Self-Driving Cars

Lecture 5 - Vehicle Dynamics

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MPI-IS / University of Tübingen

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University of Tübingen
MPI for Intelligent Systems
Autonomous Vision Group

And the Winner is ...

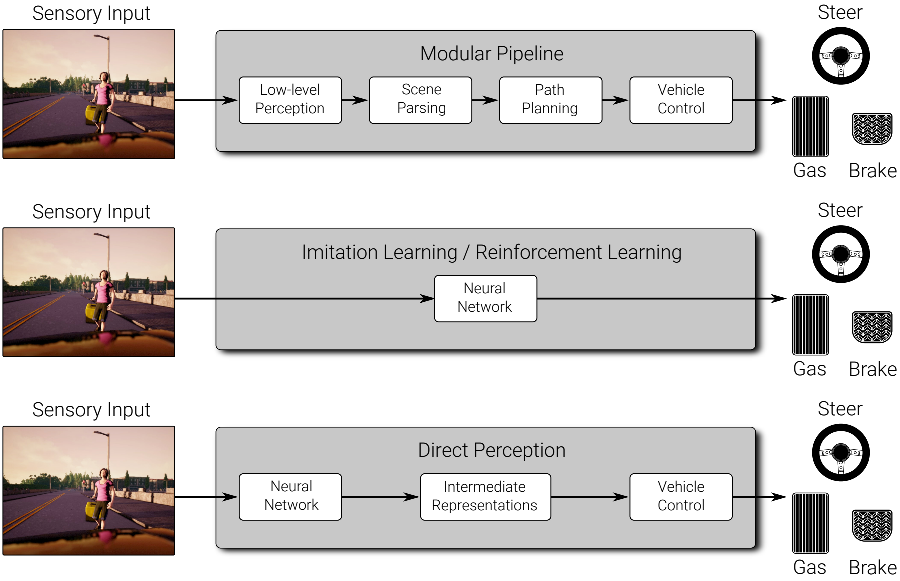
Team Member 1		Team Member 2		Score
Hille	Tobias	Mett	Arwed	553.58
Reiber	Moritz	Lahm	Marcus	511.73
Frank	Adrian	Leistner	Jonathan	460.96
Garhofer	Simon	Pfister	Maximilian	447.89
Xu	Shu	Zhu	Yifan	431.32
Geibel	Clemens	Krebs	Jan	377.33
Schwed	Mark	Varga	Leon	347.85
Spallek	Amadäus	Steinmetz	Jannik	311.04
Rist	Andreas	Marben	Joshua	300.99
Kretschmar	Floyd	Boehm	Niklas	226.47
Izadshenas	Alireza	König	Markus	209.67
Raisch	Claudio	Feil	Stefan	197.97
Runge	Arne			160.12
Kolatschek	Tamara	Nikischin	Aljoscha	92.66
Ertel	Sarah	Sohn	Tin Stribor	58.49
Hellriegel	Lucca	Speidel	Eric Raphael	30.14
Doll	Simon	Roeder	Benedict	18.43
Hobbhahn	Marius	Tomasek	Marc	12.98
Stroh	Martin	Hald	Tobias	8.7
Heidrich	Holger	Achauer	Lars	7.08
Herzig	Robert	Benbarka	Nuri	2.86
Martinovic	Igor	Chiang	Yen Chen	0

Agenda

Date	Lecture (Thursday)	Date	Exercise (Friday)
18.10.	01 - Introduction to Self-Driving Cars	19.10.	00 - Introduction Pytorch & OpenAI Gym
25.10.	02 - DNNs, ConvNets, Imitation Learning	26.10.	01 - Intro: Imitation Learning
1.11.	none (Allerheiligen)	2.11.	
8.11.	03 - Direct Perception	9.11.	01 - Q&A
15.11.	none	16.11.	
22.11.	04 - Reinforcement Learning	23.11.	01 - Discussion & 02 - Intro: Reinforcement Learning
29.11.	05 - Vehicle Dynamics	30.11.	
6.12.	06 - Vehicle Control & Localization	7.12.	02 - Q&A
13.12.	07 - Odometry & SLAM (J. Stückler)	14.12.	
20.12.	08 - Road and Lane Detection	21.12.	02 - Discussion & 03 - Intro: Modular Pipeline
10.1.	09 - Reconstruction and Motion Estimation	11.1.	
17.1.	10 - Object Detection & Tracking	18.1.	
24.1.	none	25.1.	03 - Q&A
31.1.	11 - Scene Understanding	1.2.	03 - Discussion & Announcement of Winners
7.2.	12 - Planning, Winner Talks, Q&A	8.2.	

