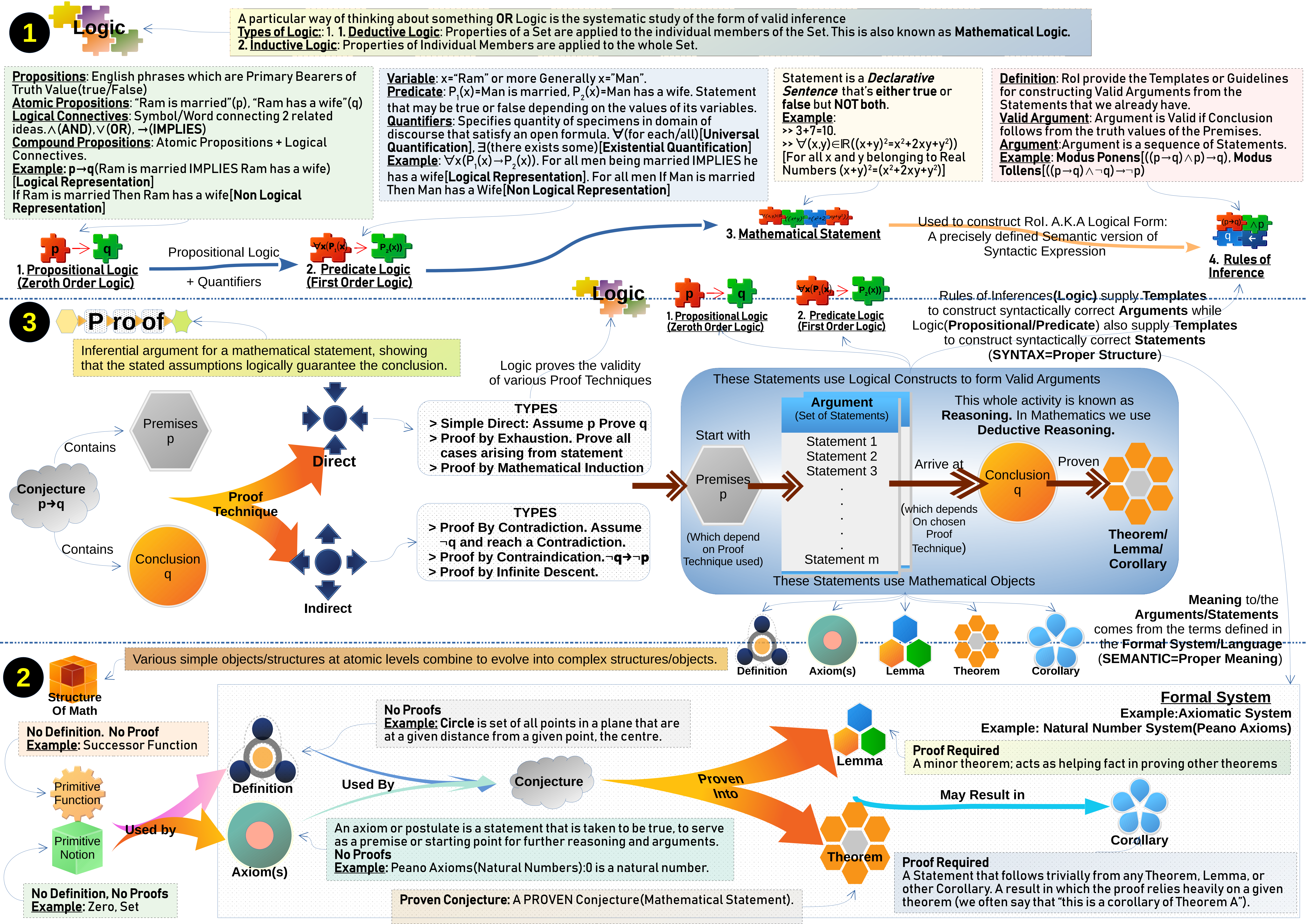

1
Logic
A particular way of thinking about something OR Logic is the systematic study of the form of valid inference
Types of Logic:: 1. 1. Deductive Logic: Properties of a Set are applied to the individual members of the Set. This is also known as Mathematical Logic.
2. Inductive Logic: Properties of Individual Members are applied to the whole Set.
Propositions: English phrases which are Primary Bearers of Truth Value(true/False)
Atomic Propositions: "Ram is married"(p), "Ram has a wife"(q)
Logical Connectives: Symbol/Word connecting 2 related ideas. ∧(AND), ∨(OR), →(IMPLIES)
Compound Propositions: Atomic Propositions + Logical Connectives.
Example: p→q(Ram is married IMPLIES Ram has a wife) [Logical Representation]
If Ram is married Then Ram has a wife[Non Logical Representation]
Variable: x="Ram" or more Generally x="Man".
Predicate: $P_1(x)$=Man is married, $P_2(x)$=Man has a wife. Statement that may be true or false depending on the values of its variables.
Quantifiers: Specifies quantity of specimens in domain of discourse that satisfy an open formula. ∀(for each/all)[Universal Quantification], ∃(there exists some)[Existential Quantification]
Example: $\forall x(P_1(x)\rightarrow P_2(x))$. For all men being married IMPLIES he has a wife[Logical Representation]. For all men If Man is married Then Man has a Wife[Non Logical Representation]
Statement is a Declarative Sentence that's either true or false but NOT both.
Example:
>> 3+7=10.
>> $\forall(x,y)\in\mathbb{R}((x+y)^2=x^2+2xy+y^2))$
[For all x and y belonging to Real Numbers $(x+y)^2=(x^2+2xy+y^2)$]
Definition: RoI provide the Templates or Guidelines for constructing Valid Arguments from the Statements that we already have.
Valid Argument: Argument is Valid if Conclusion follows from the truth values of the Premises.
Argument: Argument is a sequence of Statements.
Example: Modus Ponens[((p→q)∧p)→q), Modus Tollens[((p→q)∧¬q)→¬p)
p
q
1. Propositional Logic (Zeroth Order Logic)
Propositional Logic
+ Quantifiers
$\forall x(P_1(x))$
$P_2(x))$
2. Predicate Logic (First Order Logic)
3. Mathematical Statement
Used to construct RoI. A.K.A Logical Form: A precisely defined Semantic version of Syntactic Expression
4. Rules of Inference
3
Proof
Inferential argument for a mathematical statement, showing that the stated assumptions logically guarantee the conclusion.
Logic
