

- ③ Let  $n, m \in \mathbb{Z}$ . IF  $n^2(m+3)$  is even then  $n$  is even or  
(a)  $m$  is odd.

(Contraposition)

Proof: Assume that  $n$  is odd and  $m$  is even.

We want to show that  $n^2(m+3)$  is odd follows.

Observe that  $n = 2k_1 + 1$  for some  $k_1 \in \mathbb{Z}$  and

$m = 2k_2$  for some  $k_2 \in \mathbb{Z}$ .

Then it follows that  $n^2(m+3) = (2k_1 + 1)^2(2k_2 + 3)$ .

$$\begin{aligned} \text{Note that } (2k_1 + 1)^2(2k_2 + 3) &= (4k_1^2 + 4k_1 + 1)(2k_2 + 3) \\ &= 8k_1^2k_2 + 12k_1^2 + 8k_1k_2 + 12k_1 + \end{aligned}$$

$$2k_2 + 3$$

$$= 2(4k_1^2k_2 + 6k_1^2 + 4k_1k_2 + 6k_1 + k_2 + 1) + 1$$

Since  $4k_1^2k_2 + 6k_1^2 + 4k_1k_2 + 6k_1 + k_2 + 1 \in \mathbb{Z}$ , then we

have that  $n^2(m+3)$  is odd as desired

