Microwave Applications of Photonic Topological Insulators

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This Perspective examines the emerging applications of photonic topological insulators (PTIs) in the microwave domain. The introduction of topological protection of light has revolutionized the traditional perspective of wave propagation through the demonstration of backscatter-free waveguides in the presence of sharp bending and strong structural defects. The pseudospin degree of freedom of light enables the invention of novel topological photonic devices with useful functionalities. Here we aim to summarize the recent development and potential application directions of the PTI demonstrations mainly in the microwave regime, and explain the key features giving rise to topological protection, along with a comparison of different designs and realizations.

Topological photonics, an emerging 21^{st} century technology, has enjoyed a boost of research interest from both the physics and engineering communities[1–3]. Similar to its electronic counterpart[4], photonic topological insulators (PTIs) possess a full photonic bandgap which restricts light propagation in the bulk region of the structure. Topologically protected edgemodes are allowed to propagate at the boundary of the structure, creating an alternative type of waveguide structure. The propagation of the edgemode is free from back-scattering arising from disorder as long as it does not destroy the symmetry giving rise to the topological properties. Inside the waveguide, the robustness of edgemode propagation is demonstrated by having waves passing around sharp corners and across disordered regions. In addition, the edgemodes have new degrees of freedom that can be used to "sort" the modes, and this can be used to route them differently. Topological photonics has led to the invention of new photonic devices with novel functionalities, as well as upgrades to the performance of existing photonic devices. The plan of this article is to briefly review the physics underlying the current PTI realizations, and then to discuss the novel emerging and future microwave applications of these devices.

Topological insulators (TIs) were discovered in the context of electronic properties of many-electron condensed matter systems. However it was latter realized that the basic concepts underlying TI phenomena are quite general, do not depend in an essential way on collective electronic or quantum phenomena, and therefore apply to various classical wave phenomena[5]. A similar mapping of concepts was realized some time ago in the field of quantum chaos (or wave chaos) where ideas from nuclear physics and mesoscopic condensed matter physics crossed over to classical wave phenomena, and gave rise to many clear demonstrations of fundamental wave chaotic phenomena such as universal statistical fluctuations in microwave analogs of the Schrodinger equation[6–8]. For TI analogs one can exploit the same symmetries as those found in electronic systems to create photonic topological edge states. One can also create analogs of the spin-1/2 degree of freedom of the electron, or introduce artificial gauge fields that enable topologically-protected waveguide modes. However, the photonic analogs are Bosonic in nature and lack properties arising from the Pauli exclusion principle and Fermi-Dirac statistics. As will be discussed below, special structural designs are explored in photonics to realize topological protection.

We start by introducing selected realizations of photonic topological insulators that essentially involve a two-dimensional lattice that give rise to one-dimensional edge states. The quantum Hall (QH) effect is the pioneer example of topological systems in condensed matter physics. Under low temperature and large magnetic field, a 2D electronic system with insulating bulk can acquire a non-zero topological invariant. At the boundary formed by the interface of two insulators with different topological indices[19], the change of the topological invariant leads to the closing of the bandgap[4], while the two bulk regions remain gapped. Thus edge states are formed and are localized at the interfaces. These states typically disperse in momentum, giving rise to a finite propagation velocity. The total number of edge modes depend on the difference between the topological indices of both media.

We present in Table. I a brief summary of the core features of the major two-dimensional PTI genres. One of the first PTI realizations utilized the translation of the quantum Hall (QH) effect from the electronic to the photonic domain. The microwave photonic crystal lattice consists of gyro-magnetic ferrite rods biased by static magnetic fields[9,20]. The structure breaks time-reversal (T) symmetry, and in analogy with the QH effect, a chiral edgemode is observed to propagate uni-directionally at the boundary of the insulating bulk (shown in Fig. 1 a, b). The circulation direction of the edgemode is dictated by the magnetic bias field direction and the edgemode is demonstrated to flow around defects, such as a metallic impediment, with no back-scattering. However, gyromagnetic materials are lossy and providing a DC magnetic bias is somewhat inconvenient. It is of keen interest to search for alternative PTI realizations that do not require external magnetic field.