Logic proves the validity of various Proof Techniques
1. Propositional Logic (Zeroth Order Logic)
2. Predicate Logic (First Order Logic)
Rules of Inferences(Logic) supply Templates to construct syntactically correct Arguments while Logic(Propositional/Predicate) also supply Templates to construct syntactically correct Statements (SYNTAX=Proper Structure)
Premises p
Contains
Conjecture p→q
Contains
Conclusion q
Proof Technique
Direct
Indirect
TYPES
> Simple Direct: Assume p Prove q
> Proof by Exhaustion. Prove all cases arising from statement
> Proof by Mathematical Induction
TYPES
> Proof By Contradiction. Assume ¬q and reach a Contradiction.
> Proof by Contraindication. ¬q→¬p
> Proof by Infinite Descent.
These Statements use Logical Constructs to form Valid Arguments
Argument (Set of Statements)
This whole activity is known as Reasoning. In Mathematics we use Deductive Reasoning.
Start with
Premises p
(Which depend on Proof Technique used)
Statement 1
Statement 2
Statement 3
Statement m
Arrive at
(which depends On chosen Proof Technique)
Conclusion q
Proven
Theorem/ Lemma/ Corollary
These Statements use Mathematical Objects
Definition
Axiom(s)
Lemma
Theorem
Corollary
Meaning to/the Arguments/Statements comes from the terms defined in the Formal System/Language (SEMANTIC=Proper Meaning)
2
Structure Of Math
Various simple objects/structures at atomic levels combine to evolve into complex structures/objects.
No Definition. No Proof
Example: Successor Function
Primitive Function
Primitive Notion
No Definition, No Proofs
Example: Zero, Set
Used by
Definition
Axiom(s)
No Proofs
Example: Circle is set of all points in a plane that are at a given distance from a given point, the centre.
Used By
Conjecture
An axiom or postulate is a statement that is taken to be true, to serve as a premise or starting point for further reasoning and arguments.
No Proofs
Example: Peano Axioms(Natural Numbers):0 is a natural number.
Proven Conjecture: A PROVEN Conjecture(Mathematical Statement).
Proven Into
Lemma
Theorem
May Result in
Corollary
Formal System
Example:Axiomatic System
Example: Natural Number System(Peano Axioms)
Proof Required
A minor theorem; acts as helping fact in proving other theorems
Proof Required
A Statement that follows trivially from any Theorem, Lemma, or other Corollary. A result in which the proof relies heavily on a given theorem (we often say that "this is a corollary of Theorem A").

