

1 Logic

A particular way of thinking about something **OR** Logic is the systematic study of the form of valid inference
Types of Logic: 1. **Deductive Logic:** Properties of a Set are applied to the individual members of the Set. This is also known as **Mathematical Logic**.
2. **Inductive Logic:** Properties of Individual Members are applied to the whole Set.

Propositions: English phrases which are Primary Bearers of Truth Value(true/false)
Atomic Propositions: "Ram is married"(p), "Ram has a wife"(q)
Logical Connectives: Symbol/Word connecting 2 related ideas. \wedge (AND), \vee (OR), \rightarrow (IMPLIES)
Compound Propositions: Atomic Propositions + Logical Connectives.
Example: $p \rightarrow q$ (Ram is married IMPLIES Ram has a wife)
[Logical Representation]
If Ram is married Then Ram has a wife
[Non Logical Representation]

Variable: x="Ram" or more Generally x="Man".
Predicate: $P_1(x)$ =Man is married, $P_2(x)$ =Man has a wife. Statement that may be true or false depending on the values of its variables.
Quantifiers: Specifies quantity of specimens in domain of discourse that satisfy an open formula. \forall (for each/all) [Universal Quantification], \exists (there exists some) [Existential Quantification]
Example: $\forall x(P_1(x) \rightarrow P_2(x))$. For all men being married IMPLIES he has a wife
[Logical Representation]. For all men If Man is married Then Man has a Wife
[Non Logical Representation]

