COMP90038 Algorithms and Complexity

Graphs and Graph Concepts

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Lecture 7

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Graphs Again

One instance of the exhaustive search paradigm is graph traversal.

After this lecture we shall look at two ways of systematically visiting every node of a graph, namely depth-first and breadth-first search.

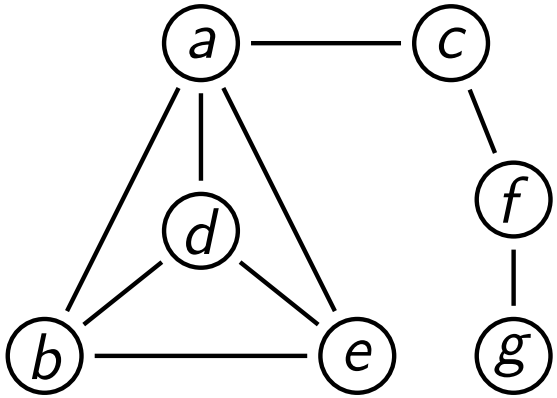
These two methods of graph traversal form the backbone of a surprisingly large number of useful graph algorithms.

The graph algorithms are useful because of the large number of practical problems we can model as graph problems, in network design, flow design, planning, scheduling, route finding, and other logistics applications.

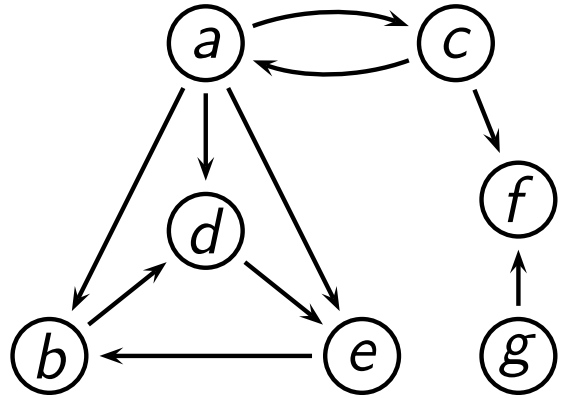
Moreover, important numeric and logic problems can be reduced to graph problems—more on this in Week 12.

Graph Concepts

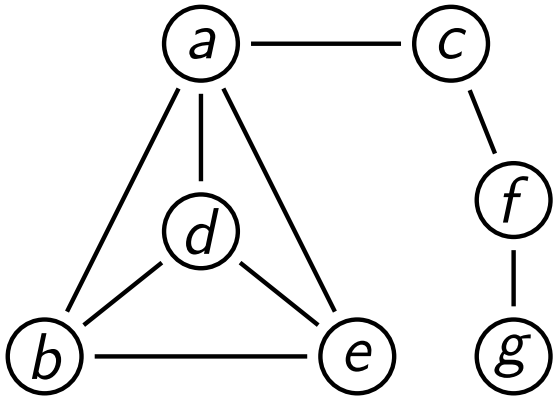
Undirected:



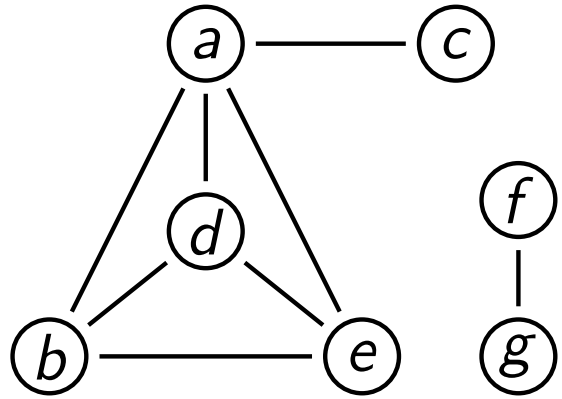
Directed:



Connected:



Not connected, two components:

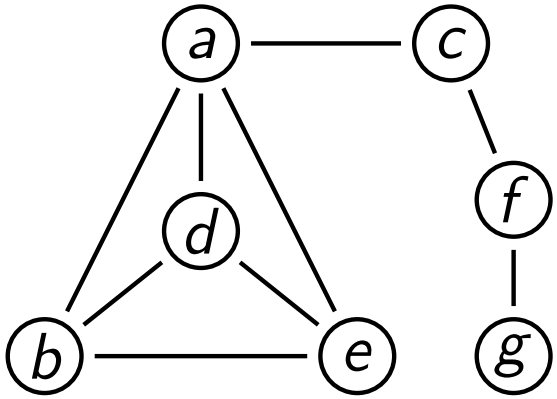


Graphs, Mathematically

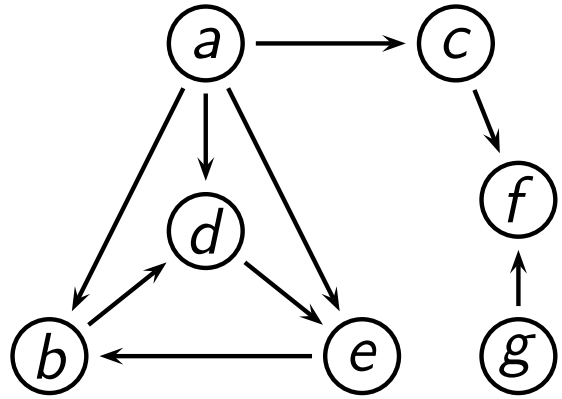
$G = \langle V, E \rangle$

V: Set of nodes or vertices

E: Set of edges (a binary relation on V)



$$
\begin{aligned}
V &= \{a, b, c, d, e, f, g\} \\
E &= \text{symmetric closure of} \\
&\quad \{(a, b), (a, c), (a, d), \\
&\quad\ (a, e), (b, d), (b, e), \\
&\quad\ (c, f), (d, e), (f, g)\}
\end{aligned}
$$



$$
\begin{aligned}
V &= \{a, b, c, d, e, f, g\} \\
E &= \{(a, b), (a, c), (a, d), \\
&\quad\ (a, e), (b, d), (c, f), \\
&\quad\ (d, e), (e, b), (g, f)\}
\end{aligned}
$$

More Graph Concepts: Degrees of Nodes

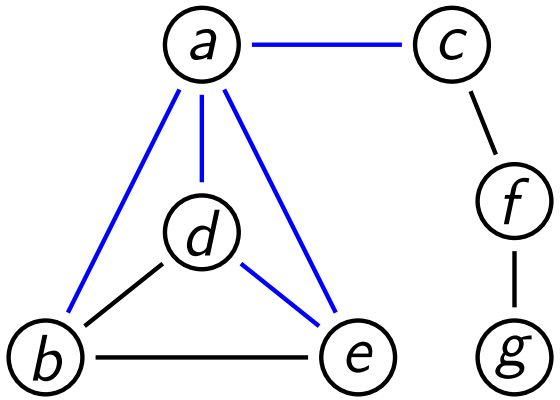
If $(v, u) \in E$ then v and u are adjacent, or neighbours.

(v, u) is incident on, or connects, v and u.

The degree of node v is the number of edges incident on v.

For directed graphs, we talk about v's in-degree (number of edges going to v) and its out-degree (number of edges going from v).

More Graph Concepts: Paths and Cycles



Path b, a, d, e, a, c shown in blue

A path in $\langle V, E \rangle$ is a sequence of nodes $v_0, v_1, \ldots, v_k$ from V, so that $(v_i, v_{i+1}) \in E$.

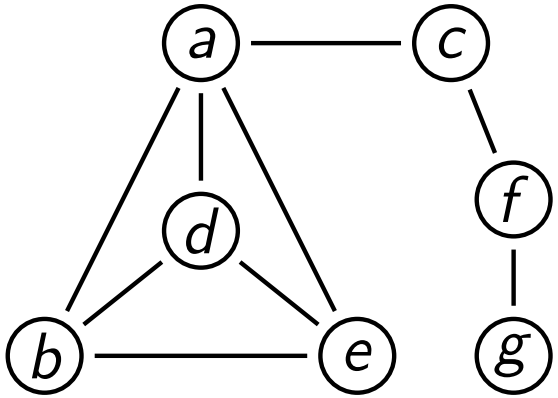
The path $v_0, v_1, \ldots, v_k$ has length k.