Another condensed matter system that shows topological edgemodes in the absence of an external magnetic

PTI Category	Example Realizations	Mechanism to Create Topological Light	Binary Degrees of Freedom
Quantum Hall (QH)	Gyro-magnetic photonic crystal with biasing magnetic field[9,10]	T-symmetry breaking by magneto-optical effect[9]	Circulation direction controlled by biased H-field direction
Quantum valley Hall (QVH)	BMW lattice[11,12]	Lattice symmetry reduction by breaking in-plane inversion symmetry	Valley degrees of freedom constituted by CP states
Quantum spin Hall (QSH)	Ring resonator lattice[13,14]	Different coupling lengths between the rings	Degenerate whispering gallery-like modes
	BMW lattice[10,15]	Bi-anisotropy effect	Combinations of degenerate TE and TM modes
	Hexagonal lattice[16,17]	Lattice symmetry reduction by distorting the unit cell	Combination of TM modes with different E_z profiles
Floquet-PTI	Single mode waveguide array modulated in z-direction[18]	Breaking z-reversal symmetry with helical waveguides modulated in the z-direction of the x-y 2D lattice.	

TABLE I. Summary of the general concepts of a selected subset of photonic topological insulator (PTI) experimental realizations in 2D and quasi-2D lattices. Binary degree of freedom (spin/valley) is observed in the quantum spin/valley-Hall PTIs. The abbreviation BMW stands for bi-anisotropic meta-waveguide.

field is the quantum spin-Hall (QSH) effect. Instead of breaking the T-symmetry as in the QH effect, QSH requires the presence of T-symmetry such that Kramers degeneracy is preserved. Electrons with opposite spins form helical edge states with opposite signs of momentum. Naturally, finding the photonic analog of the QSH topological phase provides an alternative pathway to realize PTIs which are not restricted to gyro-magnetic materials and a biasing magnetic field. The QSH requires an analog of the spin-1/2 degree of freedom and a spin-orbit (or spin-momentum locking) phenomenon to be present. The QSH-PTIs are achieved by creating an artificial spin degree of freedom (DOF) for photonic modes[2]. The early proposals of T-invariant PTIs were realized with a lattice of coupled ring resonators[13,14]. This design translates the two spin DOFs of an electron into the two circulation directions of the light in the ring (the clockwise or counter-clockwise modes). In Ref.[14], two circulating modes experience different optical paths when travelling from one site resonator to another. An effective uniform magnetic field is realized if one arranges the rings into a 2D lattice, where edge modes are excited by a laser field along the structural boundary.

A number of ways have been proposed to replicate the spin-1/2-like degree of freedom in the photonic context. Here we concentrate on a selected few of these realizations. QSH-PTIs can be realized with the so-called bianisotropic meta-waveguide (BMW) systems[15,21,22] as shown in Fig. 2. One starts with a perfect 2D hexagonal metal-post photonic crystal sandwiched between two conducting plates with carefully engineered degeneracy of the TE and TM modes at the Dirac points (K and K' points) in the 2D photonic band structure. The spin-1/2 degree of freedom is created by two orthogonal linear combinations of the TE and TM modes at the Dirac points. By breaking the inversion symmetry in the perpendicular (z) direction of the lattice, a bianisotropic coupling emerges between the transverse electric (TE) and transverse magnetic (TM) modes, which corresponds to an extra magneto-electric mixing term between electric and magnetic fields ($\mathbf{D} = \hat{\epsilon}\mathbf{E} + \hat{\chi}\mathbf{H}$ and $\mathbf{B} = \hat{\mu}\mathbf{H} + \hat{\chi}^{\dagger}\mathbf{E}$, where $\hat{\chi}$ is the magneto-electrical coupling parameter). The non-vanishing off-diagonal terms of $\hat{\chi}$ play a similar role to the off-diagonal components of a gyro-magnetic material's permeability tensor, which is responsible for emulating an artificial gauge field in QH-PTIs. In BMW systems, the bianisotropic effect introduces an artificial gauge field and Berry connection for the two spin modes and further gives rise to a QSH-PTI[21,23].

It was later shown that the BMW system is also able to host QH[10] and quantum valley-Hall (QVH)[11,12] topological phases through the inclusion of magneto-optical components, or an in-plane inversion-breaking tripod structure, respectively. An outstanding benefit of BMW-PTIs arises from the fact that they are all perturbations of the same underlying photonic crystal structure. This allows for the demonstration of composite PTI systems, where different topological phases co-exist to create unique edgemodes, and thus perform useful functionalities, such as a unique 3-port Y-junction and a full 4-port circulator[10] (shown in Fig. 1 c, d).

