(3) Let $f: A \to B$ be a function. Prove the following.

(a) Let $C, D \subseteq B$. Then $f^{-1}(C) \cup f^{-1}(D) = f^{-1}(C \cup D)$

First let's show that if $f^{-1}(C) \cup f^{-1}(D)$ then $f^{-1}(C \cup D)$. Assume that $f^{-1}(C) \cup f^{-1}(D)$ holds for some element x. We want to show that $x \in f^{-1}(C \cup D)$.
Note that $f(p_1) = C$ and $f(p_2) = D$. ~~If x~~
We also know that if x is in the union of the inverses, then $x \in p_1 \cup x \in p_2$. However ~~if we take~~ this means that $x \in p_1 \cup p_2$, which also means $x \in f^{-1}(C \cup D)$.
Now let's show that if $f^{-1}(C \cup D)$ then $f^{-1}(C) \cup f^{-1}(D)$. Assume $x \in f^{-1}(C \cup D)$. This means $x \in p_1 \cup p_2$ where $p_1, p_2 \in A$. If this holds then $x \in p_1 \cup x \in p_2$ also holds.
However this is equivalent to $x \in f^{-1}(C) \cup x \in f^{-1}(D)$.

Since double containment holds, we have the original statement is true.