Design and Analysis of Algorithms

Fundamental Concepts

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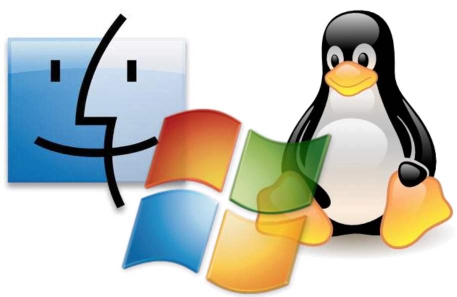
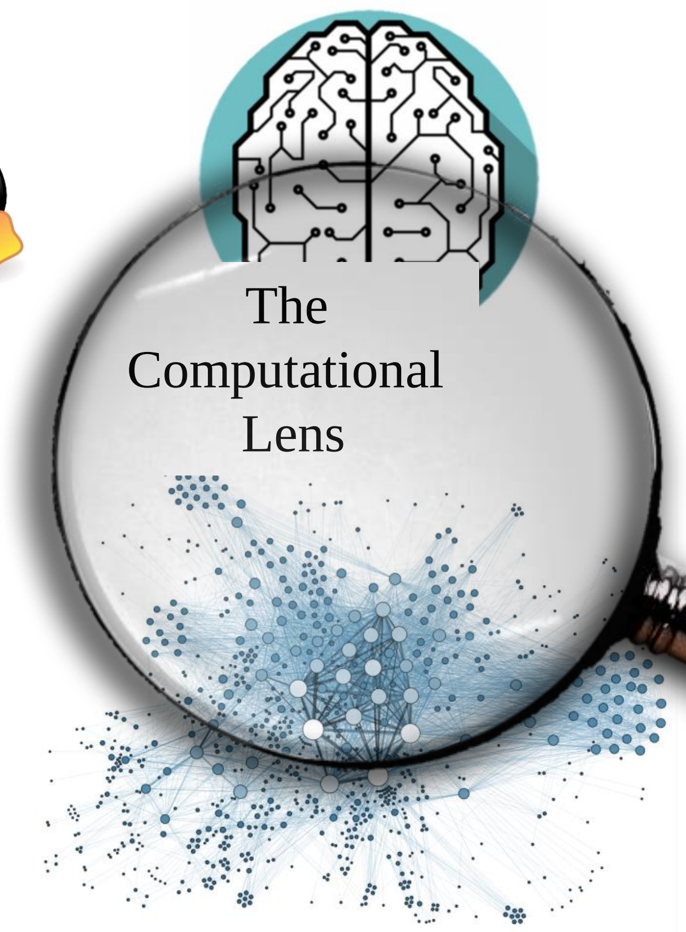
Algorithm Concept

