed in [Yager, 1986].

Interval	m_i
[1,4]	0.5
[3,5]	0.5

Table 3: The interval-based data for A and the basic probability assignments

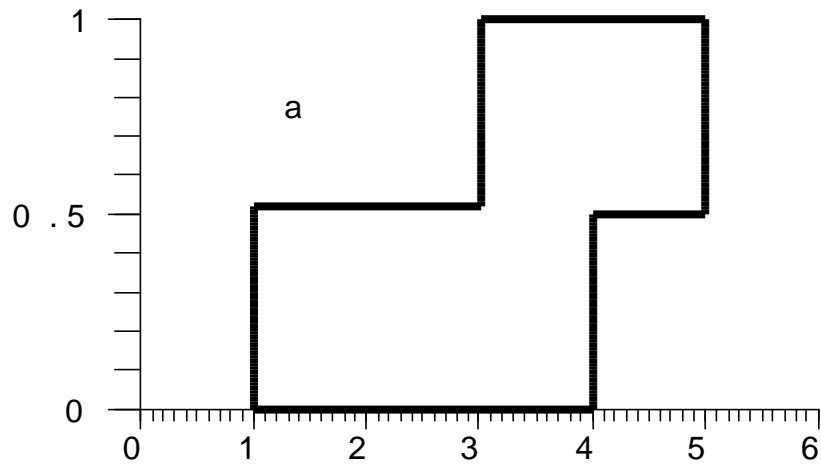


Figure 7: The gcdf of A

Interval	m_2
[1,4]	0.3333
[2,5]	0.3333
[3,6]	0.3333

Table 4: The interval-based data for B and the basic probability assignments

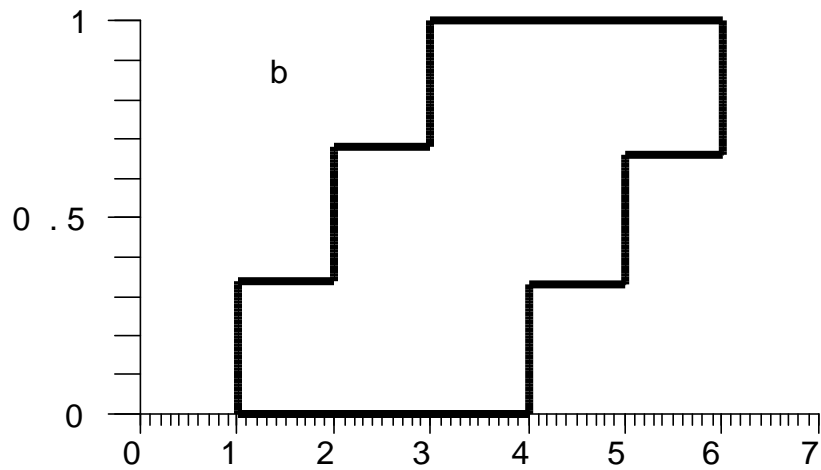


Figure 8: The gcdf of B

Interval	m_3
[6,10]	0.3333
[9,11]	0.3333
[12,14]	0.3333

Table 5: The interval-based data for C and the basic probability assignments

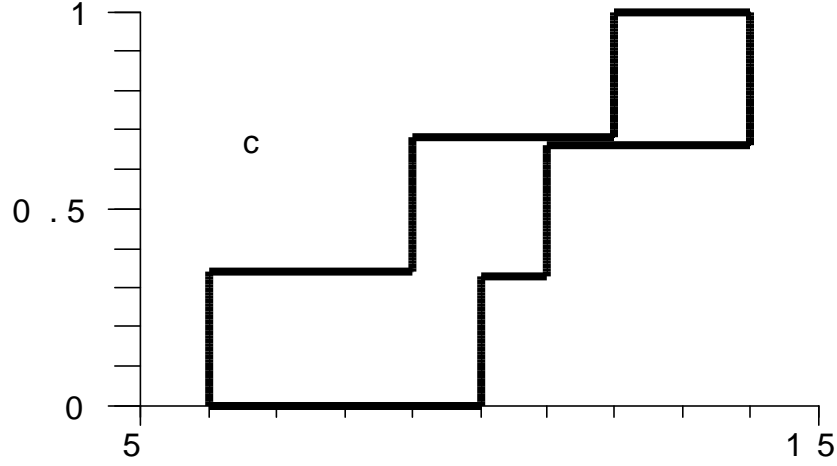


Figure 9: The gcdf of C

Without any combination operation, the gcdf's of A, B, and C are represented in Figure 10.

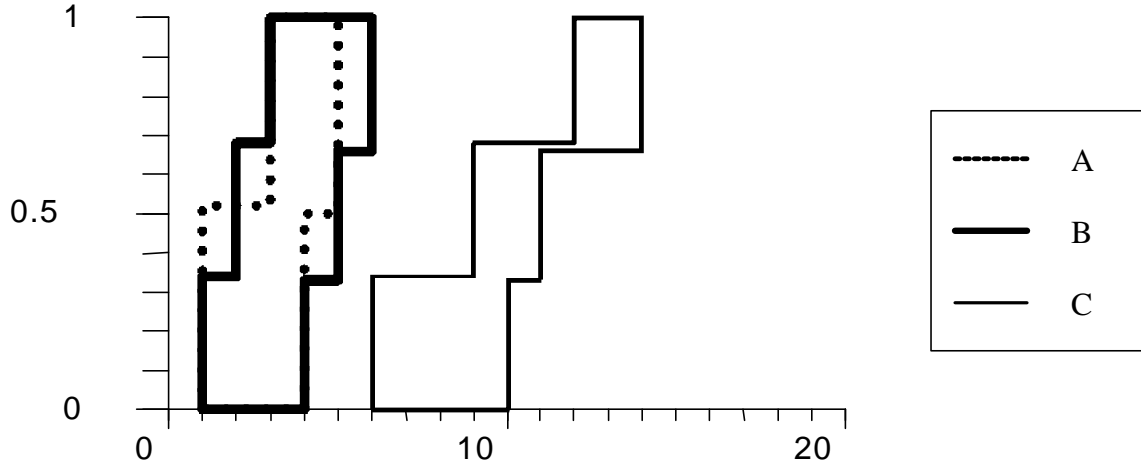


Figure 10: The gcdf's of A, B, and C without any combination operation

As is evident in Figure 10 and Tables 3,4 and 5, the data for A and B is consistent with each other. However the data for A and C are disjoint. First, we will consider the combination of consistent data (A and B) and then the combination of the disjoint data (A and C) with the combination rules discussed in Section 2.

3.2.1: Dempster's Rule

The calculation of Dempster's rule (Equation 11-13) is summarized in Table 6.

			A			
			Interval	m	Interval	m
			[1, 4]	0.5	[3, 5]	0.5
B	Interval	m				
	[1, 4]	0.33333	[1, 4]	0.16667	[3, 4]	0.16667
	[2, 5]	0.333333	[2, 4]	0.16667	[3, 5]	0.16667
	[3, 6]	0.333333	[3, 4]	0.16667	[3, 5]	0.16667

Table 6: Combination of A and B with Dempster's Rule

Note that the intersection of two intervals is defined by the maximum of the two lower bounds and the minimum of the two upper bounds corresponding to an intersection. The bpa's for like intervals are summed, i.e. [1,4] has a value for m of 0.166667; [2,4] has an m value of 0.166667; [3,4] has a value of 0.33334; and [3,5] has an m value of 0.33334.

The resulting structure of the combination of A and B using Dempster's rule is depicted in Figure 11.

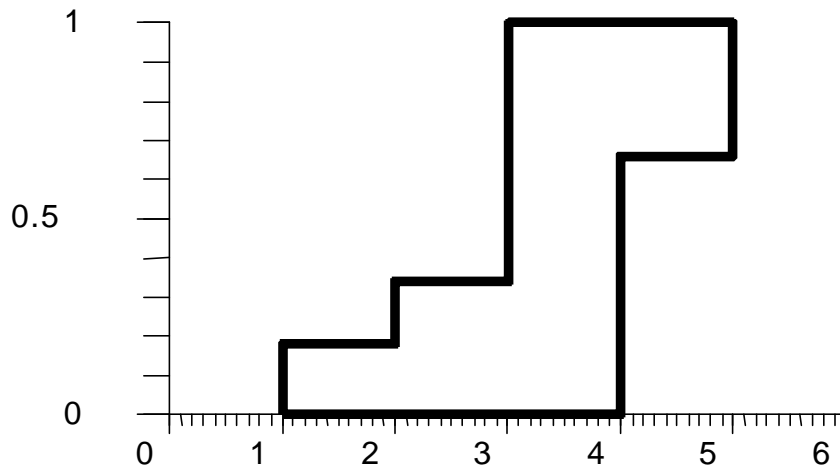


Figure 11: The gcdf of the combination of A and B using Dempster's rule

The combination of A and C using Dempster's rule is not possible due to the normalization factor.

3.2.2: Yager's Rule

As the evidence from A and B is consistent, the calculations for Yager's rule are same as in Table 6. The resulting structure of the combination of A and B using Yager's rule (Figure 12) is also the same as with Dempster's rule.

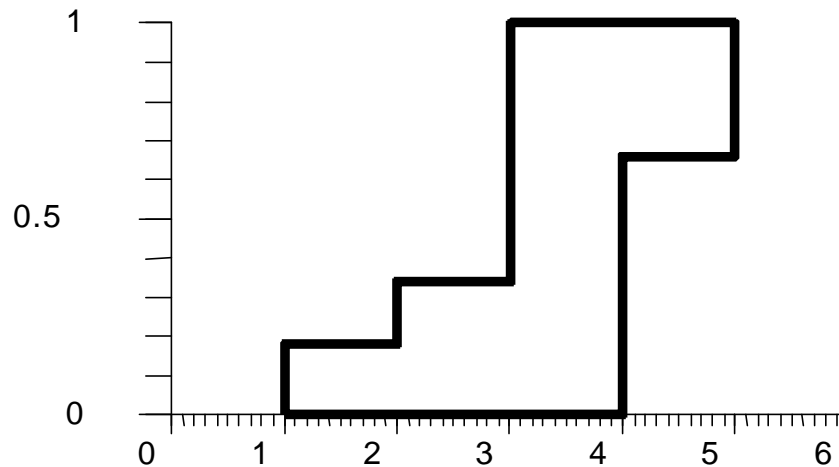


Figure 12: The combination of A and B using Yager's rule

Unlike the Dempster's case, Yager's rule can be calculated for the combination of A and C. However, since the evidence is entirely conflicting, all of the basic probability mass is attributed to the universal set. In the continuous domain this corresponds to the real line. As noted earlier, the mass allocated to the universal set is interpreted as the degree of ignorance or the degree of lack of agreement among sources.

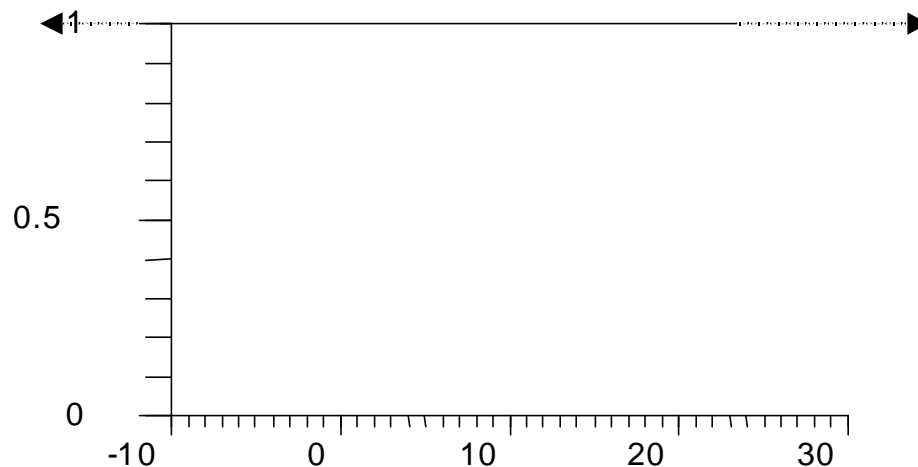


Figure 13: The gcdf of the combination of A and C using Yager's rule

3.2.3: Inagaki's Rule

Using $k=0$, we obtain the same calculations as Yager's rule and Dempster's rule.

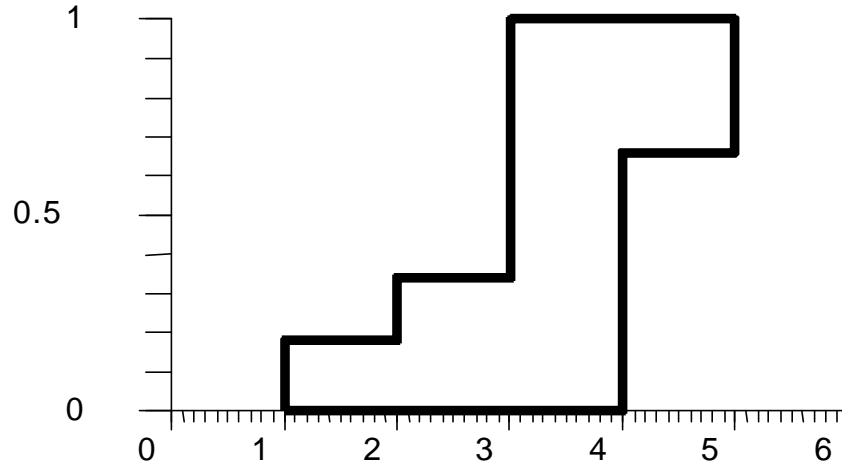


Figure 14: The Inagaki combination of A and B for $k=0$

As expected, we find the same calculations for the combination of A and B where $k=1$.

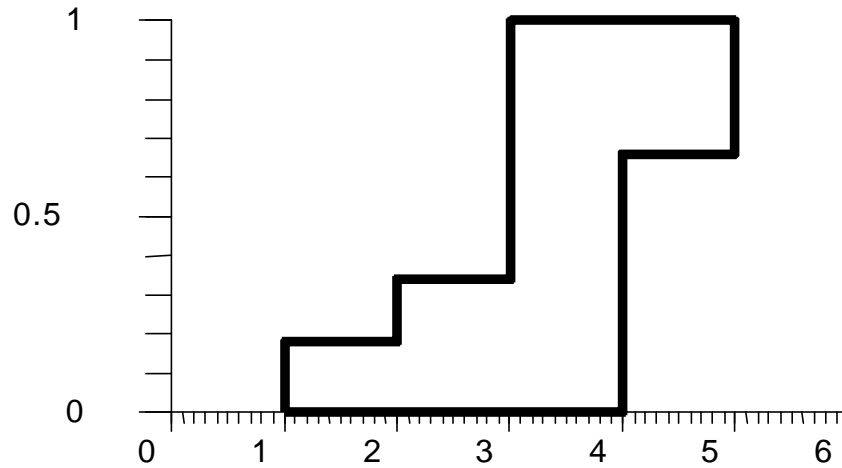


Figure 15: The Inagaki combination of A and B where $k = 1$

3.2.4: Zhang's Rule

For the step-by step calculation of Zhang's combination rule, first take the masses obtained by the simple product of the marginals.

			A			
			Interval	m	Interval	m
			[1, 4]	0.5	[3, 5]	0.5
B	Interval	m	[1, 4]	0.166667	[3, 4]	0.166667
	[1, 4]	0.333333	[2, 4]	0.166667	[3, 5]	0.166667
	[2, 5]	0.333333	[3, 4]	0.166667	[3, 5]	0.166667
	[3, 6]	0.333333	[3, 4]	0.166667	[3, 5]	0.166667

Table 7: The combination of the marginals with Zhang's rule

Next we calculate a measure of intersection. In the continuous case, we have elected to interpret interval length for the calculation of the measure of intersection.

			A			
			Interval	A length	Interval	A length
			[1, 4]	3	[3, 5]	2
B	Interval	B length	Interval	A∩B length	Interval	A∩B length
	[1, 4]	3	[1, 4]	3	[3, 4]	1
	[2, 5]	3	[2, 4]	2	[3, 5]	2
	[3, 6]	3	[3, 4]	1	[3, 5]	2

Table 8: The length of the intervals and their intersections

Then calculate the value of $r(A,B)$ from Equation 36:

			A			
			Interval	A length	Interval	A length
			[1, 4]	3	[3, 5]	2
B	Interval	B length	Interval	$r(A,B)$	Interval	$r(A,B)$
	[1, 4]	3	[1, 4]	0.333333	[3, 4]	0.166667
	[2, 5]	3	[2, 4]	0.222222	[3, 5]	0.333333
	[3, 6]	3	[3, 4]	0.111111	[3, 5]	0.333333

Table 9: Calculation of the Measure of Intersection

Multiply the basic probability masses (m) from Table 8 by the $r(a,B)$ in Table 9.

			A			
			Interval	m	Interval	m
			[1, 4]	0.5	[3, 5]	0.5
B	Interval	m	Interval	$r(A,B)*m$	Interval	$r(A,B)*m$
	[1, 4]	0.333333	[1, 4]	0.055556	[3, 4]	0.027778
	[2, 5]	0.333333	[2, 4]	0.037037	[3, 5]	0.055556
	[3, 6]	0.333333	[3, 4]	0.018519	[3, 5]	0.055556

Table 10: The product of m and $r(A,B)$

The sum of all of the masses m , scaled by $r(A,B)$ is 0.25. So the renormalization factor k is the inverse of this sum, 4. All of the masses are then renormalized by multiplying each by 4.

			A			
			Interval	m	Interval	m
			[1, 4]	0.5	[3, 5]	0.5
B	Interval	m	Interval	$r(A,B)*m$	Interval	$r(A,B)*m$
	[1, 4]	0.333333	[1, 4]	0.222222	[3, 4]	0.111111
	[2, 5]	0.333333	[2, 4]	0.148148	[3, 5]	0.222222
	[3, 6]	0.333333	[3, 4]	0.074074	[3, 5]	0.222222

Table 11: The renormalized masses with Zhang's rule

Once again, masses for like intervals are summed to obtain the final distribution, i.e. [1,4] has an m value of 0.22222; [2,4] has an m value of 0.14815; [3,4] has an m value of 0.18519; [3,5] has an m value of 0.44444.

