Midterm 1 Extra Notes

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Period

$T = \frac{2\pi}{\omega_0}$
$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$

Periodicity of Multiple Signals

$\frac{T_1}{T_2} = \frac{k}{l} \ where \ k, l \ \in \mathbb{Z}$

$T_0 = LCM(T_1, T_2, ...)$
$LCM(\frac{a_1}{b_1}, \frac{a_2}{b_2}, ..., \frac{a_n}{b_n}) = \frac{LCM(a_1, a_2, ..., a_n)}{GCD(b_1, b_2, ..., b_n)}$

Support

- Smallest closed set where signal is non-zero,
- Include boundaries and ignore simple intersections
  $supp(x) = [0, 1] \cup [2, \infty)$

Even and Odd

$x_{even}(t) = \frac{1}{2}(x(t) + x(-t))$
$x_{odd}(t) = \frac{1}{2}(x(t) - x(-t))$

Special Signals

Unit Step: CT ≈ DT

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Impulse: CT is derived from integral

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & t \neq 0 \end{cases}$$

Sifting:
$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$
$\therefore \int_{-\infty}^{\infty} x(t)\delta(t - t_0) \ dt = x(t_0)$

Sinusoids

$\omega_0 = 2\pi f_0 = \frac{2\pi}{N_0}$
DT: e^{jwn} is periodic N_0 if ω is an integer multiple of $\frac{2\pi}{N}$

Frequency Periodicity:
$e^{j\omega n} = e^{j\omega n + 2\pi}$
$e^{j\pi n} = (-1)^n$

Theorem 2.3:

$$\text{Let } N_0 \in \mathbb{Z}_{\geq 1} \text{ be a desired period. There are exactly } N_0 \text{ distinct DT}$$
$$\text{complex exponential signals of period} N_0 \text{ given by}$$
$$\phi_k[n] = e^{jk\omega_0 n}, \quad k \in \{0, 1, ..., N_0 - 1\}$$

CT Fourier Series

Existence:

$$\text{If } x \text{ has finite action, } x \in L_1^{per}$$
$$\text{then } \alpha_k \text{ are well defined and}$$
$$\lim_{k \to \pm\infty} |\alpha_k| = 0$$

Convergence:

$$\text{Let } e_k = \hat{x}_k - x \text{ be the error of approximation of } x$$
$$\hat{x} \text{ converges to } x$$
$$\text{i. pointwise if } \lim_{k \to \infty} e_k(t_0) = 0, \; t_0 \text{ is the point of interest}$$
$$\text{ii. uniformly if } \lim_{k \to \infty} ||e_k||_\infty = 0, \;\; \text{amplitude of error goes to 0}$$
$$\text{iii. in energy if } \lim_{k \to \infty} ||e_k||_2 = 0, \;\; \text{energy of error goes to 0}$$

i. if x has finite action and $\frac{d}{dt}x$ is cont. at t_o, then $\hat{x}$ converges to x
OR if there is is a finite discontinuity (left and right limits exist) $\hat{x}$ converges to the midpoint of the discontinuities, *Gibbs Phenomenon*

ii. if x has finite action and $\frac{d}{dt}x$ is cont. everywhere, then $\hat{x}$ converges to x uniformly

iii. if x has finite energy, $x \in L_2^{per}$

1. $\hat{x}$ converges to x in energy
2. $\{\alpha_k\}$ have finite energy, $\alpha \in l_2^{per}$
3. the signal and coefficients satisfy *Parsevals Relation*:

$$\frac{1}{T_0}||x||_2^2 = ||\alpha||_2^2$$

$$\frac{1}{T_0}\int_0^{T_0} |x(t)|^2 \, dt = \sum_{k=-\infty}^{\infty} |\alpha_k|^2$$

$$\text{energy of } x \text{ scaled by period} = \text{energy of } \alpha_k$$