COMP90038 Algorithms and Complexity

Time/Space Tradeoffs—String Search Revisited

Lars Kulik

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Spending Space to Save Time

Often we can find ways of decreasing the time required to solve a problem, by using additional memory in a clever way.

For example, in Lecture 6 we considered the simple recursive way of finding the nth Fibonacci number and discovered that the algorithm uses exponential time.

However, suppose the same algorithm uses a table to tabulate the function $\textsc{Fib}$ as we go: As soon as an intermediate result $\textsc{Fib}(i)$ has been found, it is not simply returned to the caller; the value is first placed in slot i of a table (an array). Each call to $\textsc{Fib}$ first looks in this table to see if the required value is there, and only if it is not, the usual recursive process kicks in.

Fibonacci Numbers with Tabulation

We assume that, from the outset, all entries of the table F are 0.

function $\textsc{Fib}(n)$
  if $n = 0$ **or** $n = 1$ **then**
    return 1
  $result \leftarrow F[n]$
  if $result = 0$ **then**
    $result \leftarrow \textsc{Fib}(n-1) + \textsc{Fib}(n-2)$
    $F[n] \leftarrow result$
  return $result$

(I show this code just so that you can see the principle; in Lecture 6 we already discovered a different linear-time algorithm, so here we don't really need tabulation.)

Sorting by Counting

Suppose we need to sort large arrays, but we know that they will hold keys taken from a small, fixed set (so lots of duplicate keys).

For example, suppose all keys are single digits:

6 3 3 8 1 0 8 7 9 2 5 3 5 3 1 8 7 6 5 1 2 1 5 3

Then we can, in a single linear scan, count the occurrences of each key in array A and store the result in a small table:

key	0	1	2	3	4	5	6	7	8	9
Occ	1	4	2	5	0	4	2	2	3	1

Now use a second linear scan to make the counts cumulative:

key	0	1	2	3	4	5	6	7	8	9
Occ	1	5	7	12	12	16	18	20	23	24

Sorting by Counting

We can now create a sorted array $S[1..n]$ of the items by simply slotting items into pre-determined slots in S (a third linear scan).

6 3 3 8 1 0 8 7 9 2 5 3 5 3 1 8 7 6 5 1 2 1 5 3

key	0	1	2	3	4	5	6	7	8	9
Occ	1	5	7	12	12	16	18	20	23	24

Place the first record (with key 6) in $S[18]$ and decrement $Occ[6]$ (so that the next '6' will go into slot 17), and so on.

for $i \leftarrow 1$ **to** n **do**
 $S[Occ[A[i]]] \leftarrow A[i]$
 $Occ[A[i]] \leftarrow Occ[A[i]] - 1$

Sorting by Counting

Note that this gives us a linear-time sorting algorithm (for the cost of some extra space).

However, it only works in situations where we have a small range of keys, known in advance.

The method never performs a key-to-key comparison.

The time complexity of key-comparison based sorting has been proven to be $\Omega(n \log n)$.

String Matching Revisited

We earlier discussed the brute-force approach to string search.

"Strings" are usually built from a small, pre-determined alphabet.

Most of the better algorithms rely on some pre-processing of strings before the actual matching process starts.

The pre-processing involves the construction of a small table (of predictable size).

Levitin refers to this as "input enhancement".

Horspool's String Search Algorithm

Comparing from right to left in the pattern.

Very good for random text strings.

S	T	R	I	N	G	S	E	A	R	C	H	E	X	A	M	P
E	X	A	M													

We can do better than just observing a mismatch here.

Because the pattern has no occurrence of I, we might as well slide it 4 positions along.

This decision is based only on knowing the pattern.

Horspool's String Search Algorithm

```
S T R I N G S E A R C H E X A M P
E X A M
        E X A M
```

Here we can slide the pattern 3 positions, because the last occurrence of E in the pattern is its first position.

```
S T R I N G S E A R C H E X A M P
E X A M
        E X A M
              E X A M
                          E X A M
                            E X A M
```

Horspool's String Search Algorithm

What happens when we have longer partial matches?

```
S  E  A  R  C  H  I  N  G|
B  I  R  C  H
   B  I  R  C  H
            B  I  R|C  H
```

The shift is determined by the last character in the pattern.