A simple path is one that has no repeated nodes.

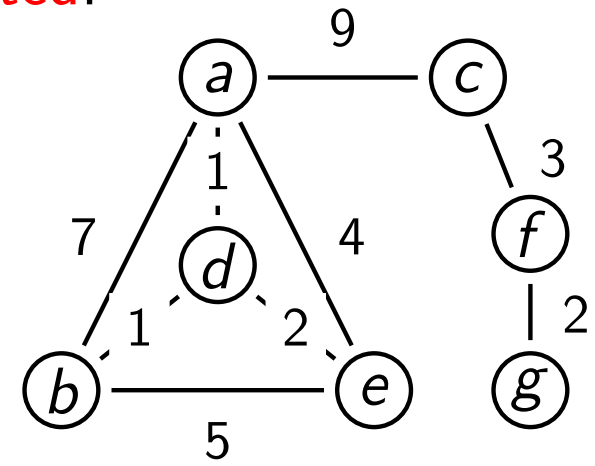
A cycle is a simple path, except that $v_0 = v_k$, that is, the last node is the same as the first node.

More Graph Concepts

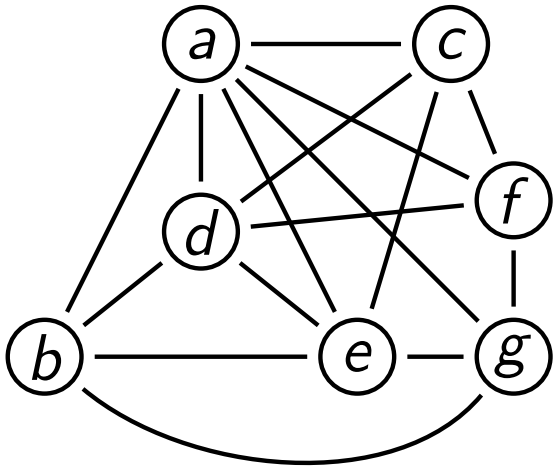
Unweighted:



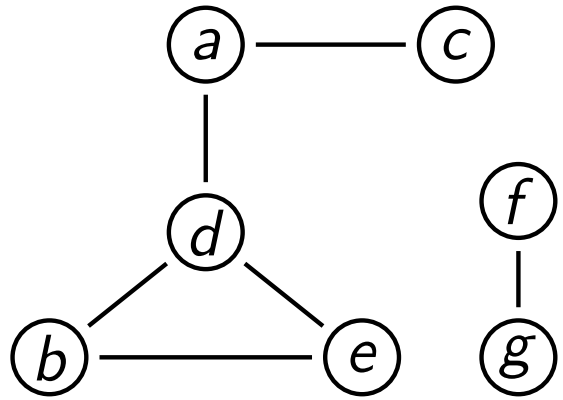
Weighted:



Dense:

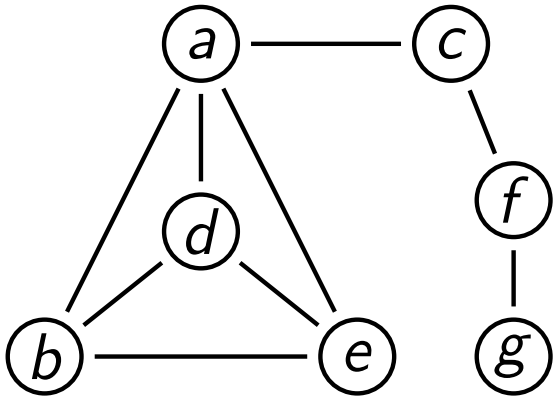


Sparse:

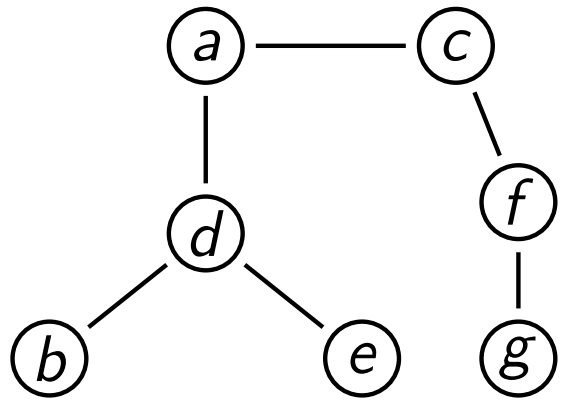


More Graph Concepts

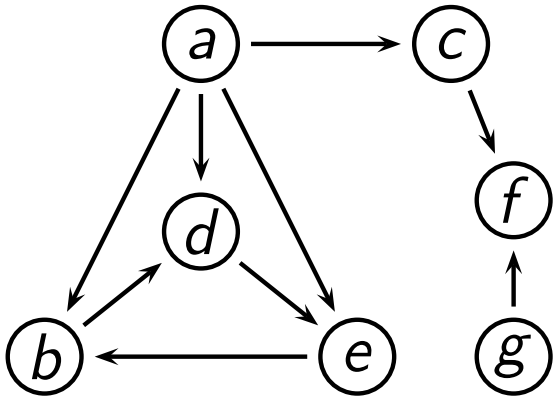
Cyclic:



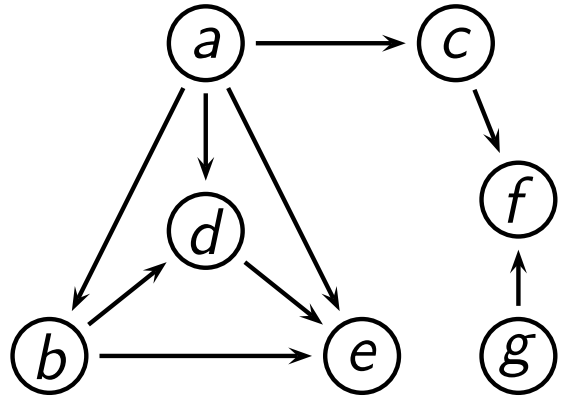
Acyclic (actually, a tree):



Directed cyclic:



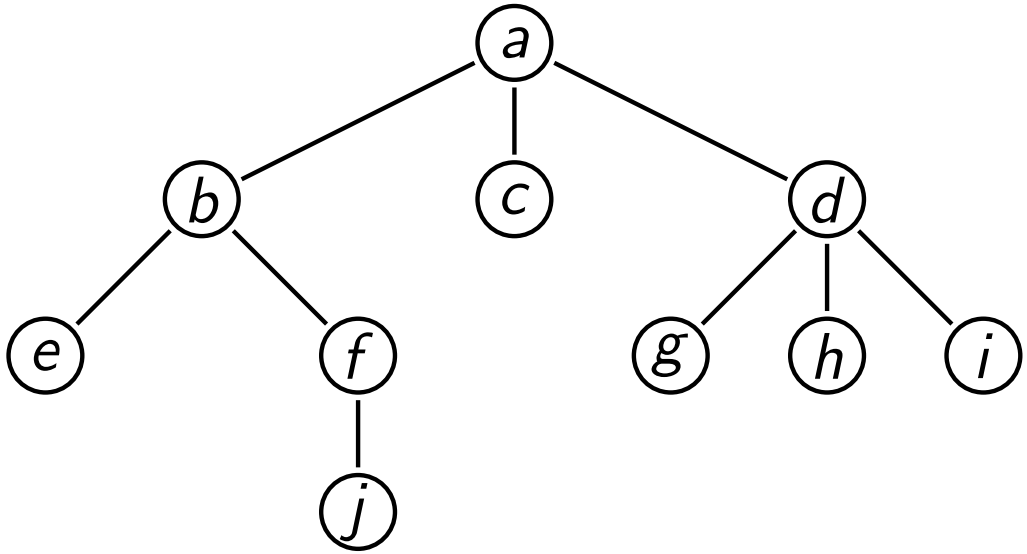
Directed acyclic (a dag):



Rooted Trees

A (free) tree is a connected acyclic graph.

A rooted tree is a tree with one node identified as special. Every other node is reachable from the node.



