

5)

$$(a) \lim_{n \rightarrow \infty} \frac{3n}{n+2} = 3.$$

proof:

Fix  $\varepsilon \in (0, \infty)$ , arbitrary.

Choose  $N := \lceil 6/\varepsilon - 2 \rceil + 1 \in \mathbb{N}$ .

Fix  $n \in \mathbb{N}$ .

Assume  $n \geq N$ .

$$n \geq \lceil 6/\varepsilon - 2 \rceil + 1$$

$$n > 6/\varepsilon - 2 + 1$$

$$n > 6/\varepsilon - 1$$

$$n+2 > 6/\varepsilon$$

$$\varepsilon > 6/(n+2)$$

$$\varepsilon > |-6/(n+2)|$$

$$\varepsilon > \left| \frac{3n}{n+2} - 3 \right|$$

$$|a_n - A| < \varepsilon$$

$$\left| \frac{3n}{n+2} - 3 \right| < \varepsilon$$

$$|-6/(n+2)| < \varepsilon$$

$$6/(n+2) < \varepsilon$$

$$6/\varepsilon < n+2$$

$$6/\varepsilon - 2 < n.$$

$$N := \lceil 6/\varepsilon - 2 \rceil + 1 \in \mathbb{N}.$$

By definition  $\frac{3n}{n+2}$  converges to 3.