These gcdf of the renormalized masses are graphed in Figure 16.

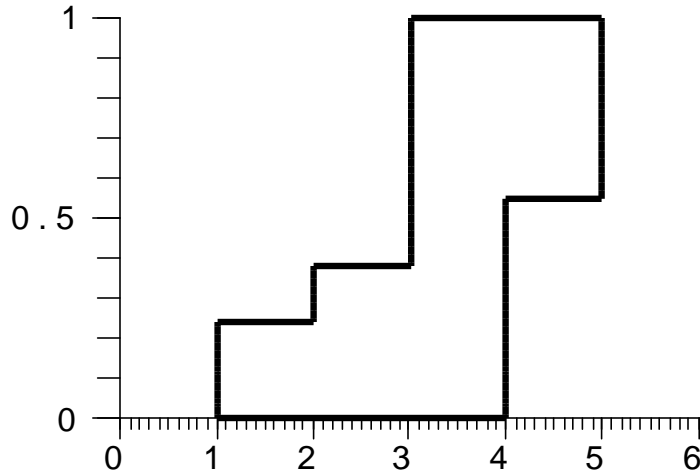


Figure 16: The Zhang combination of A and B

As can be seen in Figure 16 and its corresponding table (Table11) when compared to those of the other Dempster-type rules (Dempster's rule, Yager's rule, and Inagaki's rule for $k=0$ and $k=1$), the Zhang rule yields a slightly different answer for the combination.

As there is no overlap between the two inputs, the combination of A and C is not possible using Zhang's rule.

3.2.5: Mixing

Using Equation 40, the values for mixing (without weights) are listed in Table 12:

Sources	Initial Interval	m	Final Interval	m
Source 1	[1, 4]	0.5	[1, 4]	0.25
	[3, 5]	0.5	[3, 5]	0.25
Source 2	[1, 4]	0.333333	[1, 4]	0.166667
	[2, 5]	0.333333	[2, 5]	0.166667
	[3, 6]	0.333333	[3, 6]	0.166667

Table 12: The mixture of A and B

The masses for the like final intervals are summed: [1,4] has an m value of 0.416667; the remaining distributions remain the same. The resulting structure of the combination of A and B using mixing can be observed in the Figure 17.

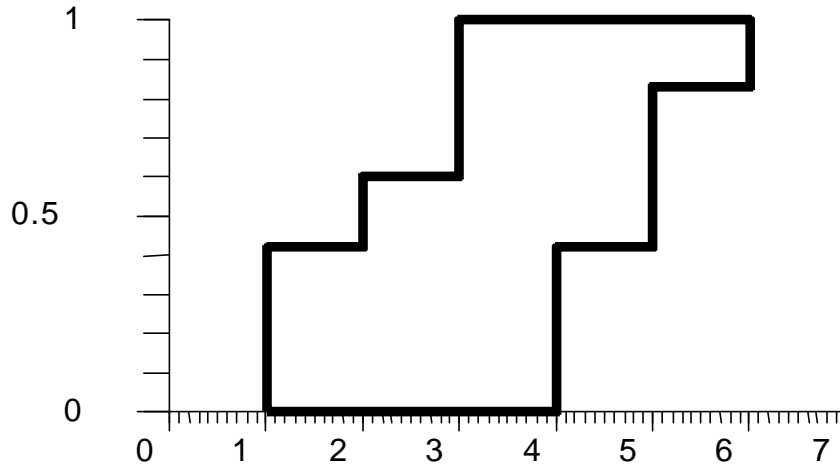


Figure 17: The mixture of A and B

The combination of A and C is possible using mixing. These calculations are summarized in Table 13.

Sources	Initial Interval	m	Final Interval	m
Source 1	[1, 4]	0.5	[1, 4]	0.25
	[3, 5]	0.5	[3, 5]	0.25
Source 2	[6, 10]	0.333333	[6, 10]	0.166667
	[9, 11]	0.333333	[9, 11]	0.166667
	[12, 14]	0.333333	[12, 14]	0.166667

Table 13: The mixture of A and C

The resulting structure of the combination of A and C using **mixing**:

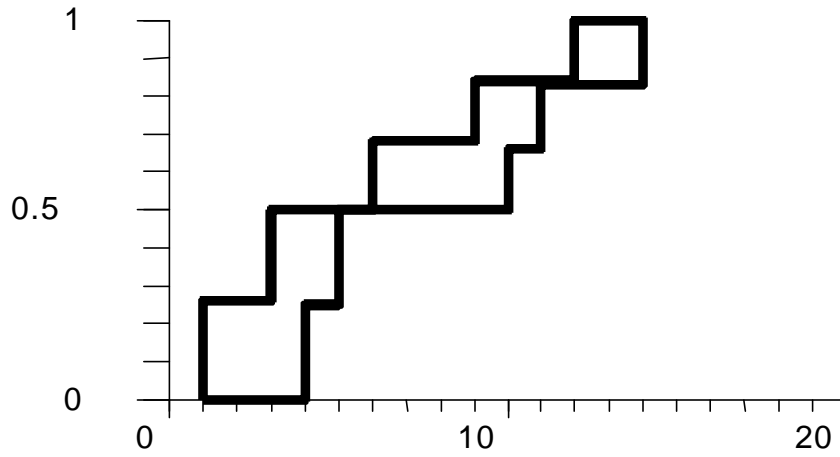


Figure 18: The mixture of A and C

3.2.6: Convolutional x-Averaging

Using Equation 42, we calculate the convolutional x-average for A and B found in Table 14.

			A			
			Interval	m	Interval	m
			[1, 4]	0.5	[3, 5]	0.5
B	Interval	m				
	[1, 4]	0.33333333	[1, 4]	0.16666667	[2, 4.5]	0.16666667
	[2, 5]	0.33333333	[1.5, 4.5]	0.16666667	[2.5, 5]	0.16666667
	[3, 6]	0.33333333	[2, 5]	0.16666667	[3, 5.5]	0.16666667

Table 14: The Combination of A and B using Convolutional x-Averaging

The resulting structure of the combination of A and B using convolutive x-averaging is depicted in Figure 19.

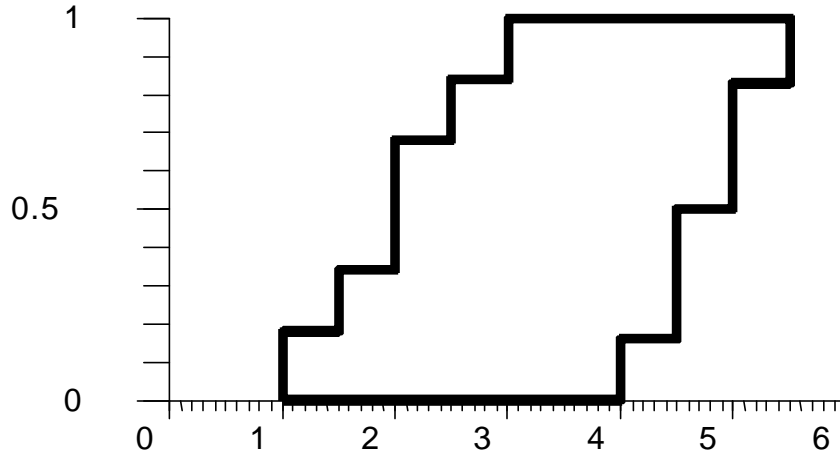


Figure 19: The gcdf of the combination of A and B using convolutive x-averaging

To see the difference between the Dempster rule (solid line) and convolutive x-averaging (dashed line) for the combination of A and B refer to Figure 20.

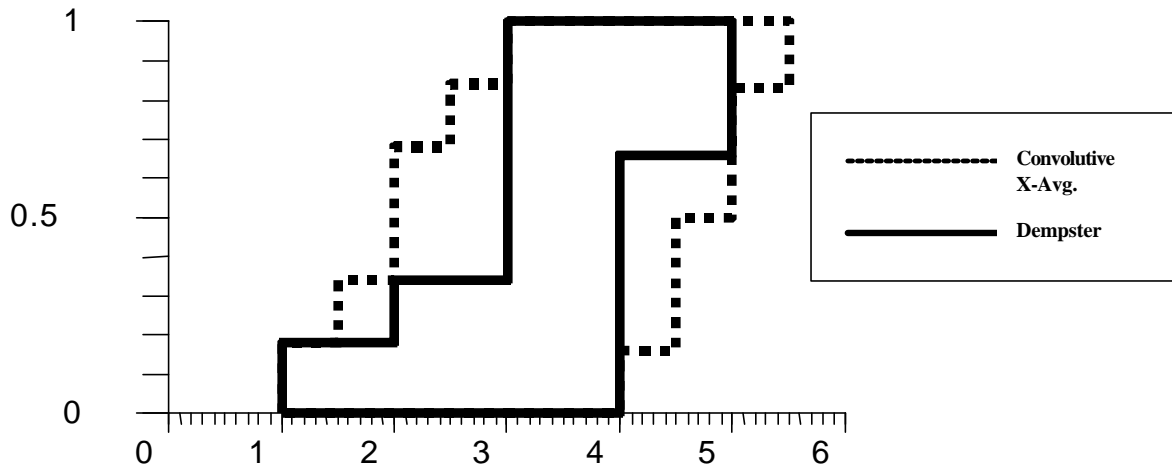


Figure 20: The Comparison of Combinations of A and B with Dempster's rule and Convolutive X-Averaging

As is readily apparent, the bound for the convolutive x-average either is equal to or is significantly larger than the bounds of the Dempster combination.

The combination for A and C can be performed though the convolutive x-average and the calculation are shown in Table 15.

			A			
			Interval	m	Interval	m
			[1, 4]	0.5	[3, 5]	0.5
C	Interval	m	[3.5, 7]	0.16666667	[4.5, 7.5]	0.16666667
	[6, 10]	0.33333333	[5, 7.5]	0.16666667	[6, 8]	0.16666667
	[9, 11]	0.33333333	[6.5, 9]	0.16666667	[7.5, 9.5]	0.16666667
	[12, 14]	0.33333333				

Table 15: The Combination of A and C using Convolutional x-Averaging

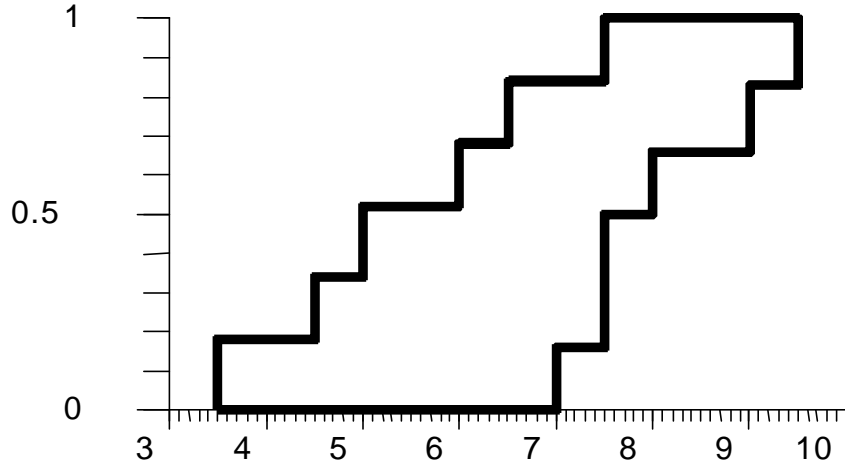


Figure 21: The gcdf of the Combination of A and C using Convolutional x-Averaging

The difference between the Yager rule and convolutional x-averaging for the combination of A and C.

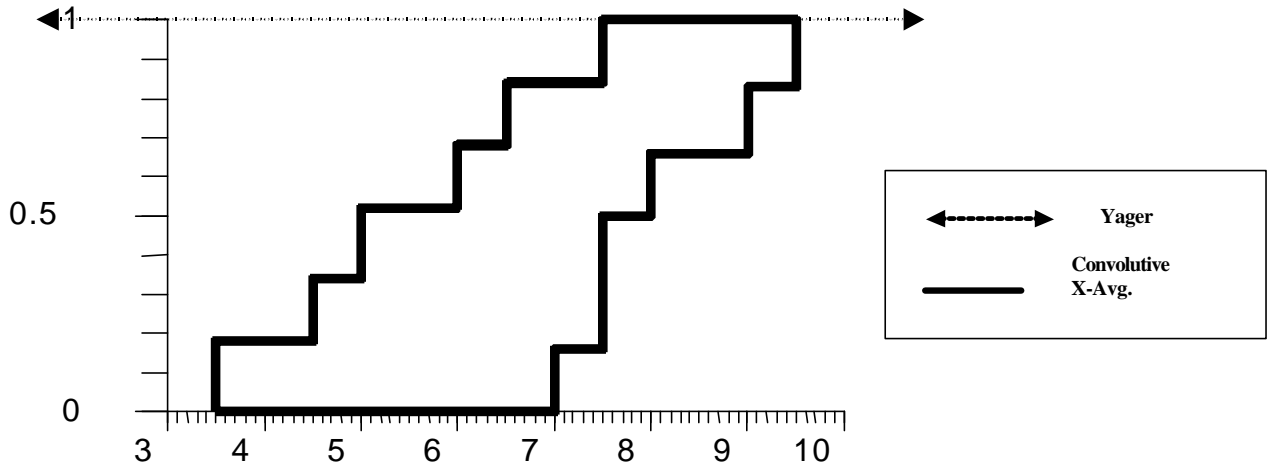


Figure 22: Comparison of Yager's rule and Convolutional x-averaging for A and C

This dramatically demonstrates the difference between the Yager combination under complete conflict (which corresponds to the whole real line) and the convolutional x-average. Yager's distribution implies that there is complete ignorance regarding the inputs, whereas the convolutional x-average simply averages them and provides a significantly narrower answer.

3.2.7 Dubois and Prade's Disjunctive Consensus

The upper and lower bounds for the disjunctive consensus are defined by the minimum of the lower bounds and the maximum of the upper bounds. The calculations for the joint of the basic probability assignments is the product of the marginals. This is also known as a convex hull of all unions. The intervals and their respective probability assignments are listed in Table 16.

			A			
			Interval	m	Interval	m
			[1, 4]	0.5	[3, 5]	0.5
B	Interval	m				
	[1, 4]	0.333333	[1, 4]	0.166667	[1, 5]	0.166667
	[2, 5]	0.333333	[1, 5]	0.166667	[2, 5]	0.166667
	[3, 6]	0.333333	[1, 6]	0.166667	[3, 6]	0.166667

Table 16: The Disjunctive Consensus Pooling of A and B

The only like interval is [1,5] where the summed m is equal to 0.33334. The other distribution remain the same as in Table 16.

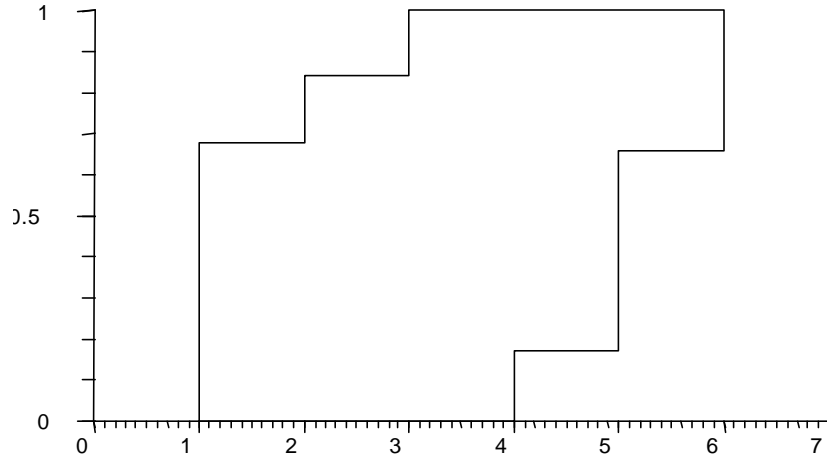


Figure 23: The Disjunctive Consensus Pooling of A and B

It is also possible to calculate the combination of A and C using disjunctive consensus pooling.

			A			
			Interval	m	Interval	m
			[1, 4]	0.5	[3, 5]	0.5
C	Interval	m				
	[6, 10]	0.333333	[1, 10]	0.166667	[3, 10]	0.166667
	[9, 11]	0.333333	[1, 11]	0.166667	[3, 11]	0.166667
	[12, 14]	0.333333	[1, 14]	0.166667	[3, 14]	0.166667

Table 17: Calculations for the Disjunctive Consensus Pooling of A and C

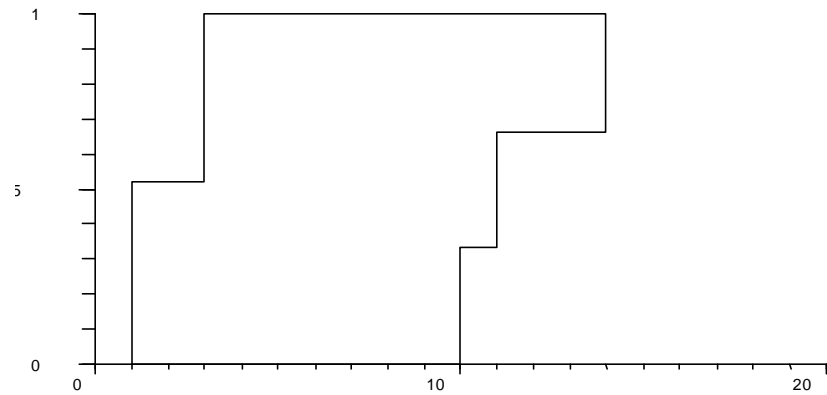


Figure 24: The gcdf for the Disjunctive Consensus Pooling of A and C

See [Ferson and Kreinovich, 2002] for a comparison of the disjunctive consensus and the envelope operation.