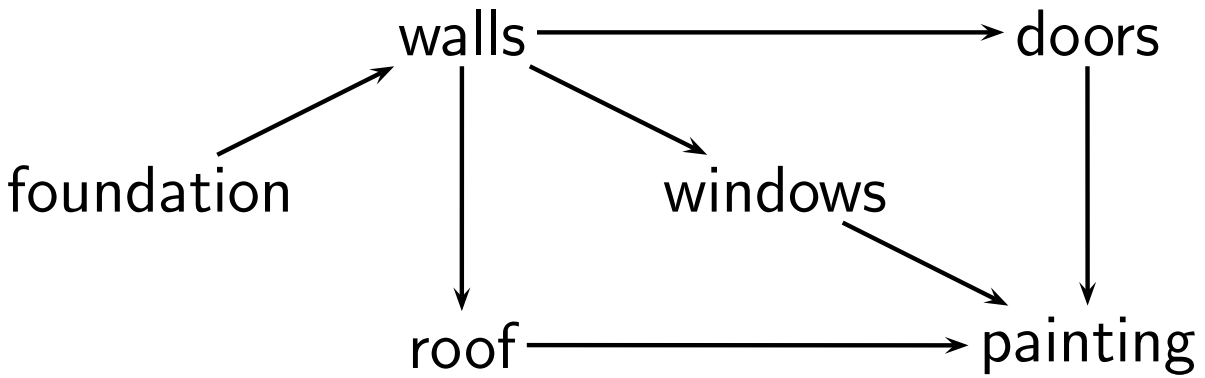
When the root is removed, a set of rooted (sub-)trees remain.

We should draw the rooted tree as a directed graph, but usually we instead rely on the layout: "parents" sit higher than "children".

Modelling with Graphs

Graph algorithms are of great importance because so many different problem types can be abstracted to graph problems.

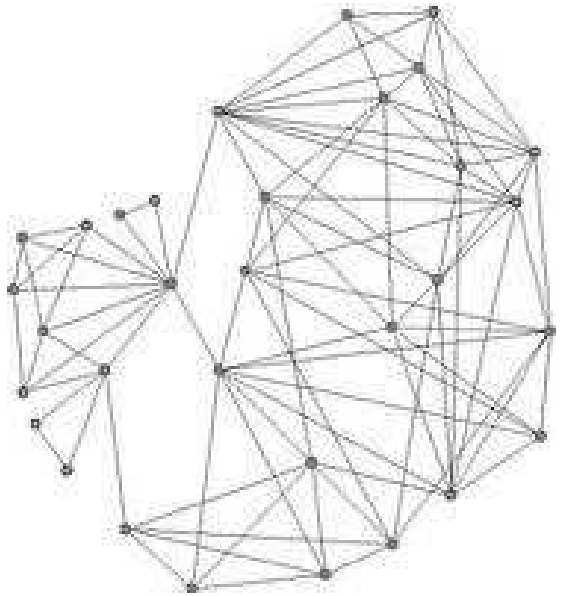
For example, directed graphs (they'd better be dags) are central in scheduling problems:



Modelling with Graphs

Graphs find use in all sorts of modelling.

For a simple example, assume you want to invite lots of friends to dinner, and you have k tables available.

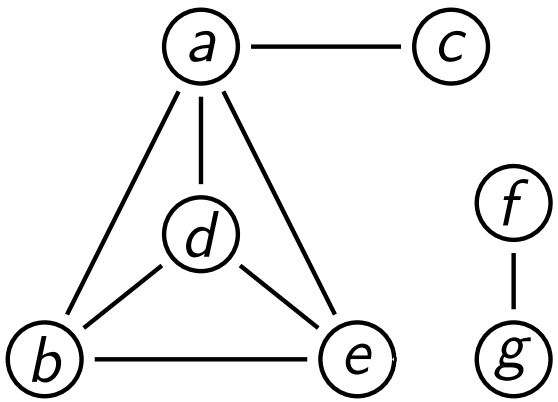


But some of your guests dislike some of the other guests; you want to make a seating plan that avoids placing foes at the same table.

