בס"ד

2) fibanaci:

for $n=1$, $gcd(f_1, f_2) = gcd(1,1) = 1$

base case $n=2$

$gcd(F_2, F_3) = gcd(1,2) = 1$ hence base case is true

inductive step: Assume true for some $n >= 2$ show also true for $n+1$

$F_{n+1} = F_n + F_{n-1}$

$F_{n+2} = F_{n+1} + F_n = 2F_n + F_{n-1}$ let,

$gcd(F_{n+2}, F_{n+1}) = g > 1$ so,

$F_n + F_{n-1} = mg$, $2F_n + F_{n-1} = ng$

$\Rightarrow$ $F_n = (n-m)g$, $F_{n-1} = (2m-n)g$

$\Rightarrow$ $gcd(F_n, F_{n-1}) \geq g > 1$ which is a contradiction since by inductive hypothesis

$gcd(F_n, F_{n-1}) = 1$ therefore $gcd(F_n, F_{n-1}) = 1 \; \forall n \in \mathbb{N}$

1) base: $7^1 - 1 = 6$ which is divisible by 6 so $P(1)$ is true

assume $P(k)$ is true, $P(k) = 7^k - 1 = 6(m)$ where m is some integer

Show $P(k+1)$ is true $7^{k+1} - 1 = (7)7^k - 1 = 7(6m+1) - 1$

$= 42m + 7 - 1 = 42m + 6 = 6(7m+1)$ which shows that $P(k+1)$ is also divesible by 6

3) a: let n be an integer Suppose $a | n$ and $a | (n+1)$. Then $as = n$ and $at = n+1$ for some $s, t \in \mathbb{Z}$. But $1 = (n+1) - n = at - as$ $= a(t-s)$ therefore a divides 1 therefore the only common divisor of n and $n+1$ are 1 and -1, a prime cant divide both n and $n+1$

3/0?

3b Suppose there are a finite number of primes $P_1, P_2, \ldots, P_n$. The number $M = (P_1 P_2 \cdots P_{N+1})$ must have a prime factor which will be one of the primes listed. But then we would have $P_i \mid M$ and $P_i \mid P_1 P_2 \ldots P_N$ contradicting there are infinitly many primes

4a) $7/4 = 1R3$

$4/3 = 1R1$

$3/1 = 3R0$

- This shows by using the Euclidean Algo that 7 and 4 are relativly prime
- according to Eulers totient thm. that $k^{\phi(n)} \equiv 1 \bmod n$ shows that $5^{\phi(28)} \equiv 1 \bmod 28$ and we know that 28 is equivlant to $7 \cdot 4$ and because both are relativly prime $\phi(28) = \phi(7) \cdot \phi(4)$
- $\phi(7) = 7^1 \cdot 7^0 = 7-1 = 6$
- $\phi(4) = 2^2 - 2^1 = 4-2 = 2$
- $\phi(28) = 6 \cdot 2 = 12$
- $5^{12} \equiv 1 \pmod{28}$
- now divide

$$\begin{array}{r} 85 \; R4 \\ 12 \overline{)1024} \\ 96 \\ \hline 64 \\ 60 \end{array}$$

- therefore $5^{1024} = 5^{12^{85}} \cdot 5^4$ which can be rewritten as $5^4 \pmod{28}$ which leaves you with a remainder of 9 when you divide so $5^{1024} \pmod{28} = 9$