3.2.8: Summary of Examples

A simple comparison of the combinations of A and B and A and C with the various rules is summarized in Table 18 and Table 19:

COMBINATION RULES	COMMENTS
Dempster's Rule	The intervals are defined by the minimum of the upper bounds and maximum of the lower bounds. The individual bpa's are calculated by multiplying the bpa's of the marginals. Where the same interval is obtained from multiple combinations, the associated bpa's are summed. No normalization step is taken in this example, as there is no mass associated with conflict.
Yager's Rule	As there is no conflict, this problem provides the same answer as Dempster's rule.
Inagaki's Rule ($k=0$)	As there is no conflict, this problem provides the same answer as Dempster's rule.
Inagaki's Rule ($k=1$)	As there is no conflict, this problem provides the same answer as Dempster's rule.
Zhang's Rule	Provides a slightly different answer than the other Dempster-type rules. The intervals are defined in the same manner but the bpa's are scaled differently because of the measure of intersection. Consequently some bpa's are larger than those obtained by Dempster's rule, while others are slightly smaller. The final masses are renormalized so all masses will add to one.
Mixing	This averaging operation provides different intervals and different bpa's than Dempster's rule. The intervals are either equal to the Dempster intervals or in most cases wider. The bpa's are more concentrated on the interval [1,4].
Convolute x-Average	The convolute x-average is quite different from Dempster's rule, Zhang's rule, and mixing in terms of the bounds of the interval and their respective bpa's. The bounds of this average are either equal to those of Dempster's rule or larger.
Disjunctive Consensus Pooling	As expected, this is by far the most imprecise of the combination methods. The intervals are defined by the maximum of the upper bounds and the minimum of the lower bounds and the bpa's are calculated in the same manner as Dempster's rule. Consequently, in this example, this method provides fewer intervals than in Dempster's rule which are either equal to or greater than the Dempster intervals.

Table 18: The Combination of A and B Comparison Table

COMBINATION RULES	COMMENTS
Dempster's Rule	No answer is possible.
Yager's Rule	To reflect the complete conflict between the two sources, Yager's rule provides the universal set or the real line as its answer.
Inagaki's Rule ($k=0$)	Provides the same answer as Yager's rule, i.e., the universal set or the real line.
Inagaki's Rule ($k=1$)	No answer is possible.
Zhang's Rule	No answer is possible.
Mixing	The mixture maintains the same intervals as the inputs but divides the bpa by 2, the number of sources. While this does provide an answer, the issue of conflict is not represented. The gcdf reflects the full scope of the input bounds.
Convolutional x-Average	The convolutional x-average provides different intervals than obtained by mixing. The upper bounds of the marginals are averaged to obtain the upper bound of the joint. The same process is repeated for the lower bound. The bpa's are the product of the marginal's masses. Consequently, this average is different than the mixing average and the gcdf is concentrated in the center of the two inputs.
Disjunctive Consensus Pooling	As a union operation, this finds the largest possible intervals obtained by the two inputs and calculates the joint bpa's by multiplying the marginal bpa's. The answer subsumes both answers provided by mixing and the convolutional x-average.

Table 19: The Combination of A and C Comparison Table

As indicated in Table 19, when the sources are completely conflicting, some rules will not apply at all (Dempster rule, Zhang's rule) or provide an answer that corresponds to complete ignorance (Yager's rule). The averaging operations will work but it may be inappropriate to average two extremes to produce an answer that neither source suggested was a possible answer.

4: CONCLUSIONS

Dempster-Shafer Theory essentially combines the Bayesian notion of probabilities with the classical idea of sets where a numerical value signifying confidence can be assigned to sets of simple events rather than to just mutually exclusive simple events. [Bogler, 1992] The theoretical basis for Dempster-Shafer Theory is an attractive one for dealing with a corpus of data that requires different degrees of resolution. From the operational perspective of Dempster-Shafer theory, we find that the aggregation of evidence from multiple sources is not straightforward, as there are a variety of possible combination rules.

As there are multiple ways of combining data, it would be desirable to develop a formal procedure by which one could select an appropriate combination operation. Although the algebraic properties may not prove to be useful in designing a

comprehensive typology of combination operators, they do provide insight into some of the behavior of the operators. Some of the algebraic properties of the combination rules discussed in this report are summarized in Table 20.

Combination Rules	Algebraic Properties			
	Idempotent	Commutative	Associative	Quasi-Associative
Dempster's Rule	No	Yes	Yes	
Yager's Rule	No	Yes	No	Yes
Inagaki's Rule	No	Yes	Depends on value of k	Depends on value of k
Zhang's Rule	No	Yes		Yes
Mixing	Yes	Yes		Yes
Convolutional x-Average	Yes	Yes		Yes
Disjunctive Consensus Pooling	No	Yes	Yes	

Table 20: Combination Rules and Their Algebraic Properties

For Dubois and Prade, combination operations cannot be discussed solely in terms of algebraic properties because the imposition of too many properties can be too restrictive to solve practical problems. As we can see with the numerous Dempster-type combination rules, they satisfy many of the same algebraic properties. Moreover, as the work of Dubois and Prade points out, [Dubois, Prade, 1992], even the definitions of the algebraic properties can be problematic and debatable. Nevertheless, understanding what are the desirable properties of a prospective combination rule can be one part of the criteria for rule selection.

Another helpful heuristic for choosing a combination rule is to identify the requirements of the situation as *disjunctive pooling*, *conjunctive pooling* or *tradeoff*. These correspond to the Dubois and Prade disjunctive pooling method, the Dempster rule, and the remaining operations of Yager's rule, Zhang's rule, discount and combine, mixing, and convolutional x-averaging, respectively. If that requirement alone cannot be determined, it may prove practical to apply Inagaki's rule for many values of k . As we have shown here, a number of these rules can be tested and their results compared. Many of the Dempster and the "Dempster-type" combination rules share a common first step, the multiplication of the marginal masses to find the joint. These rules fundamentally differ on how these joint masses are to be combined and where to allocate the mass associated with conflict in the second step.

There are a number of considerations that need to be addressed when combining evidence in Dempster-Shafer theory. Generally speaking, these include the evidence

itself, the sources of information, the context of the application, and the operation used to combine the evidence. These are depicted in Figure 25.

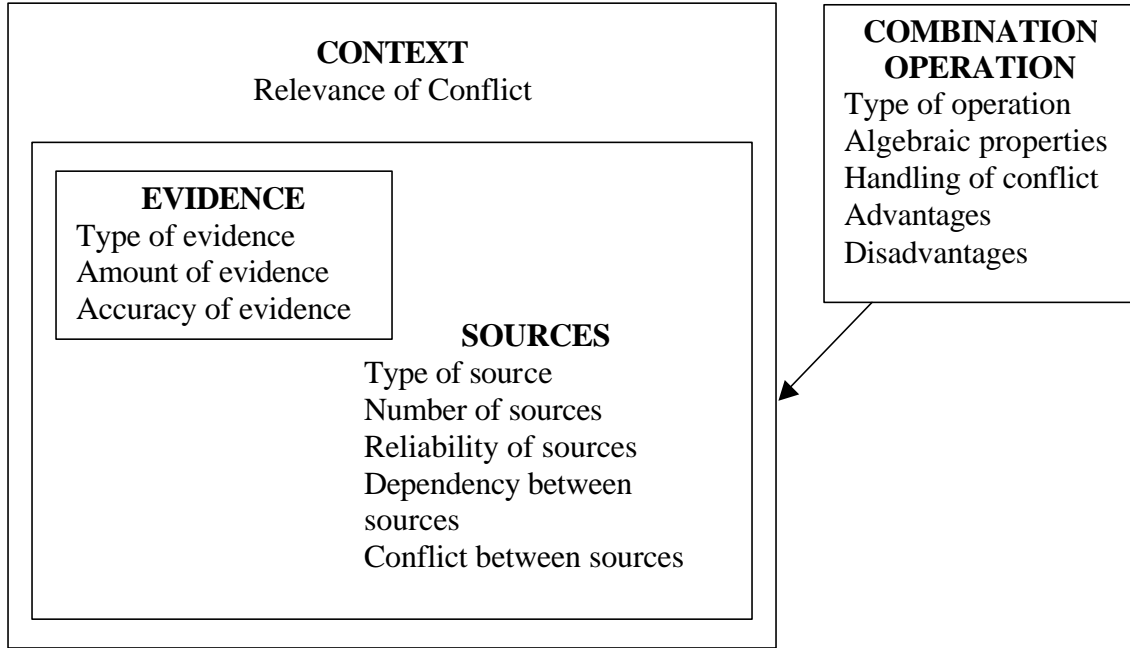


Figure 25: Important Issues in the Combination of Evidence

As the literature survey on aggregation in generalized information theory reflects, much of the research in the combination rules in Dempster-Shafer theory is devoted to advancing a more accurate mathematical representation of conflict. In Figure 25, all of the contextual considerations like the type, amount, and accuracy of evidence as well as the type and reliability of sources and their interdependencies can be interpreted as features of conflict assessment. Once values are established for degree of conflict, the most important consideration is the *relevance* of the existing conflict. *Though conflict may be present, it may not always be contextually relevant.* Take a target identification problem where there are two sensors with a small overlapping area in their respective ranges and the ultimate task is to assign priority to all detected targets. In this case, it is intuitive to assign the highest priority to the target with the largest amount of mass associated with it. We are not concerned with the mass allocated to other targets and hence, conflict is not relevant in this case. Consequently, even in a context of highly conflicting evidence, Dempster's rule might be the most appropriate rule to use as conflict is normalized out of the combination if that conflict is determined by context to be irrelevant. Dempster's rule allows for the comparative assessment the masses associated with various targets independent of their location inside or outside of the intersection of the two overlapping sets.

In conclusion of the discussion of the rules of combination in Dempster-Shafer theory we find that under situations of minimal conflict or irrelevant conflict and all of the sources can be considered reliable, a Dempster combination might be justified. As was demonstrated by the example (Section 3.2.1), when there is a situation of no conflict, two of the Dempster-type rules (Yager, Inagaki ($k=0$, $k=1$), provide the same answer as Dempster's rule. As the level of relevant conflict increases, Yager's rule might more

appropriate as the conflict is not ignored. An advantage of Yager's rule is that it represents the level conflict by the basic probability assignment of the universal set X . However, there is the possibility with Yager's rule that the basic probability mass associated with the combined result is significantly smaller than those provided by the original sources (demonstrated in Section 3.1). Inagaki's unified combination rule investigates the effects of many different values for conflict on a combined result and includes both Dempster's rule and Yager's rule. However, the procedure for the contextual determination of the value of k for Inagaki's rule is an important question that is not clearly described in this current literature survey. Zhang's rule provides a result for the bpa of the combination that is scaled by a measure of the intersection but under certain circumstances this measure can correspond to Dempster's rule and suffer the same criticisms under significant relevant conflict. If Yager's rule begins to reflect a high level of ignorance, the propriety of a combining the evidence at all should be considered. If a combination is appropriate, possible methods for this case could be disjunctive consensus pooling, the discount and combine method (when there is a qualified analyst to discount based on source reliability), or other averaging methods like mixing or convolutive \times -averaging. With all the issues that have been discussed in this report with respect to the combination of evidence in Dempster-Shafer theory, we find that most are linked to the characterization of conflict. Consequently, we identify this as the most critical concern for the specific selection of a combination operation. Specifically, what is the degree and contextual relevance of conflict and how is this handled by a particular combination rule.

REFERENCES

- Ayyub, B. M. (2001). Elicitation of Expert Opinions for Uncertainty and Risks, CRC Press.
- Bogler, P. L. (1987). "Shafer-Dempster Reasoning with Applications to Multisensor Target Identification Systems." IEEE Transactions on Systems, Man, and Cybernetics **SMC-17**(6): 968-977.
- Chokr, B and V. Kreinovich. (1994). How far are we from complete knowledge? Complexity of knowledge acquisition in the Dempster-Shafer approach. Advances in the Dempster-Shafer Theory of Evidence. R. R. Yager, J. Kacprzyk and M. Fedrizzi. New York, John Wiley & Sons, Inc.: 555-576
- Dempster, A. P. (1967). "Upper and Lower Probabilities Induced by a Multivalued Mapping." The Annals of Statistics **28**: 325-339.
- Dubois, D. and H. Prade (1986). "A Set-Theoretic View on Belief Functions: Logical Operations and Approximations by Fuzzy Sets." International Journal of General Systems **12**: 193-226.
- Dubois, D. and H. Prade (1992). "On the combination of evidence in various mathematical frameworks." Reliability Data Collection and Analysis. J. Flamm and T. Luisi. Brussels, ECSC, EEC, EAFC: 213-241.
- Ferson, S. and V. Kreinovich. (2002) "Representation, Propagation, and Aggregation of Uncertainty." SAND Report [in progress]
- Hacking, I. (1975) The Emergence of Probability. New York, Cambridge University Press.
- Hasegawa, S. and T. Inagaki (1994). Dempster-Shafer theoretic design of a decision support system for a large-complex system. 3rd IEEE International Workshop on Robot and Human Communication (RO-MAN), Nagoya, Japan.
- Helton, J. C. (1997). "Uncertainty and Sensitivity Analysis in the Presence of Stochastic and Subjective Uncertainty." Journal of Statistical Computation and Simulation **57**: 3-76.
- Inagaki, T. (1991). "Interdependence between Safety-Control Policy and Multiple-Sensor Schemes Via Dempster-Shafer Theory." IEEE Transactions on Reliability **40**(2): 182-188.
- Klir, G. (1989). "Is There More to Uncertainty Than Some Probability Theorists Might Have Us Believe?" International Journal General Systems **15**: 347-378.

- Klir, G. J. and R. M. Smith (2001). "On Measuring Uncertainty and Uncertainty-Based Information: Recent Developments." Annals of Mathematics and Artificial Intelligence **32**: 5-33.
- Klir, G. J. and M. J. Wierman (1998). Uncertainty-Based Information: Elements of Generalized Information Theory. Heidelberg, Physica-Verlag.
- Kramosil, I (2001). Probabilistic Analysis of Belief Functions. New York, Kluwer Academic/Plenum Publishers.
- Oberkampf, W., J. Helton, and Kari Sentz (2001) "Mathematical Representations of Uncertainty." AIAA Conference Proceedings: Non-Deterministic Approaches Forum, (AIAA2001-1645), Seattle, WA April, 16-19, 2001.
- Oblow, E.M. (1987). "O-Theory---A Hybrid Uncertainty Theory." International Journal of General Systems **13**: 95-106.
- Savage, L.J. (1972) The Foundations of Statistics. New York, Dover Publications.
- Shafer, G. (1976). A Mathematical Theory of Evidence. Princeton, NJ, Princeton University Press.
- Shafer, G. (1986). Probability Judgement in Artificial Intelligence. Uncertainty in Artificial Intelligence. L. N. Kanal and J. F. Lemmer. New York, Elsevier Science. **4**.
- Smets, P. (2000). Data Fusion in the Transferable Belief Model. Third International Conference on Information Fusion, Paris, France.
- Tanaka, K. and G. J. Klir (1999). "A design condition for incorporating human judgement into monitoring systems." Reliability Engineering and System Safety **65**(3): 251-258.
- Yager, R. (1986). "Arithmetic and other operations on Dempster Shafer structures." International Journal of Man-Machine Studies **25**: 357-366.
- Yager, R. (1987a). "On the Dempster-Shafer Framework and New Combination Rules." Information Sciences **41**: 93-137.
- Yager, R. R. (1987b). "Quasi-Associative Operations in the Combination of Evidence." Kybernetes **16**: 37-41.
- Yager, R. R., J. Kacprzyk, et al. (1994). Advances in the Dempster-Shafer Theory of Evidence. New York, John Wiley & Sons, Inc.

- Yao, Y. Y. and S. K. M. Wong (1994). "Representation, Propagation and Combination of Uncertain Information." International Journal of General Systems **23**: 59-83.
- Yen, J. (1989). "Can Evidence Be Combined in the Dempster-Shafer Theory." Uncertainty in Artificial Intelligence 3. L. N. Kanal and J. F. Lemmer. New York, Elsevier Science.
- Zadeh, L. A. (1984). "Review of Books: A Mathematical Theory of Evidence." The AI Magazine **5**(3): 81-83.
- Zadeh, L. A. (1986). A Simple View of the Dempster-Shafer Theory of Evidence and its Implication for the Rule of Combination. The AI Magazine. **7**: 85-90.
- Zhang, L. (1994). Representation, independence, and combination of evidence in the Dempster-Shafer theory. Advances in the Dempster-Shafer Theory of Evidence. R. R. Yager, J. Kacprzyk and M. Fedrizzi. New York, John Wiley & Sons, Inc.: 51-69.