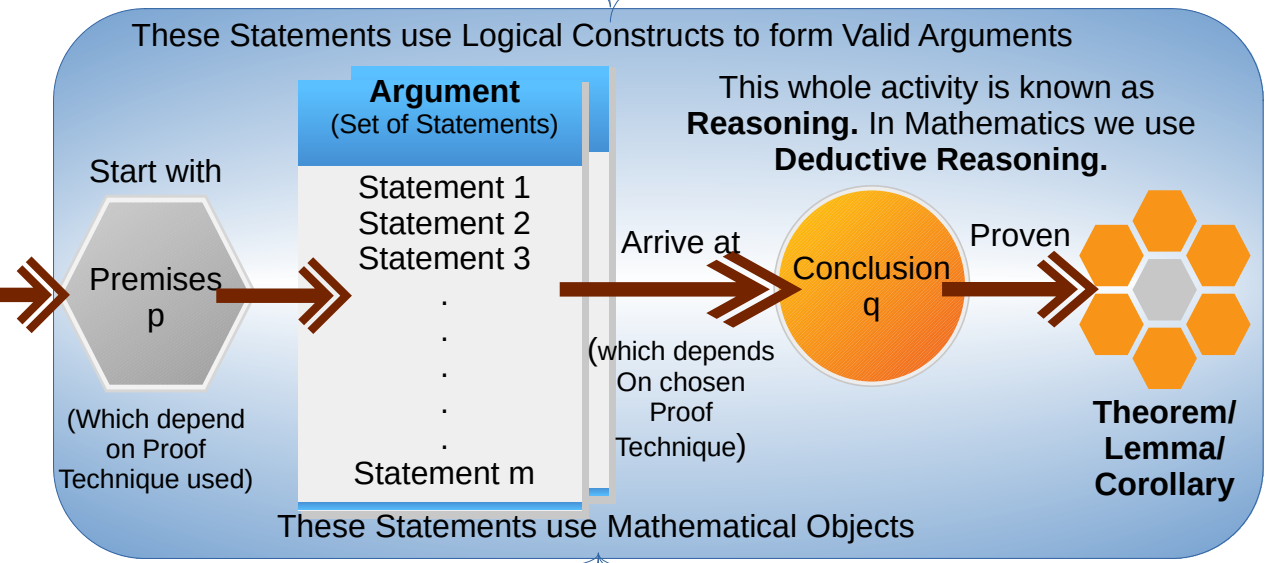
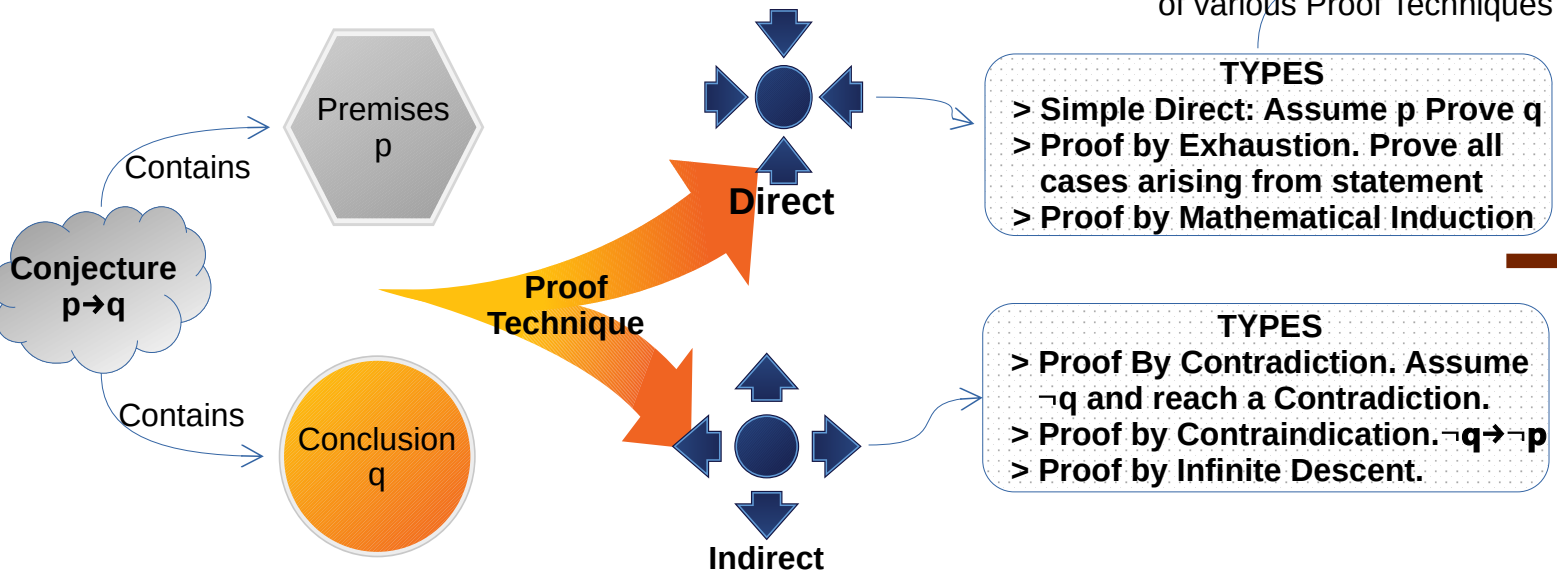
Statement is a **Declarative Sentence** that's either true or false but NOT both.
Example:
 $3+7=10$.
 $\forall (x,y) \in \mathbb{R} ((x+y)^2 = x^2 + 2xy + y^2)$
[For all x and y belonging to Real Numbers $(x+y)^2 = (x^2 + 2xy + y^2)$]

Definition: Rol provide the Templates or Guidelines for constructing Valid Arguments from the Statements that we already have.
Valid Argument: Argument is Valid if Conclusion follows from the truth values of the Premises.
Argument: Argument is a sequence of Statements.
Example: **Modus Ponens** $((p \rightarrow q) \wedge p) \rightarrow q$, **Modus Tollens** $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$