QSH-PTIs are also realized in distorted hexagonal crystalline lattices of scatterers[16,24]. The combination of TM modes with $p/d-$like symmetry E_z profiles constitutes the two pseudospins, and the expansion of the hexagonal unit cell leads to band inversion and non-trivial topological bands. The contraction of the same unit cell creates a topologically trivial insulating bulk.

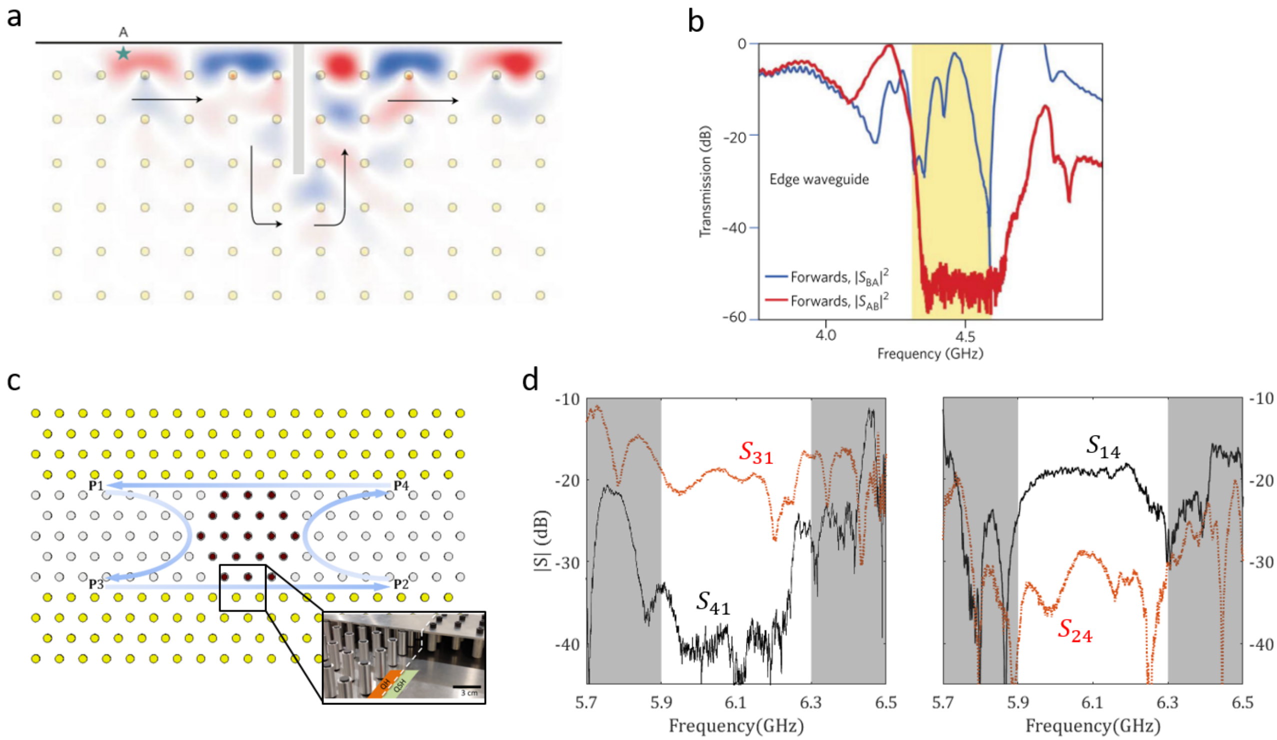

FIG. 1. **a**, The edgemode propagation is robust against the insertion of a metallic defect (grey rectangular) at the boundary of the system. The yellow circles represent the lattice sites. The blue and red colors are the out-of-plane component of the electric field. **b**, The transmission measurements of the edgemode propagation. The S_{BA} and S_{AB} are the measure of the forward and backward transmission along the edge, respectively. A $\sim 0dB$ maximum forward transmission and a $\sim 60dB$ right-left difference are observed, showing the uni-directional propagation of the edgemode inside the photonic bandgap (yellow). **c**, Schematic of the 4-port BMW-PTI circulator. The circulator is built with three QSH regions with alternating spin-Chern numbers (colored as yellow, gray and yellow), and a center QH region (black) biased by an external H-field pointing out of the plane. The edgemode will be circulating around the central QH-island from port 4→1→3→2→4, shown as blue arrows. The inset is the open-plate view of the composite QH-QSH interface. **d**, Transmission measurements of the realized 4-port circulator in **c**. In the bulk bandgap region, the outgoing waves from port 1 and 4 are directed to port 3 and 1, respectively. Panels adapted from: **a**,**b**, Ref.[9]; **c**,**d** modified using results in Ref.[10]

Localized edge states are thus created at the interface of the two regions.