Char	Shift
A	5
B	4
C	1
⋮	⋮
H	5
I	3
⋮	⋮
R	2
S	5
⋮	⋮
Z	5

Horspool's String Search Algorithm

Building (calculating) the shift table is easy.

Let a be the size of the alphabet.

function $\text{FINDSHIFTS}(P[0..m-1])$
    for $i \leftarrow 0$ to a **do**
        $Shift[i] \leftarrow m$
    for $j \leftarrow 0$ to $m-2$ **do**
        $Shift[P[j]] \leftarrow m-(j+1)$

Horspool's String Search Algorithm

function $\textsc{Horspool}(P[0..m-1], T[0..n-1])$
    $\textsc{FindShifts}(P)$
    $i \leftarrow m - 1$
    while $i < n$ **do**
        $k \leftarrow 0$
        while $k < m$ **and** $P[m-1-k] = T[i-k]$ **do**
            $k \leftarrow k + 1$
        if $k = m$ **then** ▷ We have a match
            return $i - m + 1$ ▷ Start of the match
        else
            $i \leftarrow i + Shift[T[i]]$ ▷ Slide the pattern along
    return -1

Horspool's String Search Algorithm

We can also consider posting a sentinel: Append the pattern P to the end of the text T so that a match is guaranteed.

function $\textsc{Horspool}(P[0..m-1], T[0..n-1])$
    $\textsc{FindShifts}(P)$
    $i \leftarrow m - 1$
    while True **do**
        $k \leftarrow 0$
        while $k < m$ **and** $P[m-1-k] = T[i-k]$ **do**
            $k \leftarrow k + 1$
        if $k = m$ **then**
            if $i > n$ **then**
                return -1
            else
                return $i - m + 1$
        $i \leftarrow i + Shift[T[i]]$

Horspool's String Search Algorithm

Unfortunately the worst-case behaviour of Horspool's algorithm is still $O(m \times n)$, like the brute-force method.

However, in practice, for example, when used on English texts, it is linear, and fast.

Other Important String Search Algorithms

Horspool's algorithm was inspired by the famous Boyer-Moore algorithm (BM), also covered in Levitin's book. The BM algorithm is very similar, but it has a more sophisticated shifting strategy, which makes it $O(m + n)$.

Another famous string search algorithm is the Knuth-Morris-Pratt algorithm (KMP), explained in the remainder of these slides. KMP is very good when the alphabet is small, say, we need to search through very long bit strings.

Also, we shall soon meet the Rabin-Karp algorithm (RK), albeit briefly.

While very interesting, the BM, KMP, and RK algorithms are not examinable.

Knuth-Morris-Pratt (Not Examinable)

Suppose we are searching in strings that are built from a small alphabet, such as the binary digits 0 and 1, or the nucleobases.

Consider the brute-force approach for this example:

Text:	1	0	0	0	1	0	0	0	0
Pattern:	1	0	0	0	0				

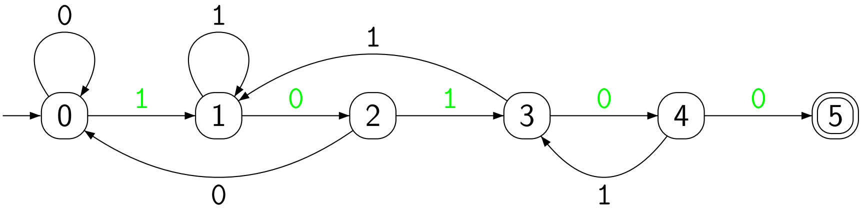
Every "false start" contains a lot of information.

Again, we hope to pre-process the pattern so as to find out when the brute-force method's index i can be incremented by more than 1.

Unlike Horspool's method, KMP works by comparing from left to right in the pattern.