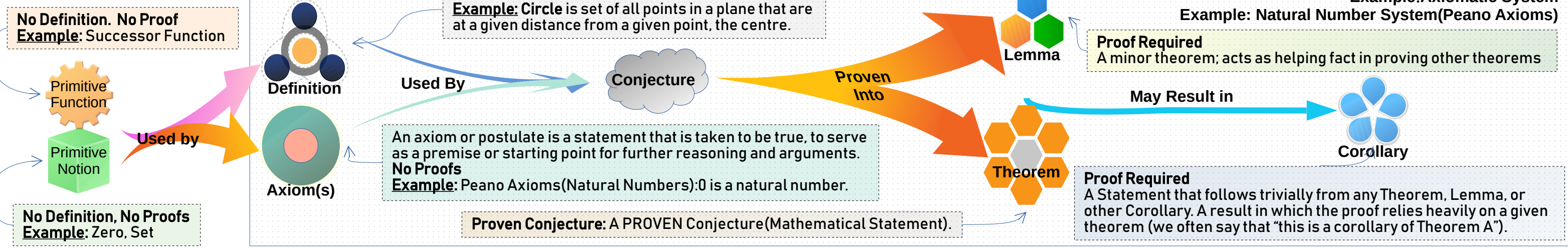
3 Proof

Inferential argument for a mathematical statement, showing that the stated assumptions logically guarantee the conclusion.



2 Structure Of Math

Various simple objects/structures at atomic levels combine to evolve into complex structures/objects.



1 Logic

1. Propositional Logic (Zeroth Order Logic)

Use to Construct

2. Predicate Logic (First Order Logic)

Used to Construct

3. Mathematical Statement

Used to construct

4. Rules of Inference

Logical/Mathematical Symbol	Logical Connectives	Propositional Logic		Predicate Logic		Mathematical Statement
		Proposition	Expression	Quantifier	Predicate Expression	
					P_1	is even
		p			$P_1(n)$	Integer n is even
					P_2	'=2k, k is some integer'
		q			$P_2(n), k \in \mathbb{Z}$	Integer n=2k, k is some integer
	\leftrightarrow					If and Only If
			$p \leftrightarrow q$		$P_1(n) \leftrightarrow P_2(n)$	Integer n is even If and Only If n=2k, k is some integer
				\exists		There exists
					$\exists k$	There exists an integer k
\in						Belongs to
$\exists, $						Such That
					$P_1(n) \leftrightarrow \exists k P_2(n), k \in \mathbb{Z}$	An integer number n is even if and only if there exists a number k such that Integer n = 2k, k is some Integer

Equivalent as Truth Values are same.

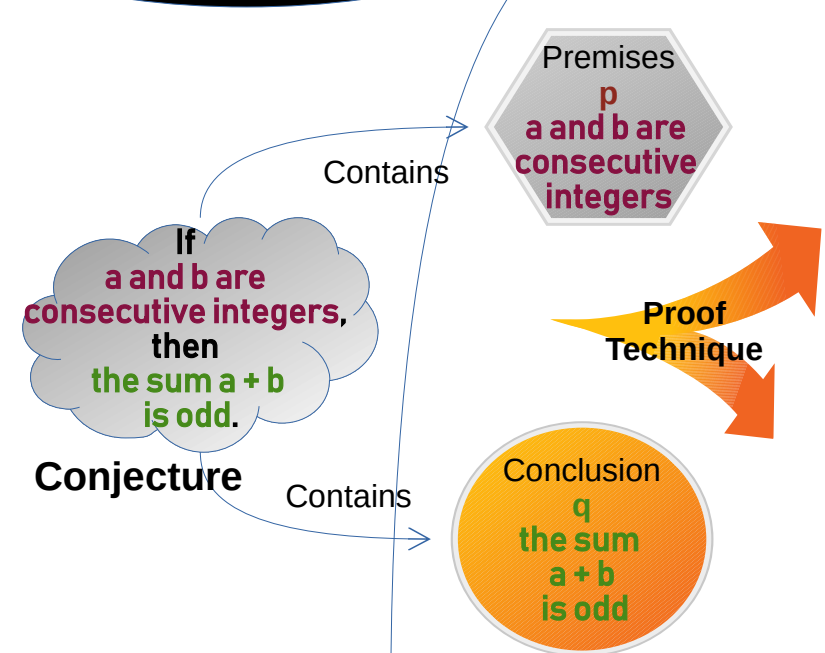
Boolean Logic/Algebra				Direct	Contrapositive
p	q	$\neg p$	$\neg q$	$p \rightarrow q (\neg p \vee q)$	$\neg q \rightarrow \neg p (q \vee \neg p)$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Validity of these two proofs is provided by Logic using Truth Table