An alternative approach to creating topologically protected modes is the so-called Floquet TIs. Floquet systems involve periodically-driven structures, either in space or in time. Consider first spatially-modulated systems in a quasi-2D lattice[18]. The system consists of a honeycomb lattice of coupled waveguides which extend in a helical manner in the perpendicular(z)-direction to the 2D lattice plane. This spatial modulation of the waveguides, effectively equivalent to applying a fast and strong temporal modulation of a strictly 2D system[25], generates a synthetic gauge field and further gives rise to the opening of a photonic bandgap. Temporal modulation has enabled various exciting photonic applications[26,27]. However the experimental demonstration of Floquet topological insulator in time awaits investigation due to the challenge of implementing sufficiently strong modulation in practice[2].

Here we summarize the common and distinct properties of the PTI realizations that were introduced above: time-reversal breaking (QH), time-reversal invariant (QSH, QVH) and the Floquet topological systems. Through careful design, each TI system possess insulating bulk regions and edge/kink states at the structure boundary. The properties of the edge state, such as the momentum, spin index and the total number of states, are based on the topological order of the bulk. Unique transmission properties are promised in various PTIs. This includes backscatter-free and uni-directional propagation involving highly localized edge states. These properties are maintained as long as the underlying symmetry is not violated.

A brief summary of practical specifications of selected microwave topological phases can be found in Table II. For the four exemplary PTI realizations, the topologically protected modes all operate in the GHz range with a relatively narrow fractional bandwidth (FBW). The

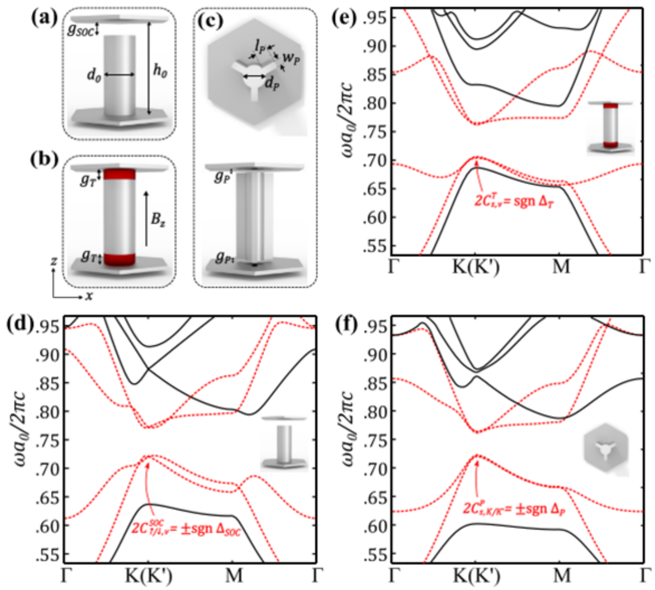

FIG. 2. **a-c**, The unitcell schematics of QSH, QH and QVH-PTIs based on the BMW system, respectively. d, h_0 and g are the diameter, unitcell height and spacing distance. B_z denotes the z-component of the magnetic field used to bias the ferrite disks (red) in a QH-BMW structure. l_P and w_P denote the structure details of the QVH tri-pods. **d-f**, The photonic band structures of the QSH, QH and QVH unitcells for a K-space excursion from Γ to $K(K')$ to M, and back to Γ. The quantities ω, a_0 and c refer to the angular frequency, the lattice constant and the speed of light. Red dotted curves are the TE and TM bands of interest. The bandgap widths are matched with careful engineered structural parameters that are shown in **a-c**. Panels adapted from Ref.[22].

physical dimension of the system as compared to the operating wavelength is another important concern for practical uses. In the coupled ring resonator system, the diameter of the ring resonators is large compared to the wavelength of the light[13,14]. On the other hand, the crystalline and BMW based PTIs are more compact in size. Apart from the microwave realizations, the concept of topological protection of modes can also be accommodated in a wide variety of physical wave systems. Examples range from acoustic systems[28], electrical circuits[29], mechanical systems[30] and optical lattices[31].