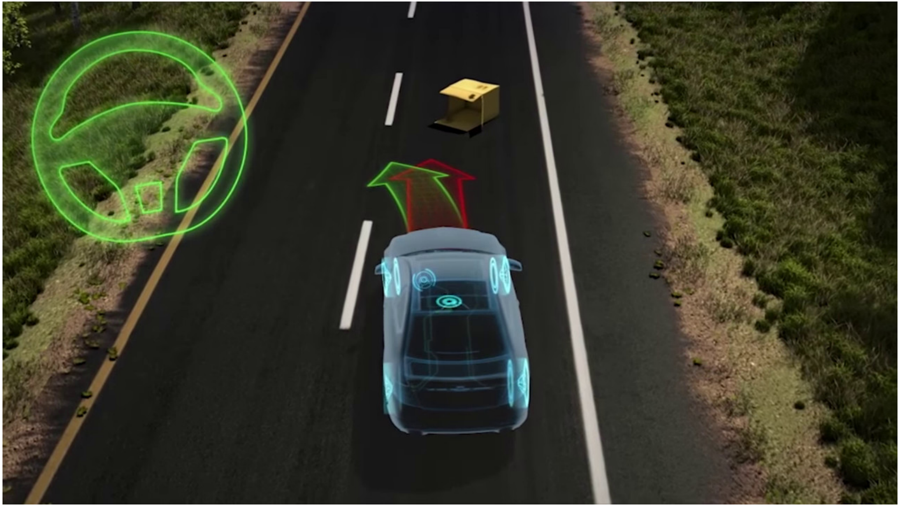
Recap

Approaches to Self-Driving



Vehicle Dynamics and Control

Electronic Stability Program



Sophisticated controllers are already in your car today!

Electronic Stability Program

Sophisticated controllers are already in your car today!

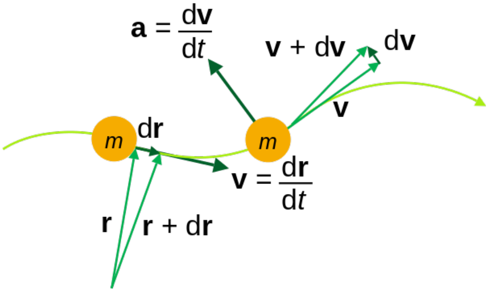
Brief History of Driver Assistance Systems

- 1926: Servo braking (Pierce-Arrow)
- 1951: Servo steering (Chrysler)
- 1958: Cruise control (Chrysler)
- 1978: Anti-lock braking system ABS (Bosch)
- 1986: Traction control system ASR (Bosch)
- 1995: Electronic stability program ESP (Bosch/BMW)
- 2000: Adaptive cruise control ACC (Mitsubishi/Toyota/Bosch)
- 2002: Emergency brake assistant (Mercedes Benz)
- 2003: Lane-keeping assistant (Honda)
- 2007: Automatic park assistant (Valeo)

Vehicle Dynamics

Kinematics

- Greek origin: “motion”, “moving”
- Branch of classical mechanics
- Describes motion of points and bodies
- Does not consider forces ($\Rightarrow$ Kinetics)
- Considers position, velocity, acceleration, ..
- Examples:
  - Celestial bodies, particle systems
  - Robotic arm, human skeleton



Holonomic vs. Non-Holonomic Systems

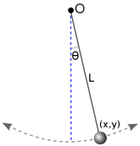
Holonomic Systems

- Controllable degrees of freedom equal to total degrees of freedom
- Can freely move in any direction
- Constraints **can** be described by $f(x_1, \ldots, x_N, t) = 0$ with coordinates x_i and time t

Example:

