Department of Computing and Information Systems
COMP90038 Algorithms and Complexity Tutorial Week 2

The exercises

1. Consider the usual (unsigned) binary representation of integers. For example, `10110010` represents 178, and `000011` represents 3.

   (a) If we call the bits in an n-bit word $x_{n-1}, x_{n-2}, \ldots, x_2, x_1, x_0$ (so x_0 is the *least significant* bit), which natural number is denoted by $x_{n-1}x_{n-2}\cdots x_2x_1x_0$?

   (b) Describe, in English, an algorithm for converting from binary to decimal notation.

   (c) Write the algorithm in (pseudo-) code.

   (d) Describe, in English, how to convert the decimal representation to binary.

2. Which of the following can be considered an algorithm for computing the area of a triangle whose side lengths are given positive numbers a, b, and c?

   (a) $S = \sqrt{p(p-a)(p-b)(p-c)}$, where $p = (a+b+c)/2$

   (b) $S = \frac{1}{2}bc\sin A$, where A is the angle between sides b and c

   (c) $S = \frac{1}{2}ah_a$, where h_a is the height to base a

3. (a) Show the stack after each operation of the following sequence that starts with the empty stack:

   `push(a), push(b), pop, push(c), push(d), pop`

   (b) Show the queue after each operation of the following sequence that starts with the empty queue:

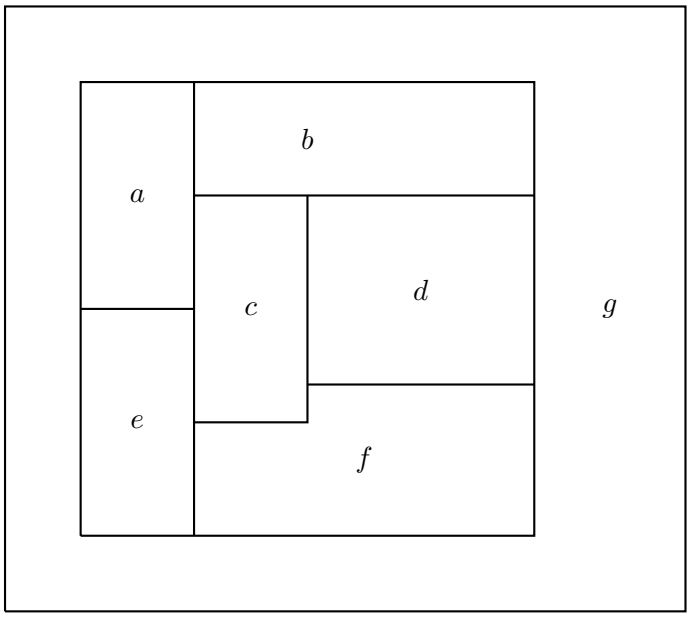
   `enqueue(a), enqueue(b), dequeue, enqueue(c), enqueue(d), dequeue`

4. Consider the following problem: You are to design an algorithm to determine the best route for a subway passenger to take from one station to another in a city such as Kolkata or Tokyo.

   (a) Discuss ways of making the problem statement less vague. In particular, what is "best" supposed to mean?

   (b) How would you model this problem by a graph?

5. Consider the following map:



   (a) A cartographer wants to colour the map so that no two neighbouring countries have the same colour. How few colours can she get away with?

   (b) Show how to reduce the problem to a graph-colouring problem.

6. You have to search for a given number n in a *sorted* list of numbers.

   (a) How can you take advantage of knowing that the list is represented as a linked (and sorted) list?

   (b) How can you take advantage of knowing the list is represented as an array?

7. Let A be the adjacency matrix of an undirected graph. How can you tell from the matrix whether the graph

   (a) is complete?

   (b) has a loop, that is, an edge connecting a node to itself?

   (c) has an isolated node?

8. Design an algorithm to check whether two given words are anagrams, that is, whether one can be obtained from the other by permuting its letters. For example, *garner* and *ranger* are anagrams.