We highlight an example system, namely Bi-anisotropic Meta-Waveguide PTIs. The BMW-PTIs are based on a photonic crystal formed from an unperturbed unit cell consisting of a metallic cylindrical rods sandwiched between two metallic plates[21,23]. The unit cells are arranged into a hexagonal 2D lattice. The dimensions of the structure are carefully engineered so that the TE and TM modes are degenerate at the Dirac points in the photonic crystal band structure[21]. A variety of perturbations are able to break this mode degeneracy to create a bandgap in the bulk, and to give rise to different topological states for the material. The synthetic spin DOFs are formed by the in-phase and out-of-phase combination of the TE and TM modes[21]. The propagation of these spin-labelled modes is along the K and K' directions of the lattice. With the introduction of inversion-symmetry-breaking in the vertical (z) direction of the lattice (Fig. 2(d)), an effective spin-orbit coupling (SOC) interaction is created and non-trivial topological indices are assigned to the two spin modes. The QSH analog photonic topological systems are thus realized without the application of a global magnetic field. The propagation of edgemodes in QSH-BMWs are experimentally observed at the interface between PTIs with different topological indices in various experiments[15,23]. Like its electronic counterpart, the momentum of a photonic edgemode is locked to its synthetic spin polarization. The edgemode propagation is free from back-scattering as long as the underlying T-symmetry of the system is preserved and severe loss is absent.

Recent studies reveal the possibilities of emulating both QH and QVH topological phases using the BMW architecture[11,12,22,23]. The QVH-BMWs (Fig. 2(f)) are realized by substituting the cylindrical center rod into a carefully designed tri-pod which creates the in-plane parity symmetry breaking[11,12] (e.g., reflect the tri-pod with respect to the x-direction). The valley refers to the vicinity of two high symmetry points K/K'. The QH-BMWs (Fig. 2(e)) are introduced with the application of magneto-optical materials biased by magnetic fields[10] which breaks the T-symmetry and introduces a bandgap. The topological index of the QSH, QVH and QH BMW modes are defined with the spin-Chern number $2C^{SOC}_{s,v} = \pm sgn(\Delta_{SOC})$, valley-Chern number $2C^{P}_{s,v} = \pm sgn(\Delta_P)$, and a global Chern number $2C^{T}_{s,v} = sgn(\Delta_T)$, respectively[22]. Here $s = \uparrow / \downarrow$ is the spin label and $v = K/K'$ is the valley label in the above expressions. The Δ_{SOC}, Δ_P and Δ_T are the overlap integrals of the unperturbed modes inside the perturbed volume of the unit cell, whose values set the scale for the width of the bulk bandgap. The three subscripts of Δ represent the specific types of symmetry-breaking which lift the mode degeneracy at the Dirac points and further give rise to non-trivial topological phases.

As mentioned before, heterogeneous PTI structures have recently been demonstrated with QSH, QVH and QH BMW lattices as building blocks[10,11]. A key requirement for a successful composite topological structure is the matching of the above Δ perturbation integrals. This ensures the reflection-free propagation of edgemodes when travelling through a heterogeneous interface between different PTI phases. Inside a composite topological device, the propagation properties of an edgemode is dictated by its spin index[10]: the number of edgemodes are determined by the difference in Chern numbers between two neighboring media, and the propagating direction of the edgemode is defined by both its spin index and the polarization direction of the QH-BMW region. Practical photonic devices, such as a 4-port circulator, have been experimentally realized with the combination of QH and QSH BMW topological phases[10] (Fig. 1 c and d). The structure consists of four I/O channels made by QSH-QSH waveguides and a center QH island to shuttle and dispatch the flow of edgemodes. High isolation is promised by the topologically protected propagation of edgemodes. Real-time switching of circulation direction is also achievable with the simple inversion of the biasing H-field. Apart from being a demonstration of a PTI-based practical application, this device is also the first-ever experimental realization of a composite physical material with both QSH and QH topological phases, in either an electronic or photonic setting. Recently a BMW-based compact and scalable delay line structure is experimentally demonstrated[32] and a directional coupler with variable coupling has been theoretically proposed[33].

