Reachable Configurations A configuration is reachable from another configuration, if the former may be required for the evaluation of the latter after any number of steps.

$$\mathcal{R}each \; : \; Module \times state \times Stmts \longrightarrow \mathcal{P}(Stmts \times state)$$

In figure 2 we define the function $\mathcal{R}each$ by cases on the structure of the expression, and depending on the execution of the statement. The set $\mathcal{R}each(M, \sigma, \texttt{stmts})$ collects all configurations reachable during execution of $\sigma, \texttt{stmts}$. Note that the function $\mathcal{R}each(M, \sigma, \texttt{stmts})$ is defined, even when the execution should diverge; of course then it may be an infinite set. The definedness of $\mathcal{R}each(M, \sigma, \texttt{stmts})$ is important, because it allows us to give meaning to capability policies without requiring termination.

Lemma 3 ($\mathcal{R}each$ and $\leadsto$)**.** *For all M, M', σ, σ', σ',and $\texttt{stmt}$, $\texttt{stmt}'$, and $\texttt{stmt}''$:*

- *If $M, \sigma, \texttt{stmt} \leadsto \sigma'$, then $(\_, \sigma') \in \mathcal{R}each(M, \sigma, \texttt{stmt})$.*
- *If $(\texttt{stmt}', \sigma') \in \mathcal{R}each(M, \sigma, \texttt{stmt})$, and $(\texttt{stmt}'', \sigma'') \in \mathcal{R}each(M, \sigma', \texttt{stmt}')$, then $(\texttt{stmt}''', \sigma'') \in \mathcal{R}each(M, \sigma, \texttt{stmt})$.*
- *If $M{*}M'$ is defined, and $(\texttt{stmt}', \sigma') \in \mathcal{R}each(M, \sigma, \texttt{stmt})$, then $(\texttt{stmt}', \sigma') \in \mathcal{R}each(M * M', \sigma, \texttt{stmt})$.*
- *If $M * M'$ is defined, then $\mathcal{A}rising(M) \subseteq \mathcal{A}rising(M * M')$.*

Proof By structural induction on $\leadsto$ and the definition of $\mathcal{R}each$ and $\mathcal{A}rising$. $\square$

Notation We shall use $\sigma' \in \mathcal{R}each(M, \sigma, \texttt{stmt})$ as a shorthand for $(\_, \sigma') \in \mathcal{R}each(M, \sigma, \texttt{stmt})$ and $\sigma' \in \mathcal{A}rising(M)$ as a shorthand for $(\_, \sigma') \in \mathcal{A}rising(M)$.