APPENDIX A
References to Applications of Dempster-Shafer Theory

SUBJECT	Related Subject Headings	Pages
Cartography	Geography, Map building, Image Processing	A-2
Classification, Identification, Recognition	Pattern Recognition, Speaker Identification, Voice Recognition, Decision-Making, Radar, Target Identification, Optimization, Fault Detection, Artificial Vision, Image Processing, Multiple Sensors	A-2 to A-7
Decision-Making	Classification, Identification, Recognition, Risk Management, Expert Systems, Image Processing, Robotics	A-7 to A-11
Engineering and Optimization	Expert Systems, Decision Making	A-11 to A-12
Expert Systems	Knowledge-based Systems, Identification, Fault Diagnosis, Geography, Control Systems, Decision-Making	A-13 to A-14
Fault Detection and Failure Diagnosis	Identification, Risk, Reliability, Classification, Sensors	A-14 to A-15
Image Processing	Object Recognition, Expert Systems, Geography, Cartography, Radar, Target Identification, Biomedical Engineering	A-16 to A-21
Medical Applications	Expert Systems, Image Processing, Control Systems	A-21 to A-23
Miscellaneous	Databases, Autonomous Vehicle Navigation, Expert Systems, Forecasting, Finance, Manufacturing, Document Retrieval, Simulation, Decision-Making, Climatology, Expert Opinion Pooling, Optimization	A-23 to A-27
Multiple Sensors	Autonomous Vehicles, Target Identification, Pattern Recognition, Classification, Simulation, Artificial Vision, Satellites, Robotics	A-27 to A-29
Risk and Reliability	Fault Diagnosis, Expert Systems, Decision-Making	A-30
Robotics	Sensors, Decision-Making, Target Identification, Artificial Vision	A-31 to A-33
Signal Processing	Sensors, Target Identification, Recognition, Classification, Radar, Detection, Expert Systems, Sensitivity Analysis	A-33 to A-34

CARTOGRAPHY				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Binaghi, E., L. Luzi, et al.	“Slope instability zonation: A comparison between certainty factor and fuzzy Dempster-Shafer approaches.”	<u>Natural Hazards</u> 17 (1): 77-97 (1998).	Cartography, Instability Maps, Evaluation	This paper presents a comparison between two methodologies for the evaluation of slope instability and the production of instability maps, using a probabilistic approach and a hybrid possibilistic and credibilistic approach. The first is the Certainty Factor method, and the second is based on Fuzzy Logic integrated with the Dempster-Shafer theory.
Leduc, F., B. Solaiman, et al.	“Combination of fuzzy sets and Dempster-Shafer theories in forest map updating using multispectral data.”	<u>Proceedings of SPIE The International Society for Optical Engineering</u> 4385 : 323-335, (2001).	Cartography, Forest Map Updating	This paper explains a new approach to change detection and interpretation in a context of forest map updating. The analysis of remotely sensed data always necessitates the use of approximate reasoning. For this purpose, we use fuzzy logic to evaluate the objects' membership values to the considered classes and the Dempster-Shafer theory to analyse the confusion between classes and to find the more evident class to which an object belongs.
Tirumalai, A. P., B. G. Schunck, et al.	“Evidential Reasoning for Building Environment Maps.”	<u>IEEE Transactions on Systems Man and Cybernetics</u> 25 (1): 10-20, (1995).	Cartography, Environmental Science	We address the problem of building a map of the environment utilizing sensory depth information obtained from multiple viewpoints. We present an approach for multi-sensory depth information assimilation based on Dempster-Shafer theory for evidential reasoning.
CLASSIFICATION, IDENTIFICATION, AND RECOGNITION				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Altincay, H. and M. Demirekler	“Novel rank-based classifier combination scheme for speaker identification.”	<u>ICASSP, IEEE International Conference on Acoustics, Speech and Signal Processing Proceedings</u> 2 : 1209-1212 (2000).	Classification, Speaker Identification,	In this paper, we propose a novel rank-based classifier combination scheme under uncertainty for speaker identification (SI). The combination is based on a heuristic method that uses Dempster-Shafer theory of evidence under some conditions.
Bauer, M.	“A Dempster-Shafer Approach to Modeling Agent Preferences for Plan Recognition.”	<u>User Modeling and User-Adapted Interaction</u> 5 (3-4): 317-348 (1995).	Plan Recognition, Modeling	In this paper, an approach to the quantitative modeling of the required agent-related data and their use in plan recognition is presented. It relies on the Dempster-Shafer Theory and provides mechanisms for the initialization and update of corresponding numerical values.

CLASSIFICATION, IDENTIFICATION, AND RECOGNITION (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Chibelushi, C. C., F. Deravi, et al.	"Audio-visual person recognition: An evaluation of data fusion strategies."	<u>IEEE Conference Publication</u> (437): 26-30, (1997).	Person Recognition, Multimedia, Decision-Making	Audio-visual person recognition promises higher recognition accuracy than recognition in either domain in isolation. To reach this goal, special attention should be given to the strategies for combining the acoustic and visual sensory modalities. This paper presents a comparative assessment of three decision-level data fusion techniques for person identification: Bayesian, Dempster-Shafer and possibilistic approaches.
Dekorvin, A., V. Espino, et al.	"Using Multiple Sources of Information to Recognize and Classify Objects."	<u>Stochastic Analysis and Applications</u> 10 (5): 573-589 (1992).	Object recognition, Classification, Identification	The authors discuss an object recognition problem in which the characteristic features of the object are reported by remote sensors. We then extend the method to a more general class of selection problems and consider several different scenarios. Fuzzy sets are used to represent vague information. Information from independent sources is combined using the Dempster-Shafer approach adapted to the situation in which the focal elements are fuzzy as in the recent paper by J. Yen.
Dekorvin, A., R. Kleye, et al.	"The Object Recognition Problem When Features Fail to Be Homogeneous."	<u>International Journal of Approximate Reasoning</u> 8 (2): 141-162, (1993).	Object Recognition, Identification, Classification	The goal of the present work is to obtain a reasonable solution to the problem of object identification. Sensors report on certain independent feature values of an object. The Dempster-Shafer theory is used to integrate the information coming from these independent sources.
Denoeux, T.	"Evidence-theoretic neural network classifier."		Pattern Recognition, Classification	A new classifier based on the Dempster-Shafer theory of evidence is presented. The approach consists in considering the similarity to prototype vectors as evidence supporting certain hypotheses concerning the class membership of a pattern to be classified. The different items of evidence are represented by basic belief assignments over the set of classes and combined by Dempster's rule of combination.
Denoeux, T.	"K-nearest neighbor classification rule based on Dempster-Shafer theory."	<u>IEEE Transactions on Systems, Man and Cybernetics</u> 25 : 804-813 (1995).	Classification	In this paper, the problem of classifying an unseen pattern on the basis of its nearest neighbors in a recorded data set is addressed from the point of view of Dempster-Shafer theory. Each neighbor of a sample to be classified is considered as an item of evidence that supports certain hypotheses regarding the class membership of that pattern.

CLASSIFICATION, IDENTIFICATION, AND RECOGNITION (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Denoeux, T.	"Function approximation in the framework of evidence theory: A connectionist approach."	<u>IEEE International Conference on Neural Networks Conference Proceedings</u> 1 : 199-203 (1997).	Functional Regression, Prediction	We propose a novel approach to functional regression based on the Transferable Belief Model, a variant of the Dempster-Shafer theory of evidence. This method uses reference vectors for computing a belief structure that quantifies the uncertainty attached to the prediction of the target data, given the input data.
Denoeux, T.	"Reasoning with imprecise belief structures."	<u>International Journal of Approximate Reasoning</u> 20 (1): 79-111, (1999).	Pattern Classification	This paper extends the theory of belief functions by introducing new concepts and techniques, allowing to model the situation in which the beliefs held by a rational agent may only be expressed (or are only known) with some imprecision. Central to our approach is the concept of interval-valued belief structure (IBS), defined as a set of belief structures verifying certain constraints. An application of this new framework to the classification of patterns with partially known feature values is demonstrated.
Denoeux, T.	"A neural network classifier based on Dempster-Shafer theory."	<u>IEEE Transactions on Systems Man and Cybernetics Part a-Systems and Humans</u> 30 (2): 131-150, (2000).	Pattern Classification	A new adaptive pattern classifier based on the Dempster-Shafer theory of evidence is presented. This method uses reference patterns as items of evidence regarding the class membership of each input pattern under consideration.
Denoeux, T. and L. M. Zouhal	"Handling possibilistic labels in pattern classification using evidential reasoning."	<u>Fuzzy Sets and Systems</u> 122 (3): 409-424, (2001).	Pattern Classification, Decision-Making	A category of learning problems in which the class membership of training patterns is assessed by an expert and encoded in the form of a possibility distribution is considered. Two approaches are proposed, based either on the transformation of each possibility distribution into a consonant belief function, or on the use of generalized belief structures with fuzzy focal elements. In each case, a belief function modeling the expert's beliefs concerning the class membership of each new pattern is obtained.
Dillard, R. A.	"Tactical Inferencing with the Dempster-Shafer Theory of Evidence."	<u>Conference Record Asilomar Conference on Circuits, Systems & Computers</u> 17th : 312-316, (1984).		

CLASSIFICATION, IDENTIFICATION, AND RECOGNITION (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Foucher, S., J. M. Boucher, et al.	"Multiscale and multisource classification using Dempster-shafer theory."	<u>IEEE International Conference on Image Processing 1</u> : 124-128, (1999).	Classification, Radar	We propose to use evidential reasoning in order to relax bayesian decisions given by a multiscale markovian classification algorithm (ICM). The Dempster-shafer rule of combination enables us to fuse decisions in a local spatial neighbourhood which we further extend to be multiscale and multisource. This approach enables us to more directly fuse multiscale information. Application to the classification of very noisy radar images produce interesting results.
Gang, T. and L. Wu	"Technique of multi-source information fusion and defects recognition in ultrasonic detection."	<u>Jixie Gongcheng Xuebao/Chinese Journal of Mechanical Engineering 35</u> : 11-14, (1999).	Recognition, Decision-Making	According to the Dempster-Shafer theory, the information fusion method and classification decision strategies in ultrasonic detection were studied. On this basis, the primary experimental research on the classification and recognition of the defects based on the information fusion has been carried out.
Horiuchi, T.	"Decision rule for pattern classification by integrating interval feature values."	<u>IEEE Transactions on Pattern Analysis and Machine Intelligence 20</u> (4): 440-448, (1998).	Pattern Classification, Decision-Making	In this paper, a pattern classification theory using feature values defined on closed interval is formalized in the framework of Dempster-Shafer measure. Then, in order to make up lacked information, an integration algorithm is proposed, which integrates information observed by several information sources with considering source values.
Kawade, M.	"Object recognition system in a dynamic environment."	<u>IEEE International Conference on Fuzzy Systems 3</u> : 1285-1290, (1995).	Object Recognition	In this paper, we propose an object recognition system in a dynamic environment based on fuzzy logic and Dempster-Shafer's Theory which can integrate various inferences.
Khalaf, S., P. Siy, et al.	"2-D and 3-D touching part recognition using the theory of evidence."	<u>Proceedings IEEE International Symposium on Circuits and Systems 2</u> : 992-994, (1990).	Recognition, Identification, Decision-Making	A unified approach is presented for solving the 2-D and 3-D touching part recognition problem. The problem is formulated as a Dempster-Shafer evidence accumulation process.

CLASSIFICATION, IDENTIFICATION, AND RECOGNITION (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Nigro, J. M., S. Lorientte Rougegrez, et al.	"Driving situation recognition in the CASSICE project towards an uncertainty management."	<u>IEEE Conference on Intelligent Transportation Systems, Proceedings, ITSC</u> : 71-76, (2000).	Recognition, Driving Maneuver, Sensors	We interested in the recognition of the maneuvers performed by the driver, specially the overtaking maneuver. We consider a maneuver as a sequence of events. Then, according to the inputs obtained from the system's sensors at different times, the goal is to evaluate the confidence of which particular maneuver is in progress. In this paper, the confidence is modeled by a distribution of mass of evidence as proposed in the Dempster-Shafer's theory.
Peddle, D. R.	"Knowledge Formulation for Supervised Evidential Classification."	<u>Photogrammetric Engineering and Remote Sensing</u> 61 (4): 409-417, (1995).	Classification, Land Cover Classification	The Dempster-Shafer Theory of Evidence provides an appropriate framework for overcoming problems associated with the analysis, integration, and classification of modern, multisource data sets. However, current methods for generating the prerequisite evidence are subjective and inconsistent. To address this, a more objective approach is presented for deriving evidence from histogram bin transformations of supervised training data frequency distributions. The procedure is illustrated by an example application in which evidential land-cover classification.
Vasseur, P., C. Pegard, et al.	"Perceptual organization approach based on Dempster-Shafer theory."	<u>Pattern Recognition</u> 32 (8): 1449-1462, (1999).	Pattern Recognition, Object Recognition, Identification, Optimization	In this paper, we propose an application of the perceptual organization based on the Dempster-Shafer theory. This method is divided into two parts which rectify the segmentation mistakes by restoring the coherence of the segments and detects objects in the scene by forming groups of primitives. We show how we apply the Dempster-Shafer theory, usually used in data fusion, in order to obtain an optimal adequation between the perceptual organization problem and this tool.
Xu, L., A. Krzyzak, et al.	"Methods of Combining Multiple Classifiers and Their Applications to Handwriting Recognition."	<u>IEEE Transactions on Systems, Man, and Cybernetics</u> 22 (3): 418-435, (1992).	Classification, Pattern Recognition	Method of combining the classification powers of several classifiers is regarded as a general problem in various applications areas of pattern recognition, and a systematic investigation has been made. Possible solutions to the problem can be divided into three categories according to the levels of information available from the various classifiers. Four approaches are proposed based on different methodologies for solving this problem.

CLASSIFICATION, IDENTIFICATION, AND RECOGNITION (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Zhu, D. P., R. W. Conners, et al.	"A prototype vision system for analyzing CT imagery of hardwood logs."	<u>IEEE Transactions on Systems Man and Cybernetics Part B-Cybernetics</u> 26 (4): 522-532, (1996).	Artificial Vision, Image, Defect Detection, Classification, Object Identification	To fully optimize the value of material produced from a hardwood log requires information about type and location of internal defects in the log, This paper describes a prototype vision system that automatically locates and identifies certain classes of defects in hardwood logs. To further help cope with the above mentioned variability, the Dempster-Shafer theory of evidential reasoning is used to classify defect objects.
DECISION-MAKING				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Attoh-Okine, N. O. and J. Gibbons	"Use of belief function in brownfield infrastructure redevelopment decision making."	<u>Journal of Urban Planning and Development-Asce</u> 127 (3): 126-143 (2001).	Decision-Making, Urban Development	The Dempster-Shafer theory of combination is used to combine independent evidence from various issues to determine the overall uncertainty in redevelopment decision-making.
Bauer, M.	"Approximation algorithms and decision making in the Dempster-Shafer theory of evidence : An empirical study."	<u>International Journal of Approximate Reasoning</u> 17 (2-3): 217-237 (1997).	Decision-Making	This article reviews a number of algorithms based on a method of simplifying the computational complexity of DST.
Beynon, M., D. Cosker, et al.	"An expert system for multi-criteria decision making using Dempster Shafer theory."	<u>Expert Systems with Applications</u> 20 (4): 357-367 (2001).	Decision-Making	This paper outlines a new software system we have developed that utilises the newly developed method (DS/AHP) which combines aspects of the Analytic Hierarchy Process (AHP) with Dempster-Shafer Theory for the purpose of multi-criteria decision making (MCDM).
Beynon, M., B. Curry, et al.	"The Dempster-Shafer theory of evidence: an alternative approach to multicriteria decision modelling."	<u>Omega-International Journal of Management Science</u> 28 (1): 37-50 (2000).	Decision-Making	We discuss recent developments of Dempster-Shafer theory including analytical and application areas of interest. We discuss developments via the use of an example incorporating DST with the Analytic Hierarchy Process (AHP).

DECISION-MAKING (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Bharadwaj, K. K., Neerja, et al.	“Hierarchical Censored Production Rules (Hcprs) System Employing the Dempster-Shafer Uncertainty Calculus.”		Decision-Making	The Dempster-Shafer Theory is used to formalize Variable Precision Logic (VPL) type inference provides a simple, intuitive notion of the precision of an inference which relates it to the amount of information found. This formalism allows the ignorance in the evidence to be preserved through the reasoning process and expressed in the decision.
Bosse, E. and J. Roy	“Fusion of identity declarations from dissimilar sources using the Dempster-Shafer theory.”	<u>Optical Engineering</u> 36 (3): 648-657 (1997).	Object Identification, Decision-Making	The problem of fusing identity declarations emanating from different sources is explored and decision makers are offered a quantitative analysis based on statistical methodology rooted in the Dempster-Shafer theory of evidence that can enhance their decision making processes regarding the identity of detected objects.
Caselton, W. F. and W. B. Luo	“Decision-Making with Imprecise Probabilities : Dempster-Shafer Theory	<u>Water Resources Research</u> 28 (12): 3071-3083 (1992).	Decision-Making, Water Resources Management	A water resources example of an application of the Dempster-Shafer approach is presented, and the results contrasted with those obtained from the closest equivalent Bayesian scheme.
Chang, Y. C., J. R. Wright, et al.	“Evidential reasoning for assessing environmental impact.”	<u>Civil Engineering Systems</u> 14 (1): 55-77 (1996).	Decision-Making,	This research proposes a formal methodology for integrating subjective inferential reasoning and geographic information systems (GIS) into a decision support system for use in these problem domains. The rationale for inferential spatial models, and the structure and function of a spatial modeling environment based on the Dempster-Shafer theory of evidence are presented.
Class, F., A. Kaltenmeier, et al.	“Soft-decision vector quantization based on the Dempster/Shافر theory.”	<u>(Proceedings ICASSP, IEEE International Conference on Acoustics, Speech and Signal Processing</u> 1 : 665-668, (1991).	Speech Recognition, Decision-Making	The authors describe an algorithm for soft-decision vector quantization (SVQ) implemented in the acoustic front-end of a large-vocabulary speech recognizer based on discrete density HMMs (hidden Markov models) of small phonetic units.
deKorvin, A., S. Hashemi, et al.	“Evaluating policies based on their long term average cost.”	<u>Stochastic Analysis and Applications</u> 18 (6): 901-919 (2000).	Decision-Making, Policy selection	We use the Dempster-Shafer theory together with techniques of Norton and Smets to approximate the transition probabilities for an application in policy selection from a set of possible policies in the long term.

DECISION-MAKING (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Dekorvin, A. and M. F. Shipley	"A Dempster-Shafer-Based Approach to Compromise Decision-Making with Multiattributes Applied to Product Selection."	<u>IEEE Transactions on Engineering Management</u> 40(1): 60-67 (1993).	Decision-Making, Product Selection	The Dempster-Shafer theory of evidence is applied to the technology assessment problem of selecting computer software
Denoeux, T.	"Modeling vague beliefs using fuzzy-valued belief structures."	<u>Fuzzy Sets and Systems</u> 116(2): 167-199 (2000).	Decision-Making, Classification	We introduce the concepts of interval-valued and fuzzy-valued belief structures and discuss the application of this framework in the areas of decision making under uncertainty and classification of fuzzy data.
Denoeux, T. and M. S. Bjanger	"Induction of decision trees from partially classified data using belief functions."	<u>Proceedings of the IEEE International Conference on Systems, Man and Cybernetics</u> 4: 2923-2928 (2000).	Decision-Making, Classification	A new tree-structured classifier based on the Dempster-Shafer theory of evidence is presented.
Drakopoulos, E. and C. C. Lee	"Decision Rules for Distributed Decision Networks with Uncertainties."	<u>IEEE Transactions on Automatic Control</u> 37(1): 5-14, (1992).	Decision-Making	A binary hypothesis testing problem is solved using some simple concepts of Dempster-Shafer theory. Each Decision Maker in a distributed decision networks employs Dempster's combining rule to <u>aggregate its input information for a decision.</u>
Ducey, M. J.	"Representing uncertainty in silvicultural decisions: an application of the Dempster-Shafer theory of evidence."	<u>Forest Ecology and Management</u> 150(3): 199-211 (2001).	Decision-Making	This paper presents examples of silvicultural decision-making using belief functions for the case of no data, sparse data, and adaptive management under increasing data availability.
Engemann, K. J., H. E. Miller, et al.	"Decision making with belief structures: An application in risk management."	<u>International Journal of Uncertainty Fuzziness and Knowledge-Based Systems</u> 4(1): 1-25 (1996).	Decision-Making, Risk Management	We then propose a methodology for decision making under uncertainty, integrating the ordered weighted averaging aggregation operators and the Dempster-Shafer belief structure. The proposed methodology is applied to a real world case involving risk management at one of the nation's largest banks.

