Department of Computing and Information Systems
COMP90038 Algorithms and Complexity Tutorial Week 10

Sample answers

The exercises

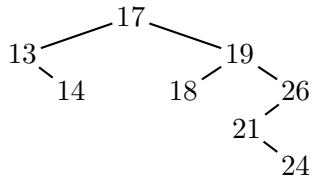
1. Give an algorithm for deciding whether an array $A[1..n]$ is a heap.

   Answer: Easy:

   ```
   function IsHeap(A[1..n])
       for i ← 1 to n/2 do
           if A[i] < A[2 × i] then
               return False
           if 2 × i < n and A[i] < A[2 × i + 1] then
               return False
       return True
   ```

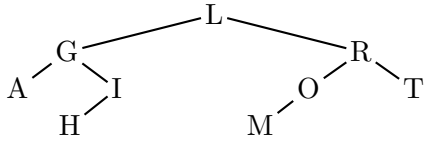
2. Construct a binary search tree (BST) by starting from an empty tree and inserting these keys, in the given order: 17, 19, 13, 26, 14, 18, 21, 24.

   Answer:

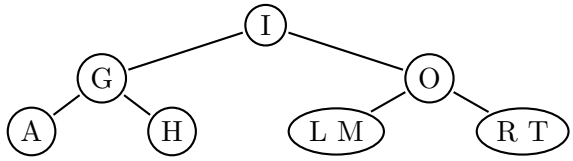
3. Construct an AVL tree from the empty tree by inserting the following keys in the given order: A, L, G, O, R, I, T, H, M.

   Answer:

4. Construct a 2–3 tree from the empty tree by inserting the following keys in the given order: A, L, G, O, R, I, T, H, M.

   Answer:

5. Consider the set of five keys (let us say they are positive integers) $\{k_1, k_2, k_3, k_4, k_5\}$, satisfying $k_1 < k_2 < k_3 < k_4 < k_5$. There are 120 different permutations of these five keys. For exactly two of the 120 permutations, the following happens, when the keys are inserted one by one, in the order given by the permutation, into an initially empty AVL tree: First an LR-rotation takes place, then an RL-rotation takes place. Which two permutations generate that behaviour?

   Answer: Without loss of generality, assume the set of keys is $\{1, 2, 3, 4, 5\}$. After the first three keys are inserted to make an AVL tree, that tree must be perfectly balanced. Hence the

first three keys inserted must cause the  zigzag path that requires an LR-rotation. After that, insertion of the two remaining keys must cause the  zigzag path that requires an RL-rotation. The only permutations that will achieve this are 3, 1, 2, 5, 4 and 5, 1, 4, 3, 2.