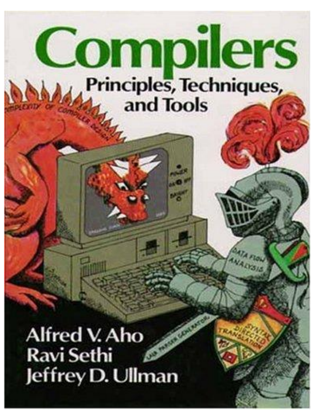


Operating Systems



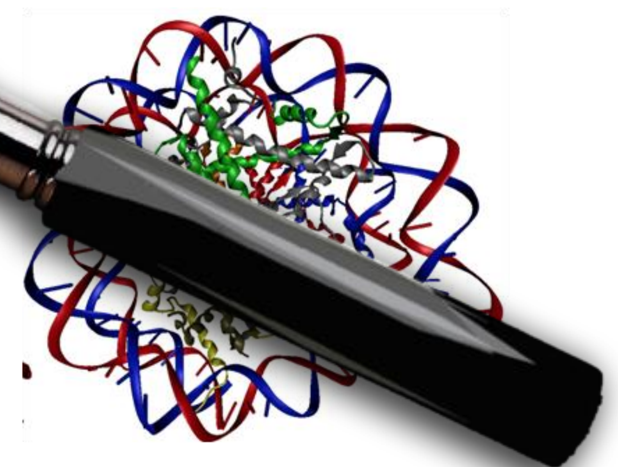
The Computational Lens

Cryptography



Compilers

Communication and Networks

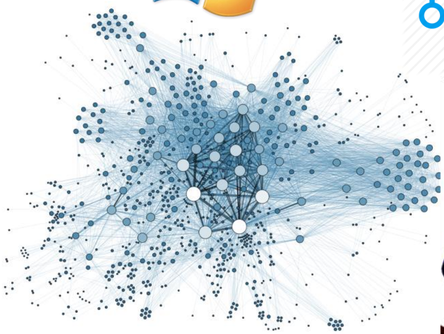


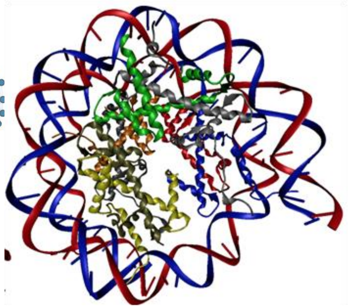
Computational Biology

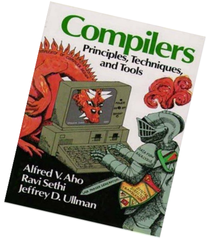
Algorithm Concept

Benefits

- All those things, without CS class numbers
- As we get more and more data and problem sizes get bigger and bigger, algorithms become more and more important.
- Will help you get a job.







Algorithm vs Program

Algorithm	Program
Design	Implementation
Domain-specific Knowledge	Programmer
English with Maths functions	Java, C++
H/W and OS	H/W and OS
Analyze	Testing

Priori Analysis

Language & Hardware independent

Posteriori Analysis

Language & Hardware dependent

Characteristics of Algorithm

- Major Characteristics are:
- Input
  - Should have 0 or more inputs
- Output
  - Should have at least one output
- Definiteness
  - Statements should be clear
- Finiteness
  - Should be finished/completed in finite time
- Effectiveness
  - Should not do the unnecessary statements

Multiplication of Integers

$$\begin{array}{r} 12 \\ \times\ 34 \\ \hline \end{array}$$

$$\begin{array}{r} 1234567895931413 \\ \times\ 4563823520395533 \\ \hline \end{array}$$

Multiplication of Integers

$$\overbrace{1233925720752752384623764283568364918374523856298}^{n}$$
$$\times\ 4562323582342395285623467235019130750135350013753$$

How would you solve this problem?
How long would it take you?

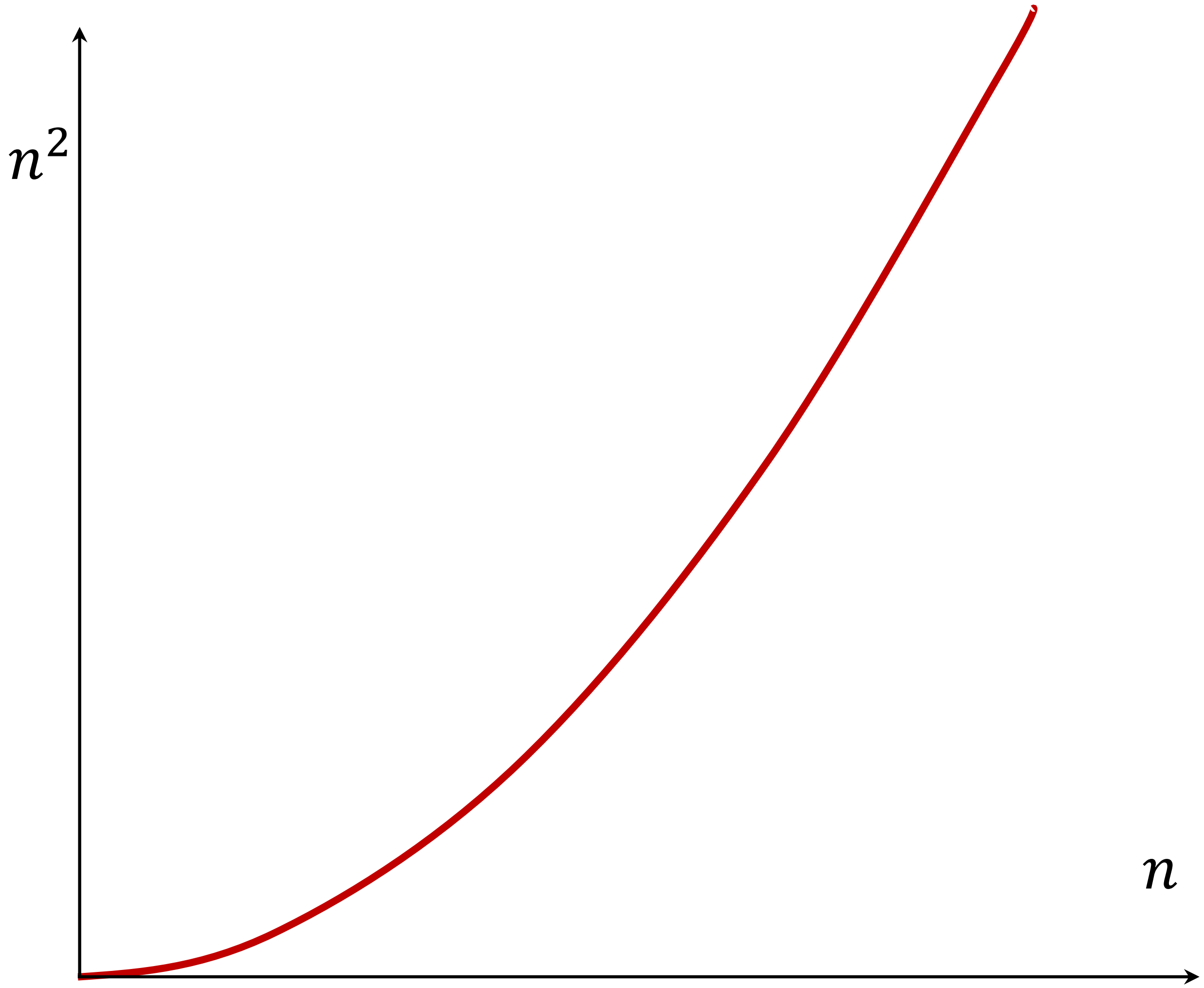
???