1 Logic

1. Propositional Logic (Zeroth Order Logic) → Use to Construct → **2. Predicate Logic (First Order Logic)** → Used to Construct → **3. Mathematical Statement** → Used to construct → **4. Rules of Inference**

Logical/Mathematical Symbol	Logical Connectives	Propositional Logic		Predicate Logic			Mathematical Statement
		Proposition	Expression	Quantifier	Predicate	Expression	
					P_1		is even
		p			$P_1(n)$		Integer n is even
					P_2		'=2k, k is some integer'
		q			$P_2(n)$, $k \in Z$		Integer n=2k, k is some integer
	$\leftrightarrow$						If and Only If
			$p \leftrightarrow q$			$P_1(n) \leftrightarrow P_2(n)$	Integer n is even If and Only If n=2k, k is some integer
				$\exists$			There exists
						$\exists k$	There exists an integer k
$\in$							Belongs to
$\ni$, |							Such That
						$P_1(n) \leftrightarrow \exists k \mid P_2(n)$, $k \in Z$	An integer number n is **even** if and only if there exists a number k such that Integer **n = 2k**, k is some Integer

Equivalent as Truth Values are same.

Boolean Logic/Algebra				Direct	Contrapositive
p	q	¬p	¬q	$p \to q (\neg p \vee q)$	$\neg q \to \neg p (q \vee \neg p)$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Validity of these two proofs is provided by Logic using Truth Table

3 Proof

Direct Proof
p→q (Constructive Proof)
If a and b are consecutive integers, then the sum a + b is odd.

1. Assume(p) that a and b are consecutive integers.
2. Because a and b are consecutive we know that b = a + 1, **(From Definition 3 below)**
3. Thus, the sum a + b may be re-written as a + (a+1) =2a+1.
4. Thus, there exists a number k such that a+b = 2k +1.
5. Hence the sum a+b is odd. **(Using point 4 above and Definition 3 below)**.
6. Since Premise(p) logically leads to Conclusion(q) hence proven that **If a and b are consecutive integers, then the sum a + b is odd.**

Indirect Proof
¬q→¬p(Proof by Contraposition)
If the sum a + b is not odd, then a and b are not consecutive integers.

1. Assume(¬q) that the sum of the integers a and b is **NOT** odd.
2. Then, there exists no integer k such that a + b = 2k + 1. **(From Contraposition of Definition 2 below)**
3. Thus, a + b ≠ k + (k + 1)[*Rewriting 2k+1*] for all integers k.
4. If k +1 is the successor of k,**(From Definition 3 below)**
5. This implies that a + b is not equal to sum of any two consecutive integers.
6. This implies(¬p) that a and b cannot be consecutive integers.
7. Since negation of Consequence leads to negation of Premise, hence the original assumption p→q is correct, i.e. **If a and b are consecutive integers, then the sum a + b is odd.**

2 Structure Of Math

DFn

1. An integer number n is even if and only if there exists a number k such that n = 2k, k is some integer.
2. An integer number n is odd if and only if there exists a number k such that n = 2k + 1.
3.Two integers a and b are consecutive if and only if b = a + 1.
4. Let n^+ be the successor of n, that is the number following n in the natural numbers, so 0^+=1, 1^+=2.
Define a + 0 = a. Define the general sum recursively by a + (b^+) = $(a + b)^+$. Hence 1 + 1 = 1 + 0^+ = $(1 + 0)^+$ = 1^+ = 2.

Uses Primitive Recursive Function

PFn Used by

PNn

Used By

We don't need to prove that **a+b=integer** because of this axiom.

Axm

1. Closure of ∗ & +: a∗b and a+b are integers. 2. Commutativity of +: a+b=b+a **3.** Associativity of +: (a+b)+c=a+(b+c) **4.** Commutativity of ∗ : a∗b=b∗a **5.** Associativity of ∗ : (a∗ b)∗ c=a∗ (b∗ c). **6.** Distributivity: a∗ (b+c)=a∗ b+a∗ c **7.** Trichotomy: Either a<0, a=0, or a>0. **8.** Well-Ordered Principle: Every non-empty set of positive integers has a least element. (This is equivalent to induction.)

Conjecture

If a and b are consecutive integers, then the *sum a + b is odd integer*.
p(Premise)=a and b are consecutive integers
q(Conclusion)=the sum a + b is odd integer.

Proof

Direct Proof (Constructive Proof)
p→q

Proof

Indirect Proof (Proof by Contraposition)
¬q→¬p
If the sum a + b is not odd, then a and b are not consecutive integers.

Proven → **Theorem**