DECISION-MAKING (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Gaglio, S., R. Minciardi, et al.	"On the Acquisition and Processing of Uncertain Information in Rule-Based Decision Support Systems."		Decision-Making	Problems relevant to the construction of a rule-based decision-support system that is based on uncertain knowledge are addressed. The representation of uncertainty and the combination of evidence are carried out by means of the Dempster-Shafer theory of evidence.
Garribba, S. F. and A. Servida	"Evidence Aggregation in Expert Judgments."	<u>Lecture Notes in Computer Science</u> 313 : 385-400, (1988).	Expert Judgments, Decision-Making	
Kohlas, J. and P.-A. Monney	"Theory of Evidence--A Survey of its Mathematical Foundations, Applications and Computational Aspects."	<u>Mathematical Methods of Operations Research</u> 39 : 35-68, (1994).	Decision Analysis, Statistical Analysis, Imaging, Project Planning, Scheduling, Risk Analysis	Evidence theory has been used to represent uncertainty in expert systems, especially in the domain of diagnostics. It can be applied to decision analysis and it gives a new perspective for statistical analysis. Among its further applications are image processing, project planning and scheduling and risk analysis. The computational problems of evidence theory are well understood and even though the problem is complex, efficient methods are available.
Shipley, M. F., C. A. Dykman, et al.	"Project management: Using fuzzy logic and the Dempster-Shafer theory of evidence to select team members for the project duration."	<u>Annual Conference of the North American Fuzzy Information Processing Society NAFIPS</u> : 640-644 (1999).	Decision-Making, Project Management	Fuzzy logic and the Dempster-Shafer theory of evidence is applied to an IS multiattribute decision making problem whereby the project manager must select project team members from candidates, none of whom may exactly satisfy the ideal level of skills needed at any point in time.
Smets, P.	"The transferable belief model for expert judgements."	<u>Analysis and Management of Uncertainty: Theory and Applications</u> . B. M. Ayyub, M. M. Gupta and L. N. Kanal. New York, North-Holland. 13 : 165-170 (1992).	Decision-Making	We show how the transferable belief model can be used to assess and to combine expert opinions. The transferable belief model has the advantage that it can handle weighted opinions and their aggregation without the introduction of any ad hoc methods.

DECISION-MAKING (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Srivastava, R. P. and T. J. Mock	"Evidential reasoning for WebTrust Assurance services."	<u>Proceedings of the Hawaii International Conference on System Sciences</u> 170 : 170 (1999).	Decision-Making	In this paper we develop an evidential network model for 'WebTrust Assurance,' a service recently proposed by the American Institute of Certified Public Accountants and the Canadian Institute of Chartered Accountants. The aggregation of evidence and the resolution of uncertainties in the model follow the approach of Dempster-Shafer theory of belief functions.
Yang, J. B. and M. G. Singh	"An Evidential Reasoning Approach for Multiple- Attribute Decision-Making with Uncertainty."	<u>IEEE Transactions on Systems Man and Cybernetics</u> 24 (1): 1-18 (1994).	Decision-Making	A new evidential reasoning based approach is proposed that may be used to deal with uncertain decision knowledge in multiple-attribute decision making (MADM) problems with both quantitative and qualitative attributes. This approach is based on an evaluation analysis model and the evidence combination rule of the Dempster-Shafer theory.
ENGINEERING AND OPTIMIZATION				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Alim, S.	"Application of Dempster- Shafer Theory for Interpretation of Seismic Parameters."	<u>Journal of Structural Engineering-Asce</u> 114 (9): 2070-2084 (1988).		
Butler, A. C., F. Sadeghi, et al.	"Computer-Aided-Design Engineering of Bearing Systems Using the Dempster-Shafer Theory."	<u>AI Edam-Artificial Intelligence for Engineering Design Analysis and Manufacturing</u> 9 (1): 1-11, (1995).	Computer Aided Design, Engineering, Expert System, Selection	Research in computer-aided design/engineering (CAD/E) has focused on enhancing the capability of computer systems in a design environment, and this work has continued in this trend by illustrating the use of the Dempster-Shafer theory to expand the computer's role in a CAD/E environment. An expert system was created using Dempster-Shafer methods that effectively modeled the professional judgment of a skilled tribologist in the selection of rolling element bearings.

ENGINEERING AND OPTIMIZATION (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Chen, L. and S. S. Rao	"A modified Dempster-Shafer theory for multicriteria optimization."	<u>Engineering Optimization</u> 30 (3-4): 177-201, (1998).	Engineering, Multicriteria Design Optimization	A new methodology, based on a modified Dempster-Shafer (DS) theory, is proposed for solving multicriteria design optimization problems. The design of a mechanism in the presence of seven design criteria and eighteen design variables is considered to illustrate the computational details of the approach. This work represents the first attempt made in the literature at applying DS theory for numerical engineering optimization.
Rao, S. S. and L. Chen	"Generalized hybrid method for fuzzy multiobjective optimization of engineering systems."	<u>AIAA Journal</u> 34 (8): 1709-1717, (1996).	Optimization, Engineering Systems	A generalized hybrid approach is presented for the multiobjective optimization of engineering systems in the presence of objectives and constraints that are partly fuzzy and partly crisp. The methodology is based on both fuzzy-set and Dempster-Shafer theories to capture the features of incomplete, imprecise, uncertain, or vague information that is often present in real-world engineering systems. The original partly fuzzy multiobjective optimization problem is first defuzzified into a crisp generalized multiobjective optimization problem using fuzzy-set theory. The resulting multiobjective problem is then transformed into an equivalent single-objective optimization problem using a modified Dempster-Shafer theory. The computational details of the approach are illustrated with a structural design example.
Yang, J. B. and P. Sen	"Multiple attribute design evaluation of complex engineering products using the evidential reasoning approach."	<u>Journal of Engineering Design</u> 8 (3): 211-230, (1997).	Evaluation, Engineering Product Selection, Decision-Making	This paper reports the application of an evidential reasoning approach to design selection of retro-fit options for complex engineering products. The particular selection problem investigated in this paper is initially modeled by means of techno-economic analysis and may be viewed as a multiple-attribute decision-making problem with a hierarchical structure of attributes which may be measured for each design option using numerical values or subjective judgments with uncertainty.

EXPERT SYSTEMS				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Berenji, H. and H. Lum, Jr.	"Application of Plausible Reasoning to AI-Based Control Systems."	<u>Proc Am Control Conf</u> : 1655-1661. (1987).	Expert Systems, Knowledge-Based Systems	The authors discuss techniques used for development of knowledge-based (e. g. , expert) systems. Specifically, the MYCIN expert system certainty factor approach, probabilistic approach, Dempster-Shafer theory of evidence (1976), possibility theory and linguistic variables, and fuzzy control are presented.
Ferrier, G. and G. Wadge	"An integrated GIS and knowledge-based system as an aid for the geological analysis of sedimentary basins."	<u>International Journal of Geographical Information Science</u> 11 (3): 281-297, (1997).	Knowledge-base, Geography	Approximate reasoning techniques to handle the vagueness and uncertainty inherent in a large amount of geological data, knowledge and reasoning are reviewed with particular emphasis on provenance analysis using subjective probability theory, Dempster-Shafer theory and fuzzy logic techniques.
Gammerman, A., B. Skullerud, et al.	"Sysex: An Expert System for Biological Identification."	<u>Proceedings of SPIE The International Society for Optical Engineering</u> 657 : 34-39, (1986).	Identification, Biology, Expert Systems	The aim of this research is to create an expert system which would help with the task of identifying a biological specimen. The Dempster-Shafer theory of evidence was used to handle uncertainty associated with the date and the expertise.
Guan, J., D. A. Bell, et al.	"Dempster-Shafer theory and rule strengths in expert systems."	(1990). <u>IEEE Colloquium</u> : 086.	Expert Systems, Fault Diagnosis	Dempster-Shafer theory is discussed, focusing on the union of the granules in a granule set. The discussion is illustrated by considering an example of fault diagnosis in a distributed vehicle monitoring system.
Shenoy, P. P.	"Using Dempster-Shafer's belief-function theory in expert systems."	<u>Proceedings of SPIE The International Society for Optical Engineering</u> : 2-14, (1992).	Expert Systems, Valuation-Based Systems	The main objective of this paper is to describe how Dempster-Shafer's (DS) theory of belief functions fits in the framework of valuation-based systems (VBS). Since VBS serves as a framework for managing uncertainty in expert systems, this facilitates the use of DS belief-function theory in expert systems.
Stephanou, H. E.	"Evidential Framework for Intelligent Control."		Control Systems, Decision-Making	The author deals with a class of knowledge-based control systems that involve two types of (not necessarily probabilistic) uncertainty: (1) an incomplete set of control rules contributed by multiple domain experts; and (2) incomplete and/or inaccurate feedback information from multiple sensors. The Dempster-Shafer theory of evidence provides the basic framework for the representation of uncertain knowledge.

EXPERT SYSTEMS (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Wunsch, G. and F. Klages	“Expert systems to assist analytical chemistry: Realization of learning ability and plausibility checking demonstrated for ICP mass spectrometry.”	<u>Journal Fur Praktische Chemie-Chemiker-Zeitung</u> 338 (7): 593-597, (1996).	Expert Systems, Analytical Chemistry, Mass Spectrometry, Decision-Making	Certain, uncertain and lacking knowledge has to be considered for intelligent counseling. In the ICP mass spectrometry the composition of the actual sample and the ionization rates are the most important parameters to be prognosticated. The way of storage and retrieval of data and of decision making should be automatically checked and improved with respect to the success of previous guesses. The Dempster-Shafer theory is used for the combination and propagation of uncertainties.
FAULT DIAGNOSIS AND FAILURE DETECTION				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Kang, H., J. Cheng, et al.	“An application of fuzzy logic and Dempster-Shafer theory to failure detection and identification.”	<u>Proceedings of the IEEE Conference on Decision and Control</u> 2 : 1555-1560, (1991).	Failure Detection, Identification, Decision-Making	A novel approach to failure detection and identification (FDI) is proposed which combines an analytic estimation method and an intelligent identification scheme in such a way that sensitivity to true failure modes is enhanced, while the possibility of false alarms is reduced. At the final stage of the algorithm, an index is computed--the degree of certainty--based on Dempster-Shafer theory, which measures the reliability of the decision. The FDI algorithm has been applied successfully to the detection of rotating stall and surge instabilities in axial flow compressors.
Parikh, C. R., M. J. Pont, et al.	“Application of Dempster-Shafer theory in condition monitoring applications: a case study.”	<u>Pattern Recognition Letters</u> 22 (6-7): 777-785, (2001).	Classification, Fault Diagnosis, Monitoring	This paper is concerned with the use of Dempster-Shafer theory in 'fusion' classifiers. We demonstrate the effectiveness of this approach in a case study involving the detection of static thermostatic valve faults in a diesel engine cooling system.

FAULT DIAGNOSIS AND FAILURE DETECTION (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Tanaka, K. and G. J. Klir	"Design condition for incorporating human judgement into monitoring systems."	<u>Reliability Engineering and System Safety</u> 65 : 251-258, (1999).	Safety Monitoring, Sensors, Failure Detection	The present article proposes two types of an automatic monitoring system not involving any human inspection or a human-machine (H-M) cooperative monitoring system with inspection. In order to compare the systems, an approach based on the Dempster-Shafer theory is proposed for uncertainty analysis. By comparing their expected losses as a result of failed dangerous failures or failed safe failures as well as the inspection errors, the condition is determined under which H-M cooperative systems incorporating human judgements are more effective than automatic monitoring systems.
Vachtsevanos G., H. Kang, et al.	"Detection and Identification of Axial-Flow Compressor Instabilities."	<u>Journal of Guidance Control and Dynamics</u> 15 (5): 1216-1223, (1992).	Identification, Failure Detection	A new approach to failure detection and identification is proposed that combines an analytic estimation method and an intelligent identification scheme in such a way that sensitivity to true failure modes is enhanced while the possibility of false alarms is reduced. We employ a real-time recursive parameter estimation algorithm with covariance resetting that triggers the fault detection and identification routine only when potential failure modes are anticipated. A possibilistic scheme based on fuzzy set theory is applied to the identification part of the algorithm with computational efficiency. At the final stage of the algorithm, an index is computed-the degree of certainty-based on Dempster-Shafer theory, which measures the reliability of the decision. The proposed algorithm has been applied successfully to the detection of rotating stall and surge instabilities in axial flow compressors.
Van Dam, K. and T. J. Mouldsley	"Extension of Dempster-Shafer theory and application to fault diagnosis in communication systems."	<u>IEEE Conference Publication</u> (395): 310-315, (1994).	Fault Diagnosis, Communication Systems	A novel method is presented for propagating uncertainty that also calculates measures of contradictions in the input data. This method can improve the performance of a Reason Maintenance System (RMS) by ranking the contradictions and resolving the most severe of these first. An example shows the application of this technique to fault diagnosis in a communication system.

IMAGE PROCESSING				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Askari, F. and B. Zerr	"Neural network architecture for automatic extraction of oceanographic features in satellite remote sensing imagery."	<u>Oceans Conference Record</u> 2: 1017-1021 (1998).	Image Processing, Oceanography, Satellite, Neural Networks	This paper discusses an approach for automatic feature detection and sensor fusion in remote sensing imagery using a combination of neural network architecture and Dempster-Shafer theory of evidence.
Aslandogan, Y. A. and C. T. Yu	"Evaluating strategies and systems for content based indexing of person images on the web."	<u>Proceedings of the ACM International Multimedia Conference and Exhibition</u> : 313-321 (2000).	Image Processing, Multimedia, Recognition, Content-based Indexing	We provide experimental evaluation of the following strategies for the content based indexing of multimedia: i) Face detection on the image followed by Text/HTML analysis of the containing page; ii) face detection followed by face recognition; iii) face detection followed by a linear combination of evidences due to text/HTML analysis and face recognition; and iv) face detection followed by a Dempster-Shafer combination of evidences due to text/HTML analysis and face recognition.
Betz, J. W., J. L. Prince, et al.	"Representation and transformation of uncertainty in an evidence theory framework."		Image Processing, Artificial Vision, Sensors	A framework is presented for deriving and transforming evidence-theoretic belief representations of uncertain variables that denote numerical quantities. Belief is derived from probabilistic models using relationships between probability bounds and the support and plausibility functions used in evidence theory. This model-based approach to belief representation is illustrated by an algorithm currently used in a vision system to label anomalous high-intensity pixels in imagery.
Bloch, I.	"Some aspects of Dempster-Shafer evidence theory for classification of multi-modality medical images taking partial volume effect into account."	<u>Pattern Recognition Letters</u> 17(8): 905-919 (1996).	Image Processing, Medicine, Classification, Medical Imaging	This paper points out some key features of Dempster-Shafer evidence theory for data fusion in medical imaging. Examples are provided to show its ability to take into account a large variety of situations, which actually often occur and are not always well managed by classical approaches nor by previous applications of Dempster-Shafer theory in medical imaging.

IMAGE PROCESSING (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Chapron, M.	"A color edge detector based on Dempster-Shafer theory."		Image Processing, Pattern Recognition	Segmentation based on contour detection is a relevant stage before image interpretation or pattern recognition. This paper is concerned with color image filtering and color edge detecting. These 2 techniques utilize the Dempster-Shafer theory.
Huber, R.	"Scene classification of SAR images acquired from antiparallel tracks using evidential and rule-based fusion."	<u>Image and Vision Computing</u> 19 (13): 1001-1010 (2001).	Image Processing, Radar, Classification	This paper presents a method for fusion of information derived from different airborne synthetic aperture radar measurement processes and from different observations of the same scene. Dempster-Shafer theory will be used to fuse radar backscatter and phase measurements.
Ip, H. H. S. and R. C. K. Chiu	"Evidential reasoning for facial gestures recognition from cartoon images."	<u>Australian and New Zealand Conference on Intelligent Information Systems Proceedings</u> : 397-401 (1994).	Image Processing, Recognition	The Dempster-Shafer theory of evidential reasoning is applied to combine evidence represented by the facial features. The study demonstrates the feasibility of applying the Dempster-Shafer theory to facial gesture recognition.
Ip, H. H. S. and J. M. C. Ng	"Human face recognition using Dempster-Shafer theory."	<u>IEEE International Conference on Image Processing</u> 1 : 292-295 (1994).	Image Processing, Recognition	In this paper, image processing techniques developed for the extraction of the set of visual evidence, the formulation of the face recognition problem within the framework of Dempster-Shafer Theory and the design of suitable mass functions for belief assignment are discussed.
Janez, F., O. Goretta, et al.	"Automatic map updating by fusion of multispectral images in the Dempster-Shafer framework."	<u>Proceedings of SPIE The International Society for Optical Engineering</u> 4115 : 245-255 (2000).	Image Processing, Cartography	In this article, we present a strategy to report in an automatic way significant changes on a map by fusion of recent images in various spectral bands. For configurations of partial overlapping between map and images, it is difficult or even impossible to formalize the approach suggested within a probabilistic framework. Thus, the Dempster-Shafer theory is shown as a more suitable formalism in view of the available information, and we present several solutions.