About n^2 one-digit operations

At most n^2 multiplications,
and then at most n^2 additions (for carries)
and then I have to add n different 2n-digit numbers…

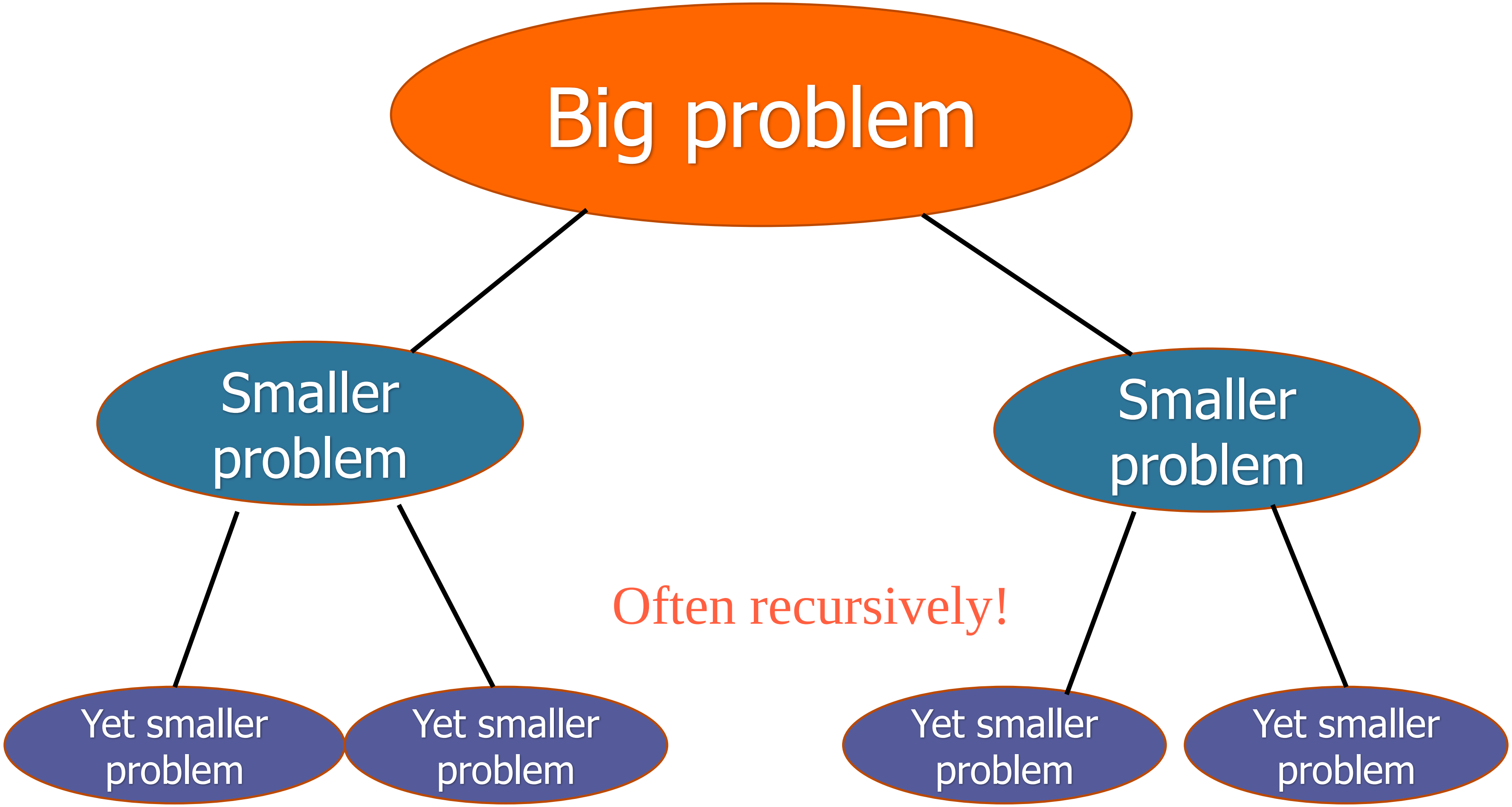
And I take 1 second to multiply two one-digit numbers and .6 seconds to add, so…