Pendulum

$x_1^2 + x_2^2 - L^2 = 0$

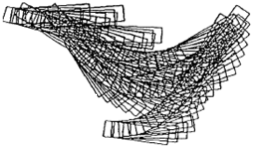


Nonholonomic Systems

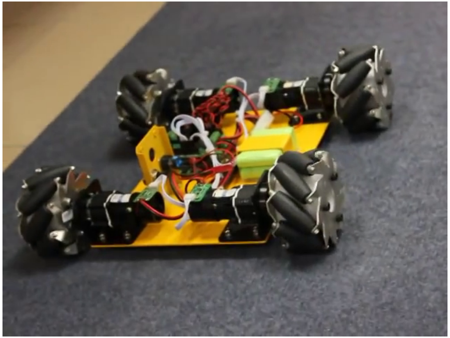
- Controllable degrees of freedom less than total degrees of freedom
- Cannot freely move in any direction
- Constraints **cannot** be described by $f(x_1, \ldots, x_N, t) = 0$ with coordinates x_i and time t

Example:

Vehicle Motion (this lecture)

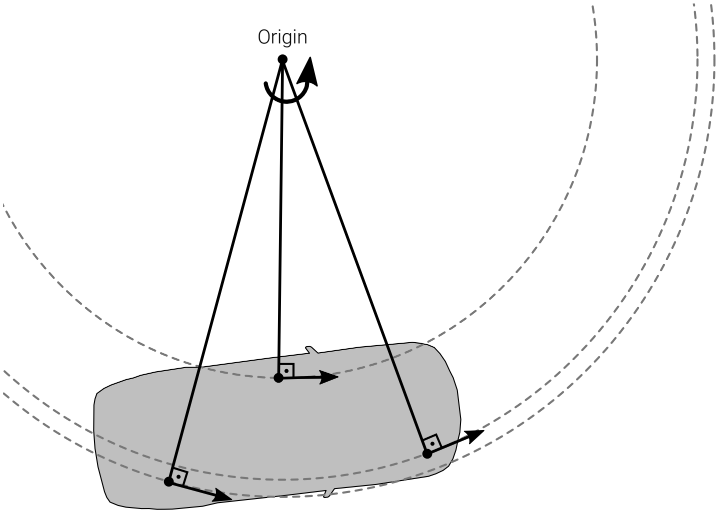


Holonomic vs. Non-Holonomic Systems



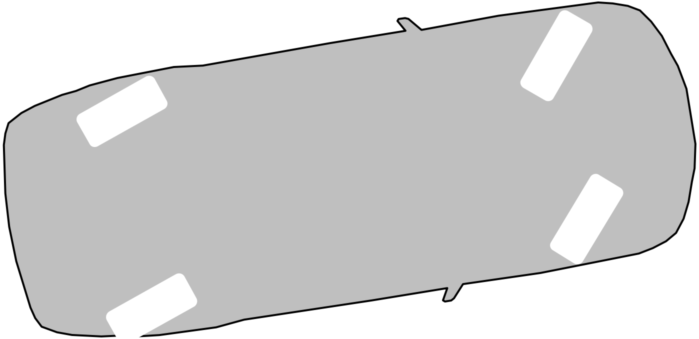
Lateral Vehicle Dynamics

Rigid Body Motion

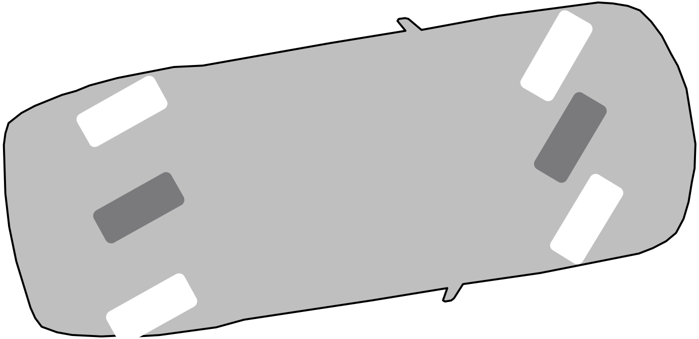


Kinematic Bicycle Model

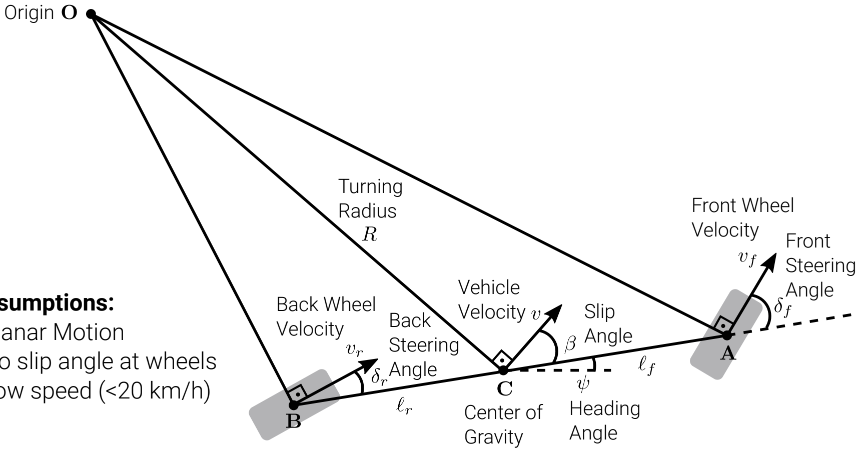
Kinematic Bicycle Model



Kinematic Bicycle Model



Kinematic Bicycle Model

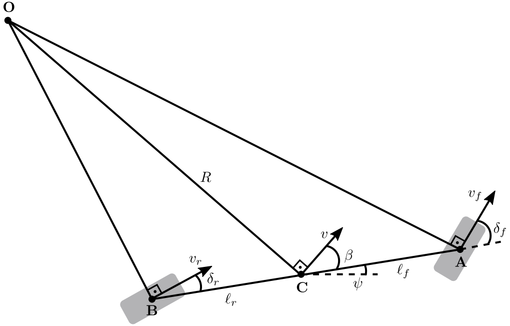


Assumptions:
- Planar Motion
