COMP90038 Algorithms and Complexity

Transform-and-Conquer

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Lecture 14

Semester 1, 2016

Transform and Conquer

- Instance simplification
- Representational change
- Problem reduction

Instance Simplification

General principle: Try to make the problem easier through some sort of pre-processing, typically sorting.

We can pre-sort input to speed up, for example

- finding the median
- uniqueness checking
- finding the mode

Uniqueness Checking, Brute-Force

The problem:
Given an unsorted array $A[0..n-1]$, is $A[i] \neq A[j]$ whenever $i \neq j$?

The obvious approach is brute-force:

```
for i ← 0 to n − 2 do
    for j ← i + 1 to n − 1 do
        if A[i] = A[j] then
            return False
return True
```

What is the complexity of this?

Uniqueness Checking, with Presorting

Sorting makes the problem easier:

SORT($A[0..n-1]$)
for $i \leftarrow 0$ to $n-2$ **do**
  if $A[i] = A[i+1]$ **then**
    return False
return True

What is the complexity of this?

Exercise: Computing a Mode

A mode is a list or array element which occurs most frequently in the list/array. For example, in

$$42, 78, 13, 13, 57, 42, 57, 78, 13, 98, 42, 33$$

the elements 13 and 42 are modes.

The problem:
Given array A, find a mode.

Discuss a brute-force approach vs a pre-sorting approach.

Mode Finding, with Presorting

$\textsc{Sort}(A[0..n-1])$
$i \leftarrow 0$
$maxfreq \leftarrow 0$
while $i < n$ **do**
    $runlength \leftarrow 1$
    while $i + runlength < n$ **and** $A[i + runlength] = A[i]$ **do**
        $runlength \leftarrow runlength + 1$
    if $runlength > maxfreq$ **then**
        $maxfreq \leftarrow runlength$
        $mode \leftarrow A[i]$
    $i \leftarrow i + runlength$
return $mode$

Again, after sorting, the rest takes linear time.

Searching, with Presorting

The problem:
Given unsorted array A, find item x (or determine that it is absent).

Compare these two approaches:

- Perform a sequential search
- Sort, then perform binary search

What are the complexities of these approaches?

Searching, with Presorting

What if we need to search for m items?

Let us do a back-of-the envelope calculation (consider worst-cases for simplicity):

Take $n = 1024$ and $m = 32$.

Sequential search: $m \times n = 32,768$.

Sorting + binsearch: $n \log_2 n + m \times \log_2 n = 10,240 + 320 = 10,560$.

Average-case analysis will look somewhat better for sequential search, but pre-sorting will still win.

Exercise: Finding Anagrams

An anagram of a word w is a word which uses the same letters as w but in a different order.

Example: ‘ate’, ‘tea’ and ‘eat’ are anagrams.

Example: ‘post’, ‘spot’, ‘pots’ and ‘tops’ are anagrams.

Example: ‘garner’ and ‘ranger’ are anagrams.

You are given a very long list of words in lexicographic order.

Devise an algorithm to find all anagrams in the list.

Binary Search Trees

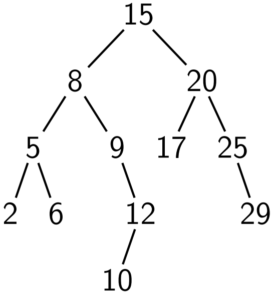
A binary search tree, or BST, is a binary tree that stores elements in all internal nodes, with each sub-tree satisfying the BST property:

Let the root be r; then each element in the left subtree is smaller than r and each element in the right sub-tree is larger than r. (For simplicity we will assume that all keys are different.)

Binary Search Trees

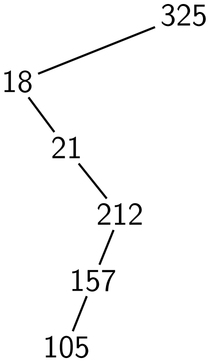
BSTs are useful for search applications. To search for k in a BST, compare against its root r. If $r = k$, we are done; otherwise search in the left or right sub-tree, according as $k < r$ or $k > r$.

If a BST with n elements is "reasonably" balanced, search involves, in the worst case, $\Theta(\log n)$ comparisons.



Binary Search Trees

If the BST is not well balanced, search performance degrades, and may be as bad as linear search:

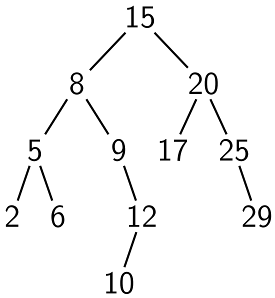


Insertion in Binary Search Trees

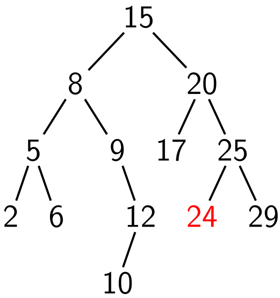
To insert a new element k into a BST, we pretend to search for k.

When the search has taken us to the fringe of the BST (we find an empty sub-tree), we insert k where we would expect to find it.

Inserting 24:



$\Rightarrow$



BST Traversal Quiz

Performing .... traversal of a BST will produce its elements in sorted order.

Next Up: Balancing Binary Search Trees

To optimise the performance of BST search, it is important to keep trees (reasonably) balanced.

Next we shall look at AVL trees and 2–3 trees.