Multiplication of Integers



Divide and Conquer

Break problem up into smaller (easier) sub-problems



Algorithm Analysis

Break up an integer:

$$1234 = 12 \times 100 + 34$$

$$1234 \times 5678$$

$$= (12 \times 100 + 34)(56 \times 100 + 78)$$

$$= (\underbrace{12 \times 56}_{1})10000 + (\underbrace{34 \times 56}_{2} + \underbrace{12 \times 78}_{3})100 + (\underbrace{34 \times 78}_{4})$$

One 4-digit multiply

Four 2-digit multiplies

Algorithm Analysis

Break up an n-digit integer:

$$[x_1 x_2 \cdots x_n] = [x_1 x_2 \cdots x_{n/2}] \times 10^{n/2} + [x_{n/2+1} x_{n/2+2} \cdots x_n]$$

$$\begin{aligned} x \times y &= (a \times 10^{n/2} + b)(c \times 10^{n/2} + d) \\ &= \underbrace{(a \times c)}_{1} 10^{n} + (\underbrace{a \times d}_{2} + \underbrace{c \times b)}_{3} 10^{n/2} + \underbrace{(b \times d)}_{4} \end{aligned}$$

One n-digit multiply → Four (n/2)-digit multiplies

How to Analyze an Algorithm

Factors to consider

- Time
- Space
- Network
- Power Consumption

How to Analyze an Algorithm

```
Algorithm swap (a, b){
      temp=a;
      a=b;
      b=temp;
}

Algorithm expression (a, b){
      x=2*a + 4*b + a/b;
      }
```

How to Analyze an Algorithm

```
A[5]={2,4,6,8,10}

Algorithm sum (A,n){
        s=0;
        for (i=0; i<n; i++)
        {
        s=s+A[i];
        }
        return s;
}
```

How to Analyze an Algorithm

A[5]={2,4,6,8,10}

Space

$A \rightarrow n$

$n \rightarrow 1$

$s \rightarrow 1$

$i \rightarrow 1$

```
Algorithm sum (A,n){
        s=0;                    --------> 1
        for (i=0; i<n; i++)     --------> n+1
        {
        s=s+A[i];               --------> n
        }
        return s;               --------> 1
}
```

Time

S(n)=n+3

f(n)=2n+3

How to Analyze an Algorithm

A[]={} & B[]={}

Space

A → n^2
B → n^2
C → n^2
n → 1
i → 1
j → 1

$S(n)=3n^2+3$

```
Algorithm sumofTwoMatrices (A,B,n)
{
    for (i=0; i<n; i++)
    {
        for (j=0; j<n; j++)
        {
        c[i,j]=A[i,j]+B[i,j];
        }
    }
}
```