- No slip angle at wheels
- Low speed (<20 km/h)

Wheelbase $L = \ell_r + \ell_f$ Course Angle $\psi + \beta$

Kinematic Bicycle Model

Model



Motion Equations

$$\dot{x} = v\cos(\psi + \beta)$$

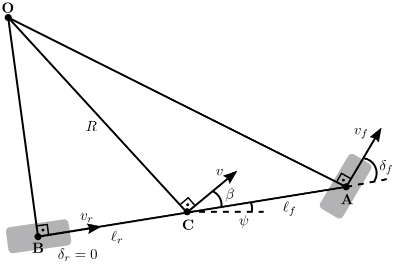
$$\dot{y} = v\sin(\psi + \beta)$$

$$\dot{\psi} = \frac{v\cos(\beta)}{\ell_f + \ell_r}\left(\tan(\delta_f) - \tan(\delta_r)\right)$$

$$\beta = \tan^{-1}\left(\frac{\ell_f \tan(\delta_r) + \ell_r \tan(\delta_f)}{\ell_f + \ell_r}\right)$$

Kinematic Bicycle Model

Model



Motion Equations

$$\dot{x} = v\cos(\psi + \beta)$$

$$\dot{y} = v\sin(\psi + \beta)$$

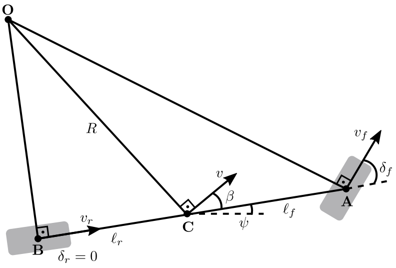
$$\dot{\psi} = \frac{v\cos(\beta)}{\ell_f + \ell_r}\tan(\delta_f)$$

$$\beta = \tan^{-1}\left(\frac{\ell_r \tan(\delta_f)}{\ell_f + \ell_r}\right)$$

(only front steering)