IMAGE PROCESSING (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Kasinski, A. and T. Piascik	"Managing processes of perceptual organization for emerging geometrical objects."	<u>Proceedings of the IEEE International Conference on Systems, Man and Cybernetics</u> 3: 1604-1609 (2000).	Image Processing, Artificial Vision	Three lower level layers of the hierarchical machine perception system are described, and experimental results are provided. Three approaches to the features fusion: one based on crisp, geometrical, heuristic conditions, the second based on fuzzyfied conditions and the third one based on Dempster-Shafer theory are addressed. The results obtained with three methods are compared and presented on the example image of the real scene.
Krishnapuram, R.	"A Belief Maintenance Scheme for Hierarchical Knowledge-Based Image-Analysis Systems."	<u>International Journal of Intelligent Systems</u> 6(7): 699-715 (1991).	Image Processing, Knowledge-base, Decision-Making	In this article, we show how the Dempster-Shafer theoretic concepts of refinement and coarsening can be used to aggregate and propagate evidence in a multi-resolution image analysis system based on a hierarchical knowledge base.
Lohmann, G.	"Evidential reasoning approach to the classification of satellite images."	<u>Forschungsbericht Deutsche Forschungsanstalt fuer Luft und Raumfahrt, DLR FB:</u> 91-29 (1991).	Image Processing, Satellite, Classification, Ecological Mapping	A new algorithm for classifying satellite images is presented. The new algorithm called EBIS (Evidence-Based Interpretation of Satellite Images) - will be used for ecological mappings. In EBIS, a feature space is regarded as a source of evidence in the sense of the Dempster-Shafer-theory, and methods of evidential reasoning are used for combining evidence stemming from several disparate sources. This makes EBIS particularly useful for integrating different data sources such as various sensors, digital elevation models or other types of ancillary data.
Mulhem, P., D. Hong, et al.	"Labeling update of segmented images using conceptual graphs and Dempster-Shafer theory of evidence."	<u>IEEE International Conference on Multi Media and Expo(II)</u> : 1129-1132 (2000).	Image Processing, Object Recognition	We propose here to use conceptual graphs (a knowledge representation formalism that allow fast processing) with Dempster-Shafer theory of evidence to update original labeling coming from a segmentation that labels image regions out of context.

IMAGE PROCESSING (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Payne, M. G., Q. Zhu, et al.	"Using the Dempster-Shafer reasoning model to perform pixel-level segmentation on color images."	Proceedings of SPIE The International Society for Optical Engineering: 26-35 (1992).	Image Processing	We present an algorithm that performs pixel-level segmentation based upon the Dempster-Shafer theory of evidence. The algorithm fuses image data from the multichannels of color spectra. Dempster-Shafer reasoning is used to drive the evidence accumulation process for pixel level segmentation of color scenes.
Peddle, D. R.	"Mercury-Circle-Plus : An Evidential Reasoning Image Classifier."	(1995). <u>Computers & Geosciences</u> 21 (10): 1163-	Image Processing, Software, Geoscience, Environment, Classification	MERCURY circle plus is a multisource evidential reasoning classification software system based on the Dempster-Shafer theory of evidence. The design and implementation of this software package is described for improving the classification and analysis of multisource digital image data necessary for addressing advanced environmental and geoscience applications. An example of classifying alpineland cover and permafrost active layer depth in northern Canada is presented to illustrate the use and application of these ideas.
Pinz, A., M. Prantl, et al.	"Active fusion : A new method applied to remote sensing image interpretation."	<u>Pattern Recognition Letters</u> 17 (13): 1349-1359 (1996).	Image Processing, Artificial Vision	In this paper, we introduce a new method, termed "active fusion", which provides a common framework for active selection and combination of information from multiple sources in order to arrive at a reliable result at reasonable costs. The implementation of active fusion on the basis of probability theory, the Dempster-Shafer theory of evidence and fuzzy sets is discussed.
Sarma, L. C. S. and V. V. S. Sarma	"A Prototype Expert-System for Interpretation of Remote-Sensing Image Data."	<u>Sadhana-Academy Proceedings in Engineering Sciences</u> 19 (pt.1): 93-111 (1994).	Image Processing, Knowledge-base, Classification, Geography	In this paper, we have critically studied visual interpretation processes for urban land cover and land use information. The Dempster-Shafer theory of evidence is used to combine evidence from various interpretation keys for identification of generic class and subclass of a logical image object. Analysis of some Indian Remote Sensing Satellite images has been done using various basic probability assignments in combination with learning.

IMAGE PROCESSING (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Tupin, F., I. Bloch, et al.	"A first step toward automatic interpretation of SAR images using evidential fusion of several structure detectors."	<u>IEEE Transactions on Geoscience and Remote Sensing</u> 37 (3/pt.1): 1327-1343 (1999).	Image Processing, Radar, Cartography	We propose a method aiming to characterize the spatial organization of the main cartographic elements of a synthetic aperture radar (SAR) image and thus giving an almost automatic interpretation of the scene. Our approach is divided into three main steps which build the whole image interpretation gradually. The first step consists of applying low-level detectors taking the speckle statistics into account and extracting some raw information from the scene. The detector responses are then fused in a second step using Dempster-Shafer theory, thus allowing the modeling of the knowledge that we have about operators, including possible ignorance and their limits. A third step gives the final image interpretation using contextual knowledge between the different classes. Results of the whole method applied to different SAR images and to various landscapes are presented.
Vancleynen-breugel, J., S. A. Osinga, et al.	"Road Extraction from Multitemporal Satellite Images by an Evidential Reasoning Approach."	<u>Pattern Recognition Letters</u> 12 (6): 371-380 (1991).	Image Processing, Satellite	Road networks extracted from multi-temporal SPOT images of the same scene are matched to collect evidence for individual road segments. The Dempster-Shafer theory is applied to find a degree of confirmation for a road segment in one network based on its corresponding lines in the other networks.
Vannoorenberghe, P., O. Colot, et al.	"Color image segmentation using Dempster-Shafer's theory."	<u>IEEE International Conference on Image Processing</u> 4 : 300-304 (1999).	Image Processing, Biomedical Engineering	In this paper, we propose a color image segmentation method based on the Dempster-Shafer's theory. The basic idea consists in modeling the color information in order to have the features of each region in the image. This model, obtained on training sets extracted from the intensity, allows for the reduction of the classification errors concerning each pixel of the image. The proposed segmentation algorithm has been applied to synthetic and biomedical images in order to illustrate the methodology.
Verly, J. G., R. L. Delanoy, et al.	"Model-Based System for Automatic Target Recognition from Forward-Looking Laser-Radar Imagery."	<u>Optical Engineering</u> 31 (12): 2540-2552 (1992).	Image Processing, Target Recognition, Radar	We describe an experimental model-based automatic target recognition (ATR) system, called XTRS, for recognizing 3-D vehicles in real or synthetic, ground-based or airborne, 2-D laser-radar range and intensity images

IMAGE PROCESSING (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Verly, J. G., B. D. Williams, et al.	“Automatic Object Recognition from Range Imagery Using Appearance Models.”		Image Processing, Radar, Object Recognition	A system for the automatic recognition of objects in real infrared-radar range imagery is described. Recognition consists of matching symbolic range silhouette descriptions against appearance models of known objects and then deciding among the possible objects using the Dempster-Shafer theory of evidence. Both contour-based and region-based silhouette extraction and recognition showed good results.
Wang, Y. and D. L. Civco	“Evidential reasoning-based classification of multi-source spatial data for improved land cover mapping.”	<u>Canadian Journal of Remote Sensing</u> 20 : 381-395 (1994).	Image Processing, Classification, Cartography	In this paper, a two-stage distribution-free classification strategy was adopted to incorporate ancillary data in remote sensing image classification.. The approach provides a scheme that can readily pool attribute information from multi-source spatial data.
Wilkinson, G. G. and J. Megier	“Evidential reasoning in a pixel classification hierarchy. A potential method for integrating image classifiers and expert system rules based on geographic context.”	<u>International Journal of Remote Sensing</u> 11 : 1963-1968, (1990).	Image Processing, Classification, Expert System, Geography	This paper presents a technique for integrating diverse sources of evidence about pixel or segment classification using the belief function approach of the Dempster-Shafer theory of evidential reasoning. A description of the algorithm is provided and a case study is given to illustrate the potential of the method for combining output from image classifiers, geographic information systems and expert system rules concerning geographic context.
Yamane, S., K. Aoki, et al.	“Model-Based Object Recognition Using Basic Probability Assignment.”	<u>Systems and Computers in Japan</u> 26 (12): 49-57 (1995).	Image Processing, Object Recognition	A method of object recognition is proposed based on Dempster-Shafer theory (DS theory), which can treat the ambiguity of image data.
MEDICAL APPLICATIONS				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Andress, K. M.	“Evidential reconstruction of vessel trees from rotational angiograms.”	<u>IEEE International Conference on Image Processing</u> 3 : 385-389, (1998).	Medicine, Digital Imagery	The Dempster-Shafer theory of evidence is used to combine information about location of the vessels from the different projections contained in the digital subtraction angiogram (DSA) sequence.

MEDICAL APPLICATIONS (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Bell, D. A., J. W. Guan, et al.	"Using the Dempster-Shafer orthogonal sum for reasoning which involves space."	<u>Kybernetes</u> 27(4-5): 511-&, (1998).	Spatial Reasoning	The objects of interest here are geometric forms, and we can encode rectangular and other shaped forms using hexadecimal numbers according to shapes and positions. Boolean algebra of such shapes can then be used directly in Dempster-Shafer-type reasoning exercised. Discusses how medical and other fields can gain from this approach.
Boston, J. R., L. Baloa, et al.	"Combination of data approaches to heuristic control and fault detection."	<u>IEEE Conference on Control Applications Proceedings</u> 1: 98-103, (2000).	Control, Detection, Medicine	Data fusion techniques for ventricular suction detection in a heart assist device based on Bayesian, fuzzy logic and Dempster-Shafer theory were evaluated. Fusion techniques based on fuzzy logic and Dempster-Shafer theory provide a measure of uncertainty in the fused result. This uncertainty measure can be used in the control process, and it can also be used to identify faults in pump operation.
Cios, K. J., R. E. Freasier, et al.	"An Expert System for Diagnosis of Coronary-Artery Stenosis Based on Ti-201 Scintigrams Using the Dempster-Shafer Theory of Evidence."	<u>Computer Applications in the Biosciences</u> 6(4): 333-342, (1990).		
Lefevre, E., O. Colot, et al.	"Knowledge modeling methods in the framework of evidence theory. An experimental comparison for melanoma detection."	<u>Proceedings of the IEEE International Conference on Systems, Man and Cybernetics</u> 4: 2806-2811, (2000).	Medical Imaging	The aim of this paper is to present modeling methods of knowledge for the initialization of belief functions from Dempster-Shafer theory. Moreover, an experimental comparison of these different modeling on real data extracted from images of dermatological lesions is presented.
Medina, R., M. Garreau, et al.	"Evidence combination approach to reconstruction of the left ventricle from two angiographic views."		Medicine, Image	A left ventricle three-dimensional reconstruction method from two orthogonal angiographic projections is described based on the cylindrical closure operation and the Dempster-Shafer Theory.

MEDICAL APPLICATIONS (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Principe, J. C., S. K. Gala, et al.	"Sleep staging automaton based on the theory of evidence."	<u>IEEE Transactions on Biomedical Engineering</u> 36 : 503-509(1989).	Medicine, Biomedical Engineering, Signal Information	The Dempster-Shafer theory of evidence is used to develop a model for automated sleep staging by combining signal information and human heuristic knowledge in the form of rules.
Suh, D. Y., R. L. Eisner, et al..	"Knowledge-based system for boundary detection of four-dimensional cardiac magnetic resonance image sequences."	<u>IEEE Transactions on Medical Imaging</u> 12 (1): 65-72 (1993).	Medicine, Knowledge-based system, Image, Multiple sources	A strategy for a knowledge-based system to detect the interior and exterior boundaries of the left ventricle from time-varying cross-sectional images obtained by ECG-gated magnetic resonance imaging uses both fuzzy set theory and Dempster and Shafer theory to manage the knowledge and to control the flow of system information.
Suh, D. Y., R. M. Mersereau, et al.	"A system for knowledge-based boundary detection of cardiac magnetic resonance image sequences."	<u>Proceedings ICASSP, IEEE International Conference on Acoustics, Speech and Signal Processing</u> 4 : 2341-2344 (1990).	Medicine, Knowledge-based system, Image, Boundary Detection	A knowledge-based system is described for boundary detection from magnetic resonance image sequences of a beating heart. It is shown that the Dempster/Shافر theory and fuzzy set theory can be used for control of the system as well as for labeling objects in the images.
MISCELLANEOUS				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Cai, D., M. F. McTear, et al.	"Knowledge discovery in distributed databases using evidence theory."	<u>International Journal of Intelligent Systems</u> 15 (8): 745-761 (2000).	Knowledge Discovery, Distributed Databases	Distributed databases allow us to integrate data from different sources which have not previously been combined. Evidential functions are suited to represent evidence from different sources. Previous work has defined linguistic summaries to discover knowledge by using fuzzy set theory and using evidence theory to define summaries. In this paper we study linguistic summaries and their applications to knowledge discovery in distributed databases.

MISCELLANEOUS (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Cortes Rello, E. and F. Golshani	“Uncertain reasoning using the Dempster-Shafer method. An application in forecasting and marketing management.”	<u>Expert Systems</u> 7: 9-18 (1990).	Forecasting, Marketing, Management, Expert Systems	The intended purpose of this article is twofold: to study techniques for uncertainty management in expert systems, particularly the Dempster-Shafer theory of belief; and to use this method in the construction of an expert system for the field of forecasting and marketing management.
Gillett, P. R.	“Monetary unit sampling: a belief-function implementation for audit and accounting applications.”	<u>International Journal of Approximate Reasoning</u> 25(1): 43-70, (2000).	Finance Applications	Audit procedures may be planned and audit evidence evaluated using monetary unit sampling (MUS) techniques within the context of the Dempster-Shafer theory of belief functions.
Golshani, F., E. Cortes Rello, et al.	“Dynamic route planning with uncertain information.”	<u>Knowledge Based Systems</u> 9: 223-232, (1996).	Autonomous Vehicle Navigation	The paper describes the design of a route planning system, called RUTA-100, that works with incomplete information obtained from many unreliable knowledge sources and plans an optimal route by minimizing both danger and distance. The Dempster-Shafer theory of belief is used as the underlying formalism to pool and represent uncertain information and reason with it.
Isaksen, G. H. and C. S. Kim	“Interpretation of molecular geochemistry data by the application of artificial intelligence technology.”	<u>Organic Geochemistry</u> 26(1-2): 1-10, (1997).	Molecular Geochemistry	This paper describes the application of fuzzy logic and Dempster-Shafer theory to the interpretation of molecular geochemistry data with respect to key exploration parameters, such as thermal maturity, organic facies (organic matter type and depositional environment of the source rock(s)), geological age, and the degree of biodegradation.
Ji, Q., M. M. Marefat, et al.	“Evidential reasoning approach for recognizing shape features.”		Manufacturing, Feature Extraction	This paper introduces an evidential reasoning based approach for recognizing and extracting manufacturing features from solid model description of objects. The main contributions of our approach include introducing the evidential reasoning (Dempster-Shafer theory) to the feature extraction domain and developing the theory of principle of association to overcome the mutual exclusiveness assumption of the Dempster-Shafer theory.

MISCELLANEOUS (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Ji, Q., M. M. Marefat, et al.	“Dempster-Shafer and Bayesian networks for CAD-based feature extraction: A comparative investigation and analysis.”	<u>Proceedings of the National Conference on Artificial Intelligence</u> 2 , (1994).	Manufacturing, Feature Extraction	The paper evaluates the performance the Dempster-Shafer theory (DS) and the Bayesian Belief Network (BBN) with regard to their ability to extract manufacturing features from the solid model description of objects.
Kawahara, A. A. and P. M. Williams	“An Application of Dempster-Shafer Theory to the Assessment of Biogas Technology.”	<u>Energy</u> 17 (3): 205-214, (1992).	Biogas Technology	We apply the Dempster-Shafer theory of belief functions to the assessment of biogas technology in rural areas of Brazil. Two case studies are discussed in detail and the results compared with a more conventional method of project appraisal. On the computational side, it is shown how local computation and dimensionality reduction, in cases where certain relations hold between variables, can increase efficiency.
Lalmas, M.	“Dempster-Shafer's theory of evidence applied to structured documents: Modelling uncertainty.”	<u>SIGIR Forum</u> : 110-118, (1997).	Document Retrieval, Document Structure, Information Retrieval, Indexing	Chiaramella et al advanced a model for indexing and retrieving structured documents. This paper adds to this model a theory of uncertainty, the Dempster-Shafer theory of evidence. It is shown that the theory provides a rule, the Dempster's combination rule, that allows the expression of the uncertainty with respect to parts of a document, and that is compatible with the logical model developed by Chiaramella et al.
Lalmas, M. and I. Ruthven	“Representing and retrieving structured documents using the Dempster-Shafer theory of evidence: Modelling and evaluation.”	<u>Journal of Documentation</u> 54 (5): 529-565, (1998).	Document Retrieval, Document Structure, Information Retrieval, Indexing	In this paper we report on a theoretical model of structured document indexing and retrieval based on the Dempster-Shafer Theory of Evidence. This includes a description of our model of structured document retrieval, the representation of structured documents, the representation of individual components, how components are combined, details of the combination process, and how relevance is captured within the model.

