

5)

$$(a) \lim_{n \rightarrow \infty} \frac{3n}{n+2} = 3.$$

proof:

Fix $\epsilon \in (0, \infty)$, arbitrary.Choose $N := \lceil 6/\epsilon - 2 \rceil + 1 \in \mathbb{N}$.Fix $n \in \mathbb{N}$.Assume $n \geq N$.

$$n \geq \lceil 6/\epsilon - 2 \rceil + 1$$

$$n > 6/\epsilon - 2 + 1$$

$$n > 6/\epsilon - 1$$

$$n+2 > 6/\epsilon$$

$$\epsilon > 6/(n+2)$$

$$\epsilon > | -6/(n+2) |$$

$$\epsilon > \left| \frac{3n}{n+2} - 3 \right|$$

$$|a_n - A| < \epsilon$$

$$\left| \frac{3n}{n+2} - 3 \right| < \epsilon$$

$$| -6/(n+2) | < \epsilon$$

$$6/(n+2) < \epsilon$$

$$6/\epsilon < n+2$$

$$6/\epsilon - 2 < n$$

$$N := \lceil 6/\epsilon - 2 \rceil + 1 \in \mathbb{N}.$$

By definition $\underline{\lim}_{n \rightarrow \infty} a_n$ converges to 3.