Kinematic Bicycle Model

Model



Motion Equations

$$\dot{x} = v\cos(\psi)$$

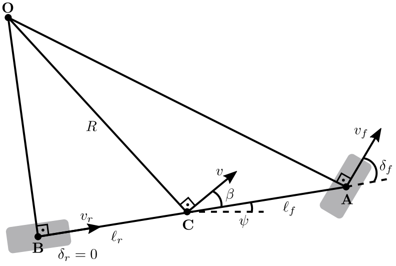
$$\dot{y} = v\sin(\psi)$$

$$\dot{\psi} = \frac{v\delta_f}{\ell_f + \ell_r}$$

(assuming β and δ_f are very small)

Kinematic Bicycle Model

Model



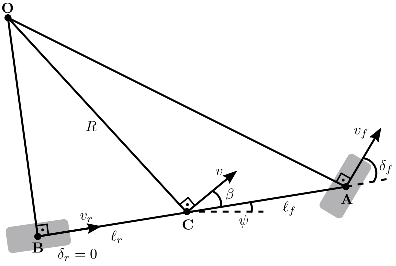
Motion Equations

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{\delta}_f \end{bmatrix} = \begin{bmatrix} \cos(\psi) \\ \sin(\psi) \\ \frac{\delta_f}{\ell_f + \ell_r} \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dot{\delta}_f$$

(two input form)

Kinematic Bicycle Model

Model



Motion Equations

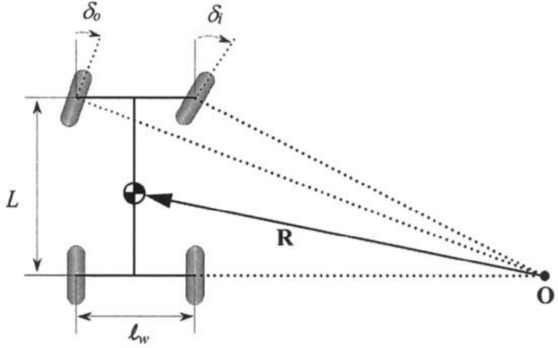
$$x_{t+1} = x_t + v\cos(\psi)\,\Delta t$$

$$y_{t+1} = y_t + v\sin(\psi)\,\Delta t$$

$$\psi_{t+1} = \psi_t + \frac{v\delta_f}{\ell_f + \ell_r}\,\Delta t$$

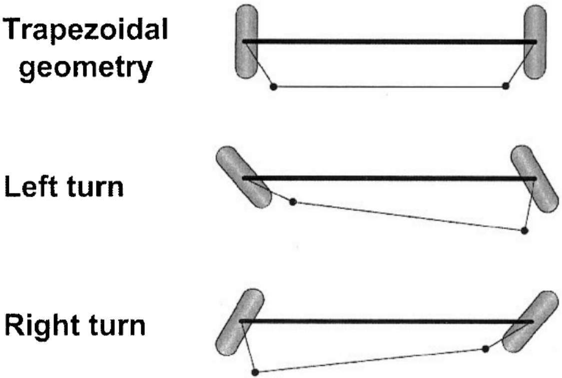
(time discretized model)

Ackermann Turning Geometry



- In practice, left and right wheel angles are not equal
- If slip angle β is small, we have: $\frac{\dot{\psi}}{v} \approx \frac{1}{R} = \frac{\delta_f}{L} \;\; \Rightarrow \;\; \delta_f = \frac{L}{R}$
- Thus the outer/inner wheel angles are given by $\delta_o = \frac{L}{R+0.5\ell_w}$ and $\delta_i = \frac{L}{R-0.5\ell_w}$

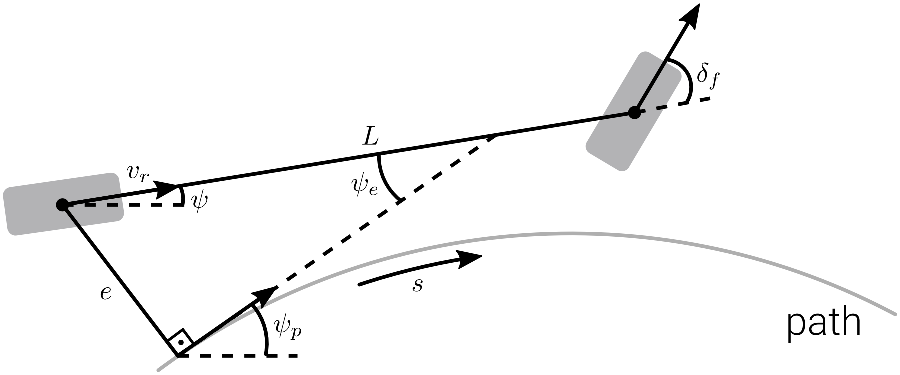
Ackermann Turning Geometry



- Difference of angles is proportional to square of avg. angle: $\delta_i - \delta_o = \delta^2 \frac{\ell_w}{L}$
- In practice, this setup is be realized using a trapezoidal tie rod arrangement