The natural model is an undirected graph, with a node for each guest, and an edge between any two guests that don't get along.

This reduces your problem to the "graph k-colouring problem": Find, if possible, a colouring of nodes so that no to connected nodes get the same colur.

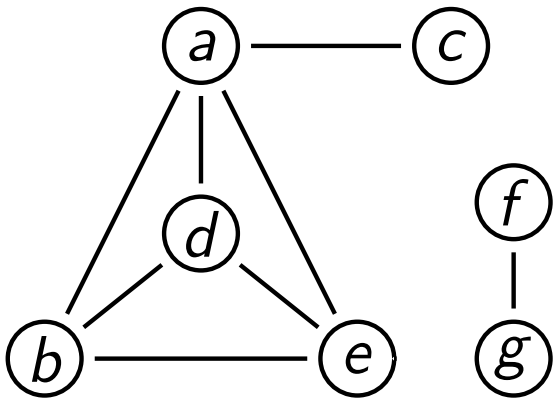
Graph Representations, Undirected Graphs



	a	b	c	d	e	f	g
a	0	1	1	1	1	0	0
b	1	0	0	1	1	0	0
c	1	0	0	0	0	0	0
d	1	1	0	0	1	0	0
e	1	1	0	1	0	0	0
f	0	0	0	0	0	0	1
g	0	0	0	0	0	1	0

The adjacency matrix for the graph.

Graph Representations, Undirected Graphs

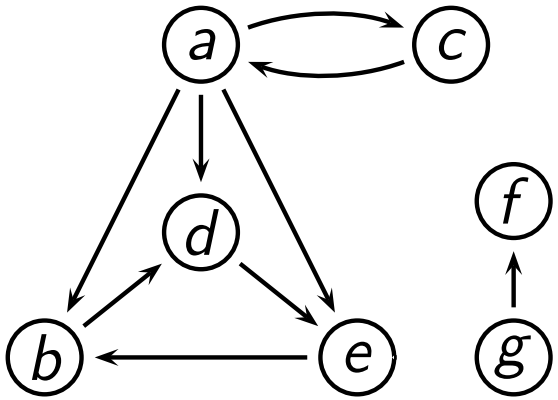


a	$\to b \to c \to d \to e$
b	$\to a \to d \to e$
c	$\to a$
d	$\to a \to b \to e$
e	$\to a \to b \to d$
f	$\to g$
g	$\to f$

The adjacency list representation.

(Assuming lists are kept in sorted order.)

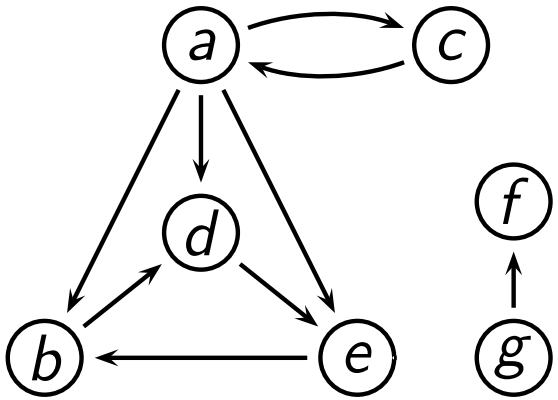
Graph Representations, Directed Graphs



	a	b	c	d	e	f	g
a	0	1	1	1	1	0	0
b	0	0	0	1	0	0	0
c	1	0	0	0	0	0	0
d	0	0	0	0	1	0	0
e	0	1	0	0	0	0	0
f	0	0	0	0	0	0	0
g	0	0	0	0	0	1	0

The adjacency matrix for the graph.

Graph Representations, Directed Graphs



a	$\rightarrow b \rightarrow c \rightarrow d \rightarrow e$
b	$\rightarrow d$
c	$\rightarrow a$
d	$\rightarrow e$
e	$\rightarrow b$
f	
g	$\rightarrow f$

The adjacency list representation.

Graph Representations

Each representation has advantages and disadvantages.

Think of some!

Up Next

Graph traversal, in which we get down to the nitty-gritty details of graph algorithms.