MISCELLANEOUS (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Laskey, K. B. and M. S. Cohen	"Applications of the Dempster-Shafer Theory of Evidence for Simulation."	<u>Winter Simul Conf Proc</u> : 440-444, (1986).	Simulation	The key feature of the Dempster-Shafer theory is that precision in inputs is required only to a degree justified by available evidence. The output belief function contains an explicit measure of the firmness of output probabilities. The authors give an overview of belief function theory, presents the basic methodology for application to simulation, and gives a simple example of a simulation involving belief functions.
Ling, X. and W. G. Rudd	"Combining opinions from several experts."	<u>Applied Artificial Intelligence</u> 3 : 439-452, (1989).	Expert Opinion, Opinion Pooling	We develop an approach for combining expert opinions that formally allows for stochastic dependence. This approach is based on an extension of the Dempster-Shafer theory, a well-known calculus for reasoning with uncertainty in artificial intelligence.
Luo, W. B. and B. Caselton	"Using Dempster-Shafer theory to represent climate change uncertainties."	<u>Journal of Environmental Management</u> 49 (1): 73-93, (1997).	Decision Analysis, Climate Change, Water Resource Projects	This paper presents, along with some elementary examples, aspects of the Dempster-Shafer approach that contribute to its appeal when dealing with weak subjective and data-based information sources that have a bearing on climate change.
Mellouli, K. and Z. Elouedi	"Pooling expert opinions using Dempster-Shafer theory of evidence."	<u>Proceedings of the IEEE International Conference on Systems, Man and Cybernetics</u> 2 : 1900-1905, (1997).	Expert Opinion, Opinion Pooling	In this paper, we propose a method based on Dempster-Shafer theory of evidence, to pool expert judgements about the hypotheses of the studied field and to get an assessment and even a ranking of the different scenarios.
Schocken, S. and R. A. Hummel	"On the use of the Dempster Shafer model in information indexing and retrieval applications."	<u>International Journal of Man Machine Studies</u> 39 (5): 843-879, (1993).	Information Retrieval, Information Indexing	This paper has two objectives: (i) to describe and resolve some caveats in the way the Dempster Shafer theory is applied to information indexing and retrieval, and (ii) to provide an intuitive interpretation of the Dempster Shafer theory, as it unfolds in the simple context of a canonical indexing model.

MISCELLANEOUS (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Sy, B. K. and J. R. Deller, Jr.	“AI-based communication system for motor and speech disabled persons: Design methodology and prototype testing.”	<u>IEEE Transactions on Biomedical Engineering</u> 36 : 565-571, (1989).	Communication System, Optimization	The device is centered on a knowledge base of the grammatical rules and message elements. A belief reasoning scheme based on both the information from external sources and the embedded knowledge is used to optimize the process of message search. The search for the message elements is conceptualized as a path search in the language graph, and a special frame architecture is used to construct and to partition the graph. Bayesian belief reasoning from the Dempster-Shafer theory of evidence is augmented to cope with time-varying evidence.
MULTIPLE SENSORS				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Basti, Egrave, et al.	“Methods for multisensor classification of airborne targets integrating evidence theory.”	<u>Aerospace Science and Technology</u> 2 (6): 401-411(1998).	Multisensor Classification, Airborne Target Classification	This paper proposes to analyze methods applied to the multisensor classification of airborne targets using Dempster-Shafer theory. Several simulations relating to an airborne target classification problem are presented.
Belloir, F., R. Huez, et al.	“A smart flat-coil eddy-current sensor for metal-tag recognition.”	<u>Measurement Science & Technology</u> 11 (4): 367-374 (2000).	Multiple Sensors, Pattern Recognition	This paper describes a smart eddy-current sensor for locating and identifying metal tags used to recognize buried pipes. Intelligent pattern-recognition methods and their combination by the Dempster-Shafer theory of evidence are briefly presented.
Braun, J. J.	“Dempster-Shafer Theory and Bayesian reasoning in multisensor data fusion.”	<u>Proceedings of SPIE The International Society for Optical Engineering</u> 4051 : 255-266 (2000).	Multiple sensors, Classification	This paper presents a Monte Carlo simulation approach for a comparative analysis of a Dempster-Shafer Theory based and a Bayesian multisensor data fusion in the classification task domain, including the implementation of both formalisms, and the results of the Monte Carlo experiments of this analysis.

MULTIPLE SENSORS (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Coombs, K., D. Freel, et al.	“Using Dempster-Shafer methods for object classification in the theater ballistic missile environment.”	<u>Proceedings of SPIE The International Society for Optical Engineering</u> 3719 : 103-113, (1999).	Sensors, Ballistic Missile Discrimination	The Dempster Shafer (DS) Theory of Evidential Reasoning may be useful in handling issues associated with theater ballistic missile discrimination. This paper highlights the Dempster-Shafer theory and describes how this technique was implemented and applied to data collected by two infrared sensors on a recent flight test.
Fabre, S., A. Appriou, et al.	“Sensor Fusion Integrating Contextual Information.”		Multiple Sensors	The Dempster-Shafer theory of evidence is used to integrate information from the context of the sensor acquisitions.
Jiang, J., J. Guo, et al.	“Multisensor multiple-attribute data association.”		Multiple Sensors, Target Identification, Simulation	A multisensor multiple-attribute data association method is presented based on Dempster and Shafer (D-S) evidence theory. This approach is illustrated by simulations involving multisensor multiple targets in a dense clutter environment.
Pigeon, L., B. Solaiman, et al.	“Dempster-Shafer theory for multi-satellites remotely-sensed observations.”	<u>Proceedings of SPIE The International Society for Optical Engineering</u> 4051 : 228-236, (2000).	Sensors, Satellites	This study suggests a slight variation of the Dempster-Shafer theory using observation qualification in multi-sensor contexts. The uncertainty is placed on the rules instead of on sources. Thus, sensor's specialization is taken into account. By this approach, the masses are not directly attributed on the frame of discernment elements, but on the rules themselves that become the sources of knowledge, in the context of Dempster combining rule. It proposes then an approach for observation qualification in a multi-sensor context, as well as it suggests a new path for the delicate task of mass attribution.
Safranek, R. J., S. Gottschlich, et al.	“Evidence accumulation using binary frames of discernment for verification vision.”	<u>IEEE Transactions on Robotics and Automation</u> 6 : 405-417, (1990).	Sensors, Artificial Vision	Vision sensor output can be processed to yield a multitude of low-level measurements, where each is inherently uncertain, which must somehow be combined to verify the locations of an object. It is shown that this combination can be accomplished via Dempster-Shafer theory using binary frames of discernment (BFODs).

MULTIPLE SENSORS (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Tang, Y. C. and C. S. G. Lee	"A Geometric Feature Relation Graph Formulation for Consistent Sensor Fusion."	<u>IEEE Transactions on Systems Man and Cybernetics</u> 22 (1): 115-129, (1992).	Identification, Sensors, Optimization, Simulation	The paper presents an effective and reliable procedure for identifying corresponding measurements of features in the presence of sensory uncertainty based on both geometric and topological constraints, and a nonlinear programming formulation for maintaining consistency in a network of relations is proposed. The Dempster-Shafer theory of belief functions is applied to make the utilization of topological constraints in achieving reliable identification.
Tang, Y. C. and C. S. G. Lee	"Optimal Strategic Recognition of Objects Based on Candidate Discriminating Graph with Coordinated Sensors."	<u>IEEE Transactions on Systems Man and Cybernetics</u> 22 (4): 647-661, (1992).	Object Recognition, Identification, Optimization, Sensors, Simulation	Reliable and knowledge-based recognition of objects is obtained by applying the Dempster-Shafer theory of belief functions. Computer simulations were performed to verify the feasibility and to analyze the performance of the optimal strategic recognition of objects.
Tchamova, A.	"Evidence reasoning theory with application to the identity estimation and data association systems."	<u>Mathematics and Computers in Simulation</u> 43 : 139-142, (1997).	Sensors, Data Association Systems, Simulation	The theory of Dempster-Shafer is discussed with emphasis placed on its use grown from the field of multisensor data fusion and data association systems. The aims of this paper are to investigate: how the structure of multisensor integration systems influences over the accuracy of objects identification process; to determine the dependence of the degree of uncertainty on the speed of receiving best evidential intervals; to determine what is the impact of increasing number of sensors on the calculation time.
Wang, G., Y. He, et al.	"Adaptive sensor management in multisensor data fusion system."	<u>Chinese Journal of Electronics</u> 8 : 136-139 (1999).	Multiple Sensors, Simulation	Sensor management has been an active research area in recent years. Based on fuzzy set theory and the Dempster-Shafer theory of mathematical evidence, adaptive sensor management schemes in multisensor data fusion system are presented by using individual sensor's performance.
Zhang, R., G. Gu, et al.	"AUV obstacle-avoidance based on information fusion of multi-sensors."		Multiple Sensors, Autonomous Vehicles	This paper presents a method of AUV (Autonomous Underwater Vehicle) obstacle avoidance based on information fusion of multi-sensors. Dempster Shafer's theory of evidence is used to judge whether an obstacle exists ahead of an AUV.

RISK AND RELIABILITY				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Cronhjort, B. T. and A. Mustonen	“Computer Assisted Reduction of Vulnerability of Data Centers.”		Risk Control, Expert Systems	The authors proceed to suggest an expert systems approach for the evaluation of EDP risks, and for risk control. A methodology based on the Dempster-Shafer Theory of Evidence is proposed, and the essential principles for the implementation of such an expert system are outlined.
Holmberg, J., P. Silvennoinen, et al.	“Application of the Dempster-Shafer Theory of Evidence for Accident Probability Estimates.”	<u>Reliability Engineering & System Safety</u> 26 (1): 47-58, (1989).	Risk Analysis	
Ibrahim, A. and B. M. Ayyub	Uncertainties in risk-based inspection of complex systems.	<u>Analysis and Management of Uncertainty: Theory and Applications</u> . B. M. Ayyub, M. M. Gupta and L. N. Kanal. New York, North-Holland. 13 : 247-262, (1992).	Risk Analysis, Complex Systems	Catastrophic industrial failures over the past decade highlight the societal need to use more explicitly risk-based methods and procedures with uncertainty analysis for these systems. Three measures of uncertainty are discussed and several examples to illustrate their applications are presented. Logic diagrams and techniques were utilized to propagate uncertainties for the process of assessing the magnitude of consequences due to failure and the uncertainty associated with them.
Inagaki, T.	“Interdependence between Safety-Control Policy and Multiple-Sensor Schemes Via Dempster-Shafer Theory.”	<u>IEEE Transactions on Reliability</u> 40 (2): 182-188, (1991).	Reliability, Safety, Fault Warning	This paper explores the application of the Dempster-Shafer theory in system reliability and safety. Inappropriate application of the Dempster-Shafer theory to safety-control policies can degrade plant safety. This is proven in two phases: 1) A new unified combination rule for fusing information on plant states given by independent knowledge sources such as sensors or human operators is developed. 2) Combination rules can not be chosen in an arbitrary manner; ie, the best choice of combination rules depends on whether the safety-control policy is fault-warning or safety-preservation.

ROBOTICS				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Clerentin, A., L. Delahoche, et al.	“Cooperation between two omnidirectional perception systems for mobile robot localization.”	<u>IEEE International Conference on Intelligent Robots and Systems 2</u> : 1499-1504 (2000).	Multiple Sensors, Robotics	In this paper, an absolute localization paradigm based on the cooperation of an omnidirectional vision system composed of a conical mirror and a CCD camera and a low cost panoramic range finder system is reported. We present an absolute localization method that uses three matching criteria fused by the combination rules of the Dempster-Shafer theory.
Graham, J. H.	“Sensory-Based Safeguarding of Robotic Systems.”	<u>International Journal of Robotics & Automation</u> 9(4): 141-148, (1994).	Robotics, Sensors, Decision-Making	This paper presents a multilevel system for contributing to robot safety by the use of sensory information in partially defined environments, including provision for sensory preprocessing, sensory fusion, and high-level decision making. Sensory fusion is achieved by using Dempster's rule of combination on a set of belief functions generated from the input sensory data.
Hughes, K. and R. R. Murphy	“Ultrasonic robot localization using Dempster-Shafer theory.”	<u>Proceedings of SPIE The International Society for Optical Engineering</u> : 2-11, (1992).	Robotics, Sensors	In this paper we present a method for ultrasonic robot localization without a priori world models utilizing the ideas of distinctive places and open space attraction. This method was incorporated into a move-to-station behavior, which was demonstrated on the Georgia Tech mobile robot. The key aspect of our approach was to use Dempster-Shafer theory to overcome the problem of the uncertainty in the range measurements returned by the sensors.
Luo, Z. and D. Li	“Multi-source information integration in intelligent systems using the plausibility measure.”	<u>IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems</u> : 403-409, (1994).	Robotics, Artificial Vision, Object Recognition, Sensors	In the paper, we develop a new multisource information fusion scheme using the plausibility measure. The method avoids using Dempster's rule of combination, so as to overcome the intractability of Dempster-Shafer computations, allowing the theory to be feasible in many more applications. A simple robotic vision system with object recognition data from multisensor is presented to highlight benefits of the new method.
Murphy, R. R.	“Dempster-Shafer theory for sensor fusion in autonomous mobile robots.”	<u>IEEE Transactions on Robotics and Automation</u> 14(2): 197-206 (1998).	Multiple Sensors, Robotics	This article discusses Dempster-Shafer (DS) theory in terms of its utility for sensor fusion for autonomous mobile robots. It exploits two little used components of DS theory: the weight of conflict metric and the enlargement of the frame of discernment.

ROBOTICS (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Murphy, R. R. and E. Rogers	"Estimating time available for sensor fusion exception handling."		Muliple Sensors, Robotics	In this paper, we consider the impact of time for teleoperation applications where a remote robot attempts to autonomously maintain sensing in the presence of failures yet has the option to contact the local for further assistance. Time limits are determined by using evidential reasoning with a novel generalization of Dempster-Shafer theory.
Puente, E. A., L. Moreno, et al.	"Analysis of data fusion methods in certainty grids application to collision danger monitoring."	<u>IECON Proceedings 2</u> : 1133-1137, (1991).	Robotics, Monitoring	The authors focus on the use of the occupancy grid representation to maintain and combine the information acquired from sensors about the environment. This information is subsequently used to monitor the robot collision danger risk and take into account that risk in starting the appropriate maneuver. The occupancy grid representation uses a multidimensional tessellation of space into cells, where each cell stores some information about its state. Two main approaches have been used to model the occupancy of a cell: probabilistic estimation and the Dempster-Shafer theory of evidence. Probabilistic estimation and some combination rules based on the Dempster-Shafer theory of evidence are analyzed and their possibilities compared.
Ribo, M. and A. Pinz	"A comparison of three uncertainty calculi for building sonar-based occupancy grids."	<u>Robotics and Autonomous Systems</u> 35 (3-4): 201-209, (2001).	Robotics, Sensors	In this paper, we describe and compare three different uncertainty calculi techniques to build occupancy grids of an unknown environment using sensory information provided by a ring of ultrasonic range-finders. These techniques are based on Bayesian theory, Dempster-Shafer theory of evidence, and fuzzy set theory.

ROBOTICS (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Utete, S. W., B. Barshan, et al.	“Voting as validation in robot programming.”	<u>International Journal of Robotics Research</u> 18 (4): 401-413, (1999).	Robotics, Target Identification, Sensors, Decision-Making	This paper investigates the use of voting as a conflict-resolution technique for data analysis in robot programming. Dispersed sensors take decisions on target type, which must then be fused to give the single group classification of the presence or absence and type of a target. Dempster-Shafer evidential reasoning is used to assign a level of belief to each sensor decision. The decisions are then fused by two means. Using Dempster's rule of combination, conflicts are resolved through a group measure expressing dissonance in the sensor views.
Wu, Y., J. Huang, et al.	“Mobile robot obstacle detection and environment modeling with sensor fusion.”	<u>Zidonghua Xuebao/Acta Automatica Sinica</u> 23 : 641-648, (1997).	Robotics, Environment Detection,	Modeling 2D environment and road detection for mobile robot by fusing color and range image information are discussed. The environment model is constructed by using multi-resolution 2D grid representation, which is proved to be a better solution to the tradeoff between accuracy and computation speed. The fusion algorithm is designed based on a generalized Dempster-Shafer theory of evidence (DSTE), which is efficient in dealing with dependent information.
SIGNAL DETECTION AND PROCESSING				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Boston, J. R.	“Signal Detection Models Incorporating Uncertainty: Sensitivity to Parameter Estimates.”	Uncertainty Modelling and Analysis: Theory and Applications. B. M. Ayyub and M. M. Gupta. New York, Elsevier. 17: 459-476(1994).	Classification, Signal Detection, Sensitivity Analysis	This chapter develops models for signal detection in noisy waveforms, based on Dempster-Shafer theory and on fuzzy logic, that classify waveforms as signal-present, signal-absent, or uncertain. The performances of the models were evaluated using simulated sensory evoked potential data and compared to a Bayesian maximum likelihood detector. The effects of errors in estimates of the statistical parameters of the wave forms are considered.

SIGNAL DETECTION AND PROCESSING (continued)				
AUTHOR(S)	TITLE	REFERENCE	GENERAL SUBJECT HEADINGS	APPLICATION
Boston, J. R.	"A signal detection system based on Dempster-Shafer theory and comparison to fuzzy detection."	<u>IEEE Transactions on Systems Man and Cybernetics Part C-Applications and Reviews</u> 30 (1): 45-51, (2000).	Classification, Signal Detection	This paper describes a signal detection algorithm based on Dempster-Shafer theory: The detector combines evidence provided by multiple waveform features and explicitly considers uncertainty in the detection decision, The detector classifies waveforms as including a signal, not including a signal, or being uncertain, in which case no conclusion regarding presence or absence of a signal is drawn.
Chao, J. J. and C. C. Lee	"An Efficient Direct-Sequence Signal Detector Based on Dempster-Shafer Theory."	<u>IEEE Transactions on Communications</u> 38 (6): 868-874, (1990).		
Chao, J. J., C. M. Cheng, et al.	"A moving target detector based on information fusion."		Target Detection, Radar, Signals	Moving target detector (MTD) related multiple-hypothesis testing is considered, and the Dempster-Shafer theory is applied to this problem. Feature parameters are extracted from radar signals, and the value of each feature parameter is interpreted in terms of Dempster-Shafer's belief or disbelief for the associated hypotheses. Using Dempster's combining rule, a generalized likelihood ratio test is derived.
Hughes, R. C. and J. N. Maksym	"Acoustic Signal Interpretation Reasoning with Non-Specific and Uncertain Information."	<u>Pattern Recognition</u> 18 : 475-483, (1984).	Identification, Expert Systems	An expert system approach to identifying the sources of underwater acoustic signals is described. In order to deal with non-specific and uncertain evidence in the presence of an unknown number of signal sources, we develop an inference network approach which is based on the Dempster-Shafer theory of evidence.
Jang, L.-W. and J.-J. Chao	"Information fusion algorithm for data association in multitarget tracking."		Target Tracking, Radar, Sonar	We employ the technique of uncertain information processing to solve problems of multitarget tracking. We consider the data association problem as a fuzzy partition. Dempster-Shafer theory is used to evaluate the plausibilities of the association events. Using the plausibilities, a fuzzy partition is performed.