From its inception, the field of topological photonics has drawn intense research interest. As compared to studying topological physics in electronic systems, photonic systems offer great convenience in designing and fabricating structures, and conducting measurements. It also allows researchers to have better control of specific physical details, such as the precise introduction of local defects in the lattice. Inventing new physical systems, such as combining multiple topological phases, is already realized with photonic systems but extremely hard to accomplish in electronic materials with current technologies. The design and optimization of new photonic topological systems are facilitated by finite-element electromagnetic numerical solvers, which produce reliable results since Maxwell's equations can be solved exactly.

The attractive properties of a topologically protected mode, such as the immunity to structural defects and being able to travel through sharp corners, are of keen interest to the field of microwave engineering[2]. In PTI-based photonic devices, the propagation of edgemodes are restricted to the boundaries of insulating bulk material[15]. In contrast to standard PC designs, mechanically flexible PTI waveguides would be less susceptible to backscattering and maintain the low return loss even as the waveguide is deformed. The introduction of local disorder along a PTI interface would only alter the shape of the original bulk boundary and leave the edgemode unperturbed[9]. Thus a PTI-based photonic device has high tolerance to structural defects, whereas the performance of more traditional microwave devices is largely limited by the fabrication techniques and the changes in the structure over time and usage. One can upgrade the performance of practical devices by either constructing purely PTI-based systems, or by combining PTI-devices with existing technologies[34].

BMW-based PTIs have been proposed for compact microwave delay lines[22,32]. Such devices can be concatenated without standing wave complications arising from slight impedance mismatches[35]. BMW PTI structures based on photonic crystals are also very suitable for high-power applications because the field levels in the hexagonal array of rods are low compared to integrated structures. Such PTIs can be used for narrow-band high-power microwave energy delivery where impedance mismatches create strong reflected waves that can create havoc. The central operating frequency f_{op} of the PTI devices can be scaled by tailoring the physical dimensions of the unit cell.

Another practical proposal is to facilitate lasing under topological protection[36–40]. Compared to topologically trivial systems, high power single mode operation is promised in a PTI based device, with high robustness to structural defects[41]. Such an approach could be applied to PTI-based microwave amplifiers and masers to improve efficiency and to provide integration with other PTI-enabled functionality. A uni-directional amplifier or detector fabricated from a PTI structure would also be of use in the quantum limit. A microwave quantum emitter placed in a PTI waveguide will experience an engineered density of electromagnetic states which can couple the quantum state of the qubit to that of the photon[42].

Aside from the good transmission properties, the synthetic spin of the edgemodes provides an extra handle to manipulate the flow of light. With the exception of polarization, this wave-steering ability is essentially unprecedented in more conventional devices. Devices with sophisticated functions can be realized by engineering the pathways of different spin DOF in a heterogeneous PTI system[10]. Spin-splitters, combiners and "logic" devices such as Boolean networks may be realized with composite topological systems. The spin-1/2-like DOF of PTI edge modes is like an effective 2-state bit. Inside a photonic logic device, information can be shuttled around and switched using QH-based composite PTI structures. Finally we note that novel antenna devices can be designed based on PTI devices by utilizing the distinct refractive properties of TE and TM kink states radiated from a BMW waveguide to free space[11].

We also note several drawbacks of the current PTI realizations. The gyro-magnetic effect, which is responsible for breaking the T-symmetry in many QH-PTI systems, typically require large magnets, and the gyromagnetic effect weakens at higher frequencies. Though the propagation of edgemodes are free from back-scattering induced from a variety of defects, the loss of the preserved system symmetry would cause reflection. A recent report studies a system where a metallic mirror is installed at the termination of a QVH-QVH waveguide[43]. Intervalley scattering of edgemodes are observed which further leads to the finite reflection of the edgemode. The propagation of a topologically protected mode would be destroyed under extreme dissipative defects[13]. Also, an inherent disadvantage of current PTI designs is that only a small portion of the structure is utilized, both in real-space and k-space. In real-space, there must be an insulating bulk region to provide a substrate for the 'edges'. Only a small fraction of the structure participates in the wave propagation. The edgemode confinement length can be estimated as $\xi \sim v_D/\Delta$, where v_D is the wave velocity at the Dirac point and Δ is the direct gap[44]. Future work may focus on minimizing the confinement length of edgemodes into the bulk, for example by in-