Rules of Inference	
p: it's raining, q: it's wet	
Modus Ponens	Modus Tollens
$p \rightarrow q$ p ----- $\therefore q$	$p \rightarrow q$ $\neg q$ ----- $\therefore \neg p$
$((p \rightarrow q) \wedge p) \rightarrow q$ If it's raining Then it's wet AND it's raining Therefore it's wet.	$((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$ If it's raining Then it's wet AND it's NOT wet Therefore it's NOT raining.

These templates are used in Proof argument

3 Proof



Direct Proof

$p \rightarrow q$ (Constructive Proof)

If a and b are consecutive integers, then the sum a + b is odd.

Direct

1. Assume(p) that a and b are consecutive integers.
2. Because a and b are consecutive we know that $b = a + 1$, (From Definition 3 below)
3. Thus, the sum $a + b$ may be re-written as $a + (a+1) = 2a+1$.
4. Thus, there exists a number k such that $a+b = 2k + 1$.
5. Hence the sum $a+b$ is odd. (Using point 4 above and Definition 3 below).
6. Since Premise(p) logically leads to Conclusion(q) hence proven that If a and b are consecutive integers, then the sum a + b is odd.

Indirect Proof

$\neg q \rightarrow \neg p$ (Proof by Contraposition)

If the sum a + b is not odd, then a and b are not consecutive integers.

Indirect

1. Assume($\neg q$) that the sum of the integers a and b is NOT odd.
2. Then, there exists no integer k such that $a + b = 2k + 1$. (From Contraposition of Definition 2 below)
3. Thus, $a + b \neq k + (k + 1)$ [Rewriting $2k+1$] for all integers k.
4. If $k+1$ is the successor of k, (From Definition 3 below)
5. This implies that $a + b$ is not equal to sum of any two consecutive integers.
6. This implies($\neg p$) that a and b cannot be consecutive integers.
7. Since negation of Consequence leads to negation of Premise, hence the original assumption $p \rightarrow q$ is correct, i.e. If a and b are consecutive integers, then the sum a + b is odd.

2 Structure Of Math

1. An integer number n is even if and only if there exists a number k such that $n = 2k$, k is some integer.
2. An integer number n is odd if and only if there exists a number k such that $n = 2k + 1$.
3. Two integers a and b are consecutive if and only if $b = a + 1$.
4. Let n^+ be the successor of n, that is the number following n in the natural numbers, so $0^+ = 1, 1^+ = 2$. Define $a + 0 = a$. Define the general sum recursively by $a + (b^+) = (a + b)^+$. Hence $1 + 1 = 1 + 0^+ = (1 + 0)^+ = 1^+ = 2$.

- Used By
- We don't need to prove that $a+b$ =integer because of this axiom.
- Conjecture
- If a and b are consecutive integers, then the sum a + b is odd integer.
- p(Premise)=a and b are consecutive integers
- q(Conclusion)=the sum a + b is odd integer.
1. Closure of * & +: $a*b$ and $a+b$ are integers.
 2. Commutativity of +: $a+b=b+a$
 3. Associativity of +: $(a+b)+c=a+(b+c)$
 4. Commutativity of *: $a*b=b*a$
 5. Associativity of *: $(a*b)*c=a*(b*c)$
 6. Distributivity: $a*(b+c)=a*b+a*c$
 7. Trichotomy: Either $a<0$, $a=0$, or $a>0$.
 8. Well-Ordered Principle: Every non-empty set of positive integers has a least element. (This is equivalent to induction.)