External Distribution

Bilal Ayyub (2)
Department of Civil Engineering
University of Maryland
College Park, MD 20742

Ivo Babuska
Texas Institute for Computational
and Applied Mathematics
Mail Code C0200
University of Texas at Austin
Austin, TX 78712-1085

Osman Balci
Department of Computer Science
Virginia Tech
Blacksburg, VA 24061

Steven Batill (2)
Dept. of Aerospace & Mechanical Engr.
University of Notre Dame
Notre Dame, IN 46556

James Berger
Inst. of Statistics and Decision Science
Duke University
Box 90251
Durham, NC 27708-0251

Chun-Hung Chen (2)
Department of Systems Engineering &
Operations Research
George Mason University
4400 University Drive, MS 4A6
Fairfax, VA 22030

Raymond Cosner (2)
Boeing-Phantom Works
MC S106-7126
P. O. Box 516
St. Louis, MO 63166-0516

Thomas A. Cruse
398 Shadow Place
Pagosa Springs, CO 81147-7610

Department of Energy (3)
Attn: William Reed, NA-114
Jamileh Soudah, NA-114
B. Pate, NA-114
1000 Independence Ave., SW
Washington, DC 20585

U. M. Diwekar (2)
Center for Energy and
Environmental Studies
Carnegie Mellon University
Pittsburgh, PA 15213-3890

Robert G. Easterling
7800 Northridge NE
Albuquerque, NM 87109

Doug Elias
Parker Center for Investment Research
Cornell University
302 Sage Hall
Ithaca, NY 14853-6201

Isaac Elishakoff
Dept. of Mechanical Engineering
Florida Atlantic University
777 Glades Road
Boca Raton, FL 33431-0991

Ashley Emery
Dept. of Mechanical Engineering
Box 352600
University of Washington
Seattle, WA 98195-2600

C. Robert Emerson
Department of Systems Science and
Industrial Engineering
Binghamton University
P.O. Box 6000
Binghamton, NY 13902-6000

Scott Ferson (5)
Applied Biomathematics
100 North Country Road
Setauket, New York 11733-1345

Mike Giltrud
Defense Threat Reduction Agency
DTRA/CPWS
6801 Telegraph Road
Alexandria, VA 22310-3398

James Glimm (2)
Dept. of Applied Math & Statistics
P138A
State University of New York
Stony Brook, NY 11794-3600

James Gran
SRI International
Poulter Laboratory AH253
333 Ravenswood Avenue
Menlo Park, CA 94025

Bernard Grossman (2)
Dept. of Aerospace &
Ocean Engineering
Mail Stop 0203
215 Randolph Hall
Blacksburg, VA 24061

Sami Habchi
CFD Research Corp.
Cummings Research Park
215 Wynn Drive
Huntsville, AL 35805

Raphael Haftka (2)
Dept. of Aerospace and Mechanical
Engineering and Engr. Science
P. O. Box 116250
University of Florida
Gainesville, FL 32611-6250

Achintya Haldar (2)
Dept. of Civil Engineering
& Engineering Mechanics
University of Arizona
Tucson, AZ 85721

Tim Hasselman
ACTA
2790 Skypark Dr., Suite 310
Torrance, CA 90505-5345

George Hazelrigg
Division of Design, Manufacturing
& Innovation
Room 508N
4201 Wilson Blvd.
Arlington, VA 22230

David Higdon
Inst. of Statistics and Decision Science
Duke University
Box 90251
Durham, NC 27708-0251

Richard Hills (2)
College of Engineering, MSC 3449
New Mexico State University
P. O. Box 30001
Las Cruces, NM 88003

F. Owen Hoffman (2)
SENES
102 Donner Drive
Oak Ridge, TN 37830

Luc Huyse
Southwest Research Institute
P. O. Drawer 28510
San Antonio, TX 78284-0510

Leo Kadanoff (2)
Research Institutes Building
University of Chicago
5640 South Ellis Ave.
Chicago, IL 60637

George Karniadakis (2)
Division of Applied Mathematics
Brown University
192 George St., Box F
Providence, RI 02912

Alan Karr
Inst. of Statistics and Decision Science
Duke University
Box 90251
Durham, NC 27708-0251

George Klir (2)
Department of Systems Science and
Industrial Engineering
Binghamton University
P.O. Box 6000
Binghamton, NY 13902-6000

Sarah Lam
Department of Systems Science and
Industrial Engineering
Binghamton University
P.O. Box 6000
Binghamton, NY 13902-6000

Harold Lewis III
Department of Systems Science and
Industrial Engineering
Binghamton University
P.O. Box 6000
Binghamton, NY 13902-6000

Robert Lust
General Motors, R&D and Planning
MC 480-106-256
30500 Mound Road
Warren, MI 48090-9055

Sankaran Mahadevan (2)
Dept. of Civil &
Environmental Engineering
Vanderbilt University
Box 6077, Station B
Nashville, TN 37235

Hans Mair
Institute for Defense Analysis
Operational Evaluation Division
1801 North Beauregard Street
Alexandria, VA 22311-1772

Gregory McRae (2)
Dept. of Chemical Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139

Michael Mendenhall (2)
Nielsen Engineering & Research, Inc.
510 Clyde Ave.
Mountain View, CA 94043

Sue Minkoff (2)
Dept. of Mathematics and Statistics
University of Maryland
1000 Hilltop Circle
Baltimore, MD 21250

Max Morris (2)
Department of Statistics
Iowa State University
304A Snedecor-Hall
Ames, IW 50011-1210

NASA/Ames Research Center (2)
Attn: Unmeel Mehta, MS T27 B-1
David Thompson, MS 269-1
Moffett Field, CA 94035-1000

NASA/Glen Research Center (2)

Attn: John Slater, MS 86-7
Chris Steffen, MS 5-11
21000 Brookpark Road
Cleveland, OH 44135

NASA/Langley Research Center (7)

Attn: Dick DeLoach, MS 236
Michael Hemsch, MS 280
Tianshu Liu, MS 238
Jim Luckring, MS 280
Joe Morrison, MS 128
Ahmed Noor, MS 369
Sharon Padula, MS 159
Hampton, VA 23681-0001

Robert Nelson

Dept. of Aerospace & Mechanical Engr.
University of Notre Dame
Notre Dame, IN 46556

Dick Neumann

8311 SE Millihanna Rd.
Olalla, WA 98359

Efstratios Nikolaidis (2)

MIME Dept.
4035 Nitschke Hall
University of Toledo
Toledo, OH 43606-3390

Dale Pace

Applied Physics Laboratory
Johns Hopkins University
11100 Johns Hopkins Road
Laurel, MD 20723-6099

Alex Pang

Computer Science Department
University of California
Santa Cruz, CA 95064

Allan Pifko

2 George Court
Melville, NY 11747

Pradeep Raj (2)

Computational Fluid Dynamics
Lockheed Martin Aeronautical Sys.
86 South Cobb Drive
Marietta, GA 30063-0685

John Renaud (2)

Dept. of Aerospace & Mechanical Engr.
University of Notre Dame
Notre Dame, IN 46556

Tim Ross (2)

Dept. of Civil Engineering
University of New Mexico
Albuquerque, NM 87131

J. Sacks

Inst. of Statistics and Decision Science
Duke University
Box 90251
Durham, NC 27708-0251

Sunil Saigal (2)

Carnegie Mellon University
Department of Civil and
Environmental Engineering
Pittsburgh, PA 15213

Len Schwer

Schwer Engineering & Consulting
6122 Aaron Court
Windsor, CA 95492

Paul Senseny

Factory Mutual Research Corporation
1151 Boston-Providence Turnpike
P.O. Box 9102
Norwood, MA 02062

Kari Sentz (50)
Department of Systems Science and
Industrial Engineering
Binghamton University
P.O. Box 6000
Binghamton, NY 13902-6000

E. Sevin
Logicon RDA, Inc.
1782 Kenton Circle
Lyndhurst, OH 44124

Mark Shephard (2)
Rensselaer Polytechnic Institute
Scientific Computation Research Center
Troy, NY 12180-3950

Tom I-P. Shih
Dept. of Mechanical Engineering
2452 Engineering Building
East Lansing, MI 48824-1226

T. P. Shivananda
Bldg. SB2/Rm. 1011
TRW/Ballistic Missiles Division
P. O. Box 1310
San Bernardino, CA 92402-1310

Don Simons
Logicon
222 W. Sixth St.
P.O. Box 471
San Pedro, CA 90733-0471

Ashok Singhal
CFD Research Corp.
Cummings Research Park
215 Wynn Drive
Huntsville, AL 35805

Bill Spencer (2)
Dept. of Civil Engineering
and Geological Sciences
University of Notre Dame
Notre Dame, IN 46556-0767

D. E. Stevenson (2)
Computer Science Department
Clemson University
442 Edwards Hall, Box 341906
Clemson, SC 29631-1906

Ben Thacker
Southwest Research Institute
P. O. Drawer 28510
San Antonio, TX 78284-0510

Fulvio Tonon (2)
Geology and Geophysics Dept.
East Room 719
University of Utah
135 South 1460
Salt Lake City, UT 84112

Robert W. Walters (2)
Aerospace and Ocean Engineering
Virginia Tech
215 Randolph Hall, MS 203
Blacksburg, VA 24061-0203

Justin Y-T Wu
Applied Research Associates
Probabilistic Engineering
811 Spring Forest Rd.
Raleigh, NC 27609

Ren-Jye Yang
Ford Research Laboratory
MD2115-SRL
P.O.Box 2053
Dearborn, MI 4812

Simone Youngblood (2)
DOD/DMSO
Technical Director for VV&A
1901 N. Beauregard St., Suite 504
Alexandria, VA 22311

Foreign Distribution

Yakov Ben-Haim (2)
Department of Mechanical Engineering
Technion-Israel Institute of Technology
Haifa 32000
ISRAEL

Gert de Cooman (2)
Universiteit Gent
Onderzoeksgroep, SYSTeMS
Technologiepark - Zwijnaarde 9
9052 Zwijnaarde
BELGIUM

Luis Eca (2)
Instituto Superior Tecnico
Department of Mechanical Engineering
Av. Rovisco Pais
1096 Lisboa CODEX
PORTUGAL

Charles Hirsch (2)
Department of Fluid Mechanics
Vrije Universiteit Brussel
Pleinlaan, 2
B-1050 Brussels
BELGIUM

Igor Kozine (2)
Systems Analysis Department
Riso National Laboratory
P. O. Box 49
DK-4000 Roskilde
DENMARK

K. Papoulia
Inst. Eng. Seismology & Earthquake
Engineering
P.O. Box 53, Finikas GR-55105
Thessaloniki
GREECE

Max Ruppert
UniBw Munich - BauV 2.2
Inst. Engng.Mech. & Struct.Mech.
D - 85577 Neuibiberg
GERMANY

Lev Utkin
Institute of Statistics
Munich University
Ludwigstr. 33
80539, Munich
GERMANY

Malcolm Wallace
National Engineering Laboratory
East Kilbride
Glasgow
G75 0QU
UNITED KINGDOM

Peter Walley
6 Jewel Close
Port Douglas
Queensland 4871
AUSTRALIA

Department of Energy Laboratories

Los Alamos National Laboratory (13)

Mail Station 5000

P.O. Box 1663

Los Alamos, NM 87545

Attn: Jane M. Booker, MS P946

Terrence Bott, MS K557

Scott Doebling, MS P946

S. Eisenhower, MS K557

James Hyman, MS B284

Cliff Joslyn, MS B265

S. Keller-McNulty, MS F600

Deborah Leishman, MS F600

Mike McKay, MS F600

Laura McNamara, MS F600

Karen I. Pao, MS B256

Alyson G. Wilson, MS F600

Gregory Wilson, MS F600

Lawrence Livermore National Laboratory (3)

7000 East Ave.

P.O. Box 808

Livermore, CA 94550

Attn: Richard Klein, MS L-023

Roger Logan, MS L-125

Cynthia Nitta, MS L-096

Argonne National Laboratory

Attn: Paul Hovland

MCS Division

Bldg. 221, Rm. C-236

9700 S. Cass Ave.

Argonne, IL 60439

Sandia Internal Distribution

1	MS 0427	2104	F. F. Dean	1	MS 0847	9120	H. S. Morgan
1	MS 0482	2109	S. E. Lott	1	MS 0555	9122	M. S. Garrett
1	MS 0759	5845	I. V. Waddoups	1	MS 0893	9123	R. M. Brannon
1	MS 0751	6117	L. S. Costin	1	MS 0847	9124	J. M. Redmond
1	MS 0708	6214	P. S. Veers	1	MS 0847	9124	K. F. Alvin
1	MS 0490	6252	J. A. Cooper	1	MS 0553	9124	T. G. Carne
1	MS 0747	6410	A. L. Camp	1	MS 0847	9124	R. V. Field
1	MS 0747	6410	G. D. Wyss	1	MS 0847	9124	S. F. Wojtkiewicz
1	MS 0748	6413	D. G. Robinson	1	MS 0557	9125	T. J. Baca
1	MS 0748	6413	R. D. Waters	1	MS 0847	9126	R. A. May
1	MS 0451	6502	T. D. Woodall	1	MS 0824	9130	J. L. Moya
1	MS 1137	6534	S. M. DeLand	1	MS 1135	9132	L. A. Gritz
1	MS 1137	6536	G. K. Froehlich	1	MS 0828	9133	M. Pilch
1	MS 1137	6536	A. L. Hodges	1	MS 0828	9133	B. F. Blackwell
1	MS 0716	6804	P. G. Kaplan	1	MS 0828	9133	K. J. Dowding
1	MS 1395	6820	D. K. Belasich	40	MS 0828	9133	W. L. Oberkamp
1	MS 1395	6820	M. J. Chavez	1	MS 0557	9133	T. L. Paez
1	MS 1395	6820	J. G. Miller	1	MS 0828	9133	V. J. Romero
1	MS 1395	6821	M. K. Knowles	1	MS 0828	9133	M. P. Sherman
1	MS 1395	6821	J. W. Garner	1	MS 0557	9133	A. Urbina
1	MS 1395	6821	E. R. Giambalvo	1	MS 0847	9133	W. R. Witkowski
1	MS 1395	6821	T. Hadgu	1	MS 1135	9134	S. Heffelfinger
1	MS 1395	6821	S. C. James	1	MS 0835	9140	J. M. McGlaun
1	MS 1395	6821	J. S. Stein	1	MS 0835	9141	S. N. Kempka
1	MS 0779	6840	M. G. Marietta	1	MS 0835	9142	J. S. Peery
1	MS 0779	6840	P. Vaughn	1	MS 0827	9143	J. D. Zepper
2	MS 0779	6849	J. C. Helton	1	MS 1110	9214	D. E. Womble
1	MS 0779	6849	L. C. Sanchez	1	MS 0847	9211	M. S. Eldred
1	MS 0778	6851	G. E. Barr	1	MS 0847	9211	A. A. Giunta
1	MS 0778	6851	R. J. MacKinnon	1	MS 0847	9211	J. R. Red-Horse
1	MS 0778	6851	P. N. Swift	1	MS 0819	9211	T. G. Trucano
1	MS 0779	6852	B. W. Arnold	1	MS 1110	9214	R. B. Lehoucq
1	MS 0779	6852	R. P. Rechar	1	MS 0310	9220	R. W. Leland
1	MS 9404	8725	J. R. Garcia	1	MS 0318	9230	P. Yarrington
1	MS 9405	8726	R. E. Jones	1	MS 0819	9231	E. A. Boucheron
1	MS 9042	8727	J. J. Dike	1	MS 0316	9233	S. S. Dosanjh
1	MS 9012	8920	P. E. Nielan	1	MS 0316	9235	J. B. Aidun
1	MS 9003	8950	C. M. Hartwig	1	MS 0660	9519	D. S. Eaton
1	MS 0841	9100	T. C. Bickel	1	MS 0660	9519	M. A. Ellis
1	MS 0825	9115	W. H. Rutledge	1	MS 0139	9900	M. O. Vahle
1	MS 0836	9116	E. S. Hertel	1	MS 0139	9904	R. K. Thomas
1	MS 0836	9116	D. Dobranich	1	MS 0428	12300	D. D. Carlson
1	MS 0836	9117	R. O. Griffith	1	MS 0428	12301	V. J. Johnson

1	MS 0421	12323	J. M. Sjulín
1	MS 0829	12323	B. M. Rutherford
1	MS 0829	12323	F. W. Spencer
1	MS 0638	12326	M. A. Blackledge
1	MS 0638	12326	D. E. Percy
1	MS 0638	12326	D. L. Knirk
1	MS 0492	12332	D. R. Olson
1	MS 0405	12333	T. R. Jones
1	MS 0405	12333	S. E. Camp
1	MS 0434	12334	R. J. Breeding
1	MS 0829	12335	K. V. Diegert
1	MS 1170	15310	R. D. Skocypec

1	MS 1176	15312	R. M. Cranwell
1	MS 1176	15312	D. J. Anderson
1	MS 1176	15312	J. E. Campbell
1	MS 1176	15312	L. P. Swiler
1	MS 1179	15340	J. R. Lee
1	MS 1179	15341	L. Lorence
1	MS 9018	8945-1	Central Technical Files
2	MS 0899	9616	Technical Library
1	MS 0612	9612	Review & Approval Desk For DOE/OSTI