Kinematic Bicycle Model wrt. Desired Path

Kinematic Bicycle Model wrt. Desired Path



- Define the cross-track error as e and the orientation error as $\psi_e = \psi - \psi_p$
- The curvature along a path is given as $c(s) = \partial\psi_p(s)/\partial s$, thus: $\dot{\psi}_p(s) = c(s)\dot{s}$
- Furthermore we have: $\dot{e} = v_r \sin(\psi_e)$ and $\dot{s} = v_r \cos(\psi_e) + \dot{\psi}_p e$

Kinematic Bicycle Model wrt. Desired Path

Substituting (s, e, ψ_e) for (x, y, ψ) into the Kinematic bicycle model equation yields:

$$\begin{bmatrix} \dot{s} \\ \dot{e} \\ \dot{\psi}_e \\ \dot{\delta}_f \end{bmatrix} = \begin{bmatrix} \frac{\cos(\psi_e)}{1-e\,c(s)} \\ \sin(\psi_e) \\ \frac{\delta_f}{L} - \frac{c(s)\cos(\psi_e)}{1-e\,c(s)} \\ 0 \end{bmatrix} v_r + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dot{\delta}_f$$

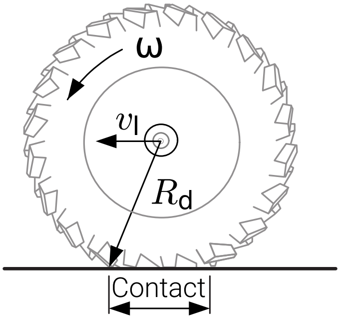
Notation:

- Path length s and path curvature $c(s)$
- Wheelbase $L = \ell_r + \ell_f$
- Front steering angle δ_f and rear wheel speed v_r
- Cross track error e
- Orientation error $\psi_e = \psi - \psi_p$

Kinematics is not enough ..

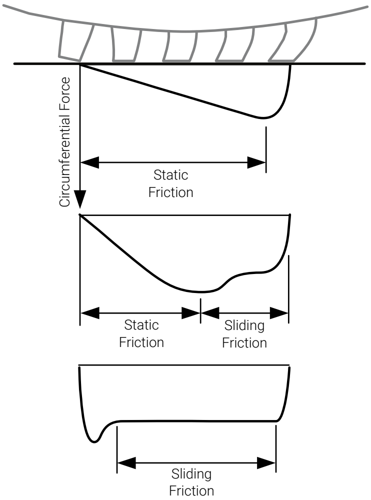
Which assumption of our model is violated in this case?

Tire Model



- Tire models describe the lateral and longitudinal forces at the tires
- There exist many different tire models at various levels of complexity
- For a qualitative description we consider the tire tread blocks
- For a quantitative description, the model by Pacejka is often used

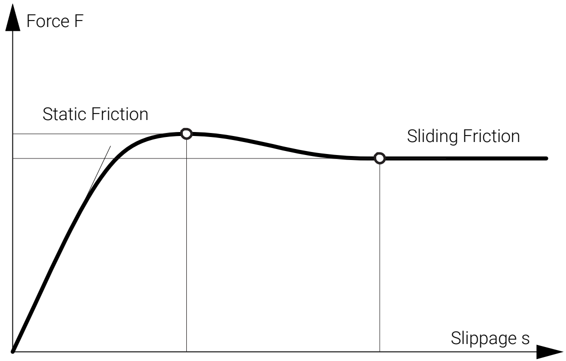
Tire Model



Longitudinal Force:

- As soon as the wheel is driven externally, the tire tread blocks start deforming and slipping
- The tire tread blocks adhere to the ground, deform and slip when loosing contact
- When the driving force increases and static friction is exceeded the blocks slip earlier
- As sliding friction is smaller than static friction, this decreases the transmitted driving force
- If the tire tread blocks start sliding at the beginning, only sliding friction can be applied

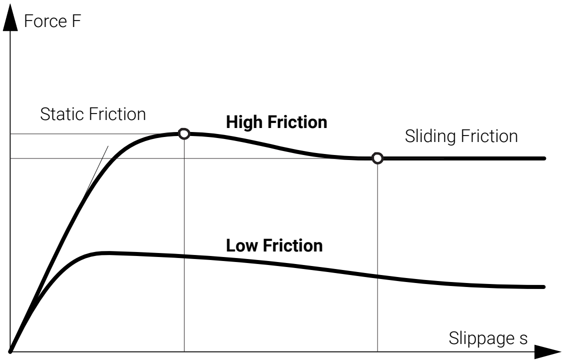
Tire Model



The tread block model explains:

- Why larger contact forces and dry ground lead to higher transmitted driving forces
- Why the force F grows linearly with the slippage s in the beginning (linear deform.)
- Why large slippage s leads to a reduction of F (sliding friction ¡ static friction)

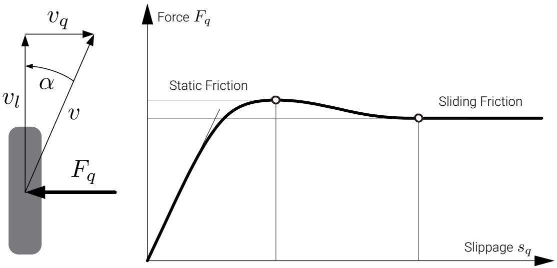
Tire Model



Quiz: How does the force curve $F(s)$ change for slippery terrain (low friction)?

- Start of the curve doesn't change as the elasticity of the blocks doesn't change
- However, the maximum reduces due to the decreased static friction, i.e., the tread blocks start sliding earlier due to a decrease in friction

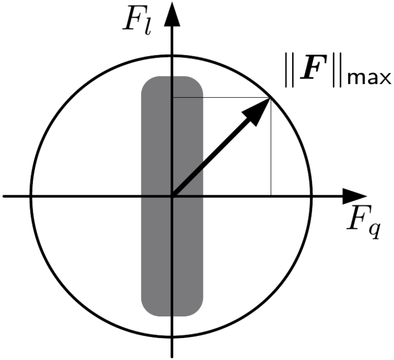
Tire Model

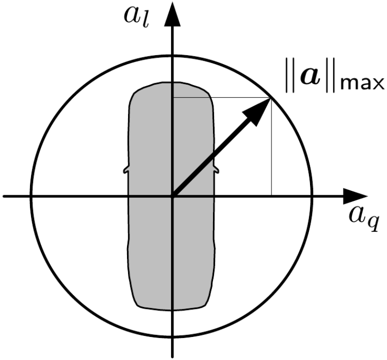


Lateral Force:

- Lateral force F_q analogous to longitudinal force F_l but blocks move laterally now
- Lateral force for small s and α given by: $F_q = c\, s_q = c\, \arctan(\alpha) \approx c\, \alpha$
- Here: v = wheel velocity, v_l = longitudinal wheel velocity, v_q = lateral wheel velocity

Tire Model



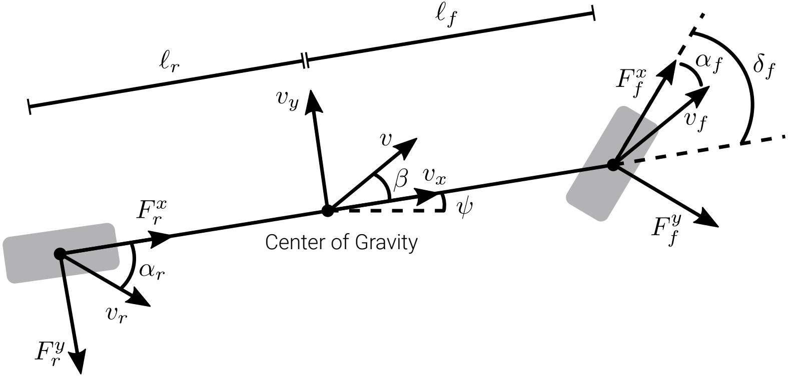


Circle of Forces:

- Lateral F_q and longitudinal F_l force cannot exceed max. friction force $\|F\|_{max}$
- More long. force implies less lat. force; max. acceleration only for straight driving
- Allows to make statements about maximal possible vehicle accelerations

Dynamic Bicycle Model

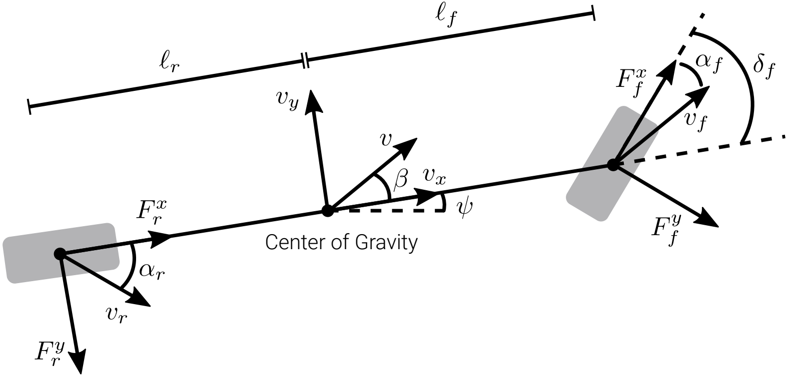
Dynamic Bicycle Model



$F = m\,a$ (force = mass x acceleration)

$\tau = I\,\ddot{\psi}$ (torque = moment of inertia x angular acceleration)

Dynamic Bicycle Model



$$F_f^y \cos(\delta_f) - F_f^x \sin(\delta_f) + F_r^y = m(\dot{v}_y + v_x \dot{\psi})$$

$$\ell_f(F_f^y \cos(\delta_f) - F_f^x \sin(\delta_f)) - \ell_r F_r^y = I_z \ddot{\psi}$$

Dynamic Bicycle Model

Exploiting the small angle assumption, this yields the following state space model:

$$\begin{bmatrix} \dot{e} \\ \ddot{e} \\ \dot{\psi}_e \\ \ddot{\psi}_e \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{c_f+c_r}{mv_x} & \frac{c_f+c_r}{m} & \frac{\ell_r c_r - \ell_f c_f}{mv_x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{\ell_r c_r - \ell_f c_f}{I_z v_x} & \frac{\ell_f c_f - \ell_r c_r}{I_z} & -\frac{\ell_f^2 c_f + \ell_r^2 c_r}{I_z v_x} \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \\ \psi_e \\ \dot{\psi}_e \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{c_f}{m} \\ 0 \\ \frac{\ell_f c_f}{I_z} \end{bmatrix} \delta_f + \begin{bmatrix} 0 \\ \frac{\ell_r c_r - \ell_f c_f}{mv_x} - v_x \\ 0 \\ -\frac{\ell_f^2 c_f + \ell_r^2 c_r}{I_z v_x} \end{bmatrix} r(s)$$

With:

- e, ψ_e: Orthogonal distance of vehicle center to path, orientation error $\psi_e = \psi - \psi_p$
- $r(s)$: Yaw rate of path $r(s) = c(s) v_x = \frac{v_x}{R}$
- m, I_z: Mass & moment of inertia of vehicle
- c_f, c_r: Cornering stiffness parameters (assumption: $F_f^y = c_f \alpha_f$, $F_r^y = c_r \alpha_r$)
- Parameters $m, I_z, \ell_f, \ell_r, c_f, c_r$ must be identified (see, e.g., Snider 2009)

Further Readings

- Rajamani: Vehicle Dynamics and Control. Springer, 2006.
- Snider: Automatic Steering Methods for Autonomous Automobile Path Tracking. CMU-RI-TR, 2009.
- Kong, Pfeiffer, Schildbach and Borrelli: Kinematic and dynamic vehicle models for autonomous driving control design. IV, 2015.

Questions?