PTI Platform	f_{op}	FBW	a_0/λ_{op} mm/mm	Potential Destructive Disorder	Special Remarks	Compatibility with Other PTIs	Potential Application
Gyro-magnetic QH-PTI[9]	4.5GHz	6%	40/66	Magnetic disorder	Magneto-optical effects are hard to emulate at higher frequencies	No	Isolators, slow-light devices, bandpass filters ...
Coupled ring resonators QSH-PTI[13]	11.3GHz	5%	83/28	Spin-flipping defects; Strong dissipative disorder	Fine fabrication techniques are required to prevent losses	No	Topological mode amplification ...
Distorted lattice QSH-PTI[17]	7.5GHz	2%	25/40	Sensitive to the ratio between lattice constant and contraction distance	Band gap opened at Γ point (time-reversal invariant momentum)	No	Applicable to both metallic and dielectric environments
BMW Composite PTI systems[10,11]	6GHz	6%	36/50	Spin-flipping or inter-valley scattering disorder	Modes propagate along K direction	Yes	Circulators, isolators, photonic logic devices, high-power devices

TABLE II. Practical metrics for a selected subset of PTI microwave realizations. The column f_{op} includes the reported central operating frequency of example PTI realization. FBW stands for the fractional bandwidth, defined as $FBW = \Delta f/f_{op}$ where Δf is the size of the bandgap. The quantities λ_{op} and a_0 represent the operating wavelength and the lattice constant of the photonic crystal.

creasing the gap Δ, allowing for the elimination of unnecessary bulk structures. Top-down structure design methods might be useful for the optimization of topological system properties[45]. It is notable that enhanced energy confinement and transport can be realized with the QSH effect in planar structures, where the surface states commonly found in two-dimensional lattices are now one-dimensional line states[46]. As shown in the photonic band structure studies and transmission measurements, the operating bandwidth of PTI systems are small compared to the width of the conducting bands (Tab. II). This phenomenon is dictated by the literal analogy to the electronic topological systems. Future generalizations of PTI wave behavior may eliminate this restriction. We note that a PTI delay line based on all-dielectric QVH random cavities presents a way to utilize the bulk regions to accomplish a practical goal. Compact delay lines are proposed utilizing the whispering-galley-like modes circulating inside the randomized bulk rather than the perimeter of the resonator[47].

We would like to conclude the paper with an outlook on topological photonics. A fundamental research interest of the field is to identify and exploit PTI realizations in new physical systems. Up to this point, many realizations of PTIs have been done in close analogy to their electronic forebearers. Hence PTI realizations based on topological states created by more general conditions and in other dimensions are also of interest. Recent progress includes finding higher-order topological states[48,49], and the demonstration of 3D PTIs either through the addition of an artificial dimension[50] or in real space[51,52]. Beyond the common photonic lattice system, topics like the experimental demonstration of topological phases in disordered systems[53,54] await further investigation. The emulation of non-Hermitian PTIs with parity-time symmetric structures opens up the invention of topological systems with co-existing gain and loss, which is difficult to realize in condensed matter systems[55], and a microwave limiter has been demonstrated using this idea[56]. The introduction of nonlinear effects into topological photonics would invoke new research directions in both the microwave and optical ranges, such as studying the edgemode response to weak and strong diode-induced nonlinearity[57]. The application of PTI technologies are also becoming increasingly coupled with other physical research. Its superior light transmission properties make PTI technology an ideal choice in various physical experiments. Potential directions include emulation of fractional quantum Hall systems[58], the creation of photon-phonon interactions[59], and better control of the flow of light through the effective Lorentz force for photons[25].

Compared to the fundamental studies, the utilization of topologically protected light in more applied fields are less well-developed, unfortunately. The recent advancement of realizing heterogeneous PTI systems presents the first demonstration of realistic photonic devices that are purely empowered by topological physics[10]. Aside from the realization of PTI-based isolators and circulators, we believe that a composite PTI system is fully capable of achieving more sophisticated functions with the combination of QSH, QH and QVH topological domains. As a future direction, miniature on-chip PTI devices would find broad application in fields including telecommunications and high performance all-optical computation. Though many PTI designs would be of fundamental interest, we are certain that the unique properties of topological photonic technologies would ensure its role in future applications.

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