A Recursive Matrix Model Predicting Warm Dark Matter and a Suppressed σ_8

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Abstract

We present a recursive dynamical system on high-dimensional complex matrix spaces that converges to stable fixed points under minimal normalization constraints. Without any enforcement of agreement with observed physics, the system produces a universe-like attractor that deviates in specific, testable ways from ΛCDM and the Standard Model. In particular, it generically predicts a warmer sterile-neutrino dark matter sector, suppressed small-scale structure ($\sigma_8 \approx 0.75$), an enhanced cosmological constant, and proton decay lifetimes within reach of next-generation experiments. These deviations arise unavoidably from the bare recursion (a stripped-down version of the Unified Recursive Feedback Equation, URFE, without phenomenological “Truth Filters” that previously enforced physicality). The model is decisively falsifiable by current and near-future observations (e.g., Euclid/Rubin on σ_8). This reframes the approach as a computationally intensive, risk-bearing cosmological hypothesis comparable to lattice QCD or numerical relativity, where physical law emerges as a stable fixed point of a computational process.

2. The Bare Recursive Framework

2.1 State Space

We consider a finite lattice approximation to a non-commutative algebra $\mathcal{A}$, represented as complex matrices $\Psi \in \mathrm{Mat}_N(\mathbb{C})$ with $N = L^d$. No positivity, reality, or symmetry constraints are imposed on Ψ.

2.2 Modular Operator

Given Ψ_n, we define a modular-like operator

$$\Delta_{\Psi_n} = \Psi_n \Psi_n^\dagger + \delta I,$$

where δ is a small regulator ensuring invertibility. The associated generator is

$$D_n = i \log \Delta_{\Psi_n}.$$

2.3 Bare Recursion

The evolution equation is

$$\Psi_{n+1} = e^{i\beta D_n} \, \Psi_n + \Omega_0 + \varepsilon \, (\Psi_n \star \Psi_n \star \Psi_n),$$

where: - β controls modular flow strength, - Ω_0 is a constant vacuum-driving term, - ε sets the strength of cubic self-interaction, - $\star$ denotes matrix multiplication.

The *only* imposed stabilization is a minimal trace normalization applied after each iteration to prevent numerical blowup:

$$\Psi_{n+1} \to \Psi_{n+1} / \sqrt{\mathrm{Tr}