Knuth-Morris-Pratt as Running an FSA

Given the pattern `1 0 1 0 0` we want to construct the following finite-state automaton:

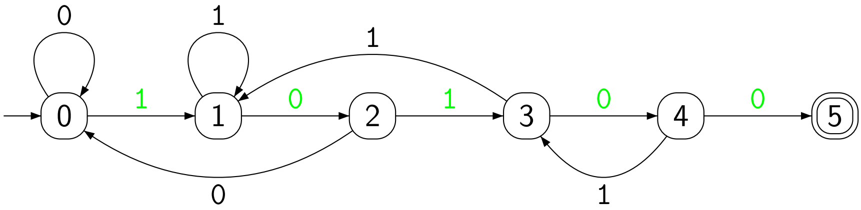


We can capture the behaviour of this automaton in a table.

Knuth-Morris-Pratt Automaton

We can represent the finite-state automaton as a 2-dimensional "transition" array T, where $T[c][j]$ is the state to go to upon reading the character c in state j.

j	$T[\texttt{'0'}][j]$	$T[\texttt{'1'}][j]$
0	0	1
1	2	1
2	0	3
3	4	1
4	5	3



Constructing the Automaton

The automaton (or the table T) can be constructed step-by-step:

Somewhat tricky but fast.

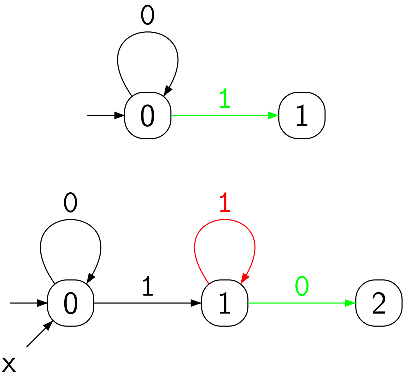
x is a "backtrack point".

For next state j:

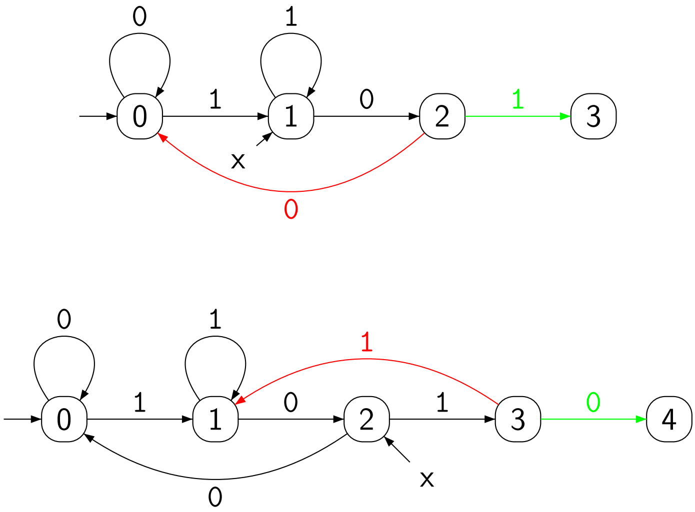
First x's transitions are copied (in red).

Then the success arc is updated, determined by $P[j]$ (in green).

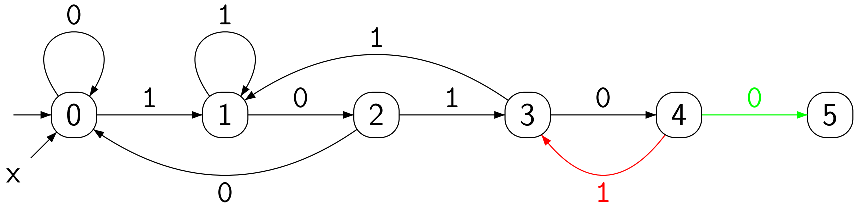
Finally x is updated based on $P[j]$.



Constructing the Automaton



Constructing the Automaton



Constructing the Automaton

$T['0'][0] \leftarrow 0$
$T['1'][0] \leftarrow 0$
$T[P[0]][0] \leftarrow 1$
$x \leftarrow 0$
$j \leftarrow 1$
while $j < m$ **do**
    $T['0'][j] \leftarrow T['0'][x]$
    $T['1'][j] \leftarrow T['1'][x]$
    $T[P[j]][j] \leftarrow j + 1$
    $x \leftarrow T[P[j]][x]$
    $j \leftarrow j + 1$

Next Up

We look at the hugely important technique of hashing, a standard way of implementing a "dictionary".

Hashing is arguably the best example of how to gain speed by using additional space to great effect.