Time

for (i=0; i<n; i++) → $\boxed{n+1}$

for (j=0; j<n; j++) → $\boxed{n*(n+1)}$

c[i,j]=A[i,j]+B[i,j]; → $\boxed{n*n}$

$f(n)=2n^2+2n+1$

How to Analyze an Algorithm

```
Algorithm matrixMultiplication (A,B,C,n)
{
      for (i=0; i<n; i++)
      {
            for (j=0; j<n; j++)
            {
                   c[i,j]=0;
                  for (k=0; k<n; k++)
                  {
                        c[i,j]=c[i,j]+A[i,k]*B[k,j];
                  }
            }
      }
}
```

How to Analyze an Algorithm

- Time Analysis

```
Algorithm Multiply(A,B,n)
{
    for(i=0; i<n; i++)                          --- n+1
    {
        for(j=0; j<n; j++)                      --- (n+1) n
        {
            c[i,j]=0;                           --- n×n
            for(k=0; k<n; k++)                  --- (n+1)×n×n
            {
                c[i,j]=c[i,j]+A[i,k]*B[k,j];    --- n×n×n
            }
        }
    }
}
```

$f(n)=2n^3+3n^2+2n+1$

How to Analyze an Algorithm

- Space Analysis

```
Algorithm Multiply(A,B,n)
{
    for(i=0; i<n; i++)
    {
        for(j=0; j<n; j++)
        {
            C[i,j]=0;
            for(k=0; k<n; k++)
            {
                C[i,j]=C[i,j]+A[i,k]*B[k,j];
            }
        }
    }
}
```

Space

A — n^2
B — n^2
C — n^2
n — 1
i — 1
j — 1
k — 1

$S(n)=3n^2+4$

$O(n^2)$

Theoretical Analysis

- Uses a high-level description of the algorithm instead of an implementation
- Characterizes running time as a function of the input size, *n*.
- Takes into account all possible inputs
- Allows us to evaluate the speed of an algorithm independent of the hardware/software environment

Counting Primitive Operations

- By inspecting the pseudo code, we can determine the maximum number of primitive/basic operations executed by an algorithm, as a function of the input size

Algorithm ***arrayMax***(***A***, ***n***)	# operations
currentMax ← *A*[0]	2
for (*i* =1; i<n; i++)	$2n$
(i=1 once, i<n n times, i++ (n-1) times: post increment)	
if *A*[*i*] > *currentMax* **then**	$2(n-1)$
currentMax ← *A*[*i*]	$2(n-1)$
return *currentMax*	1
Total	$6n-1$

Estimating Running Time

- Algorithm ***arrayMax*** executes $6\boldsymbol{n} - 1$ primitive operations in the worst case.

  Define:

  $\boldsymbol{a}$ = Time taken by the fastest primitive operation

  $\boldsymbol{b}$ = Time taken by the slowest primitive operation

- Let $\boldsymbol{T}(\boldsymbol{n})$ be worst-case time of ***arrayMax***. Then

$$\boldsymbol{a}\,(6\boldsymbol{n} - 1) \leq \boldsymbol{T}(\boldsymbol{n}) \leq \boldsymbol{b}(6\boldsymbol{n} - 1\,)$$

- Hence, the running time $\boldsymbol{T}(\boldsymbol{n})$ is bounded by two linear functions

Growth Rate of Running Time

- Changing the hardware/ software environment
  - Affects $T(n)$ by a constant factor, but
  - Does not alter the growth rate of $T(n)$
- The linear growth rate of the running time $T(n)$ is an intrinsic/basic property of algorithm ***arrayMax***

Function of Growth rate

Function	Name
c	Constant
$\log N$	Logarithmic
$\log^2 N$	Log-squared
N	Linear
$N \log N$	N log N
N^2	Quadratic
N^3	Cubic
2^N	Exponential

Functions in order of increasing growth rate

Big-Oh Notation

- To simplify the running time estimation, for a function *f(n),* we ignore the constants and lower order terms.

Example: $10n^3+4n^2-4n+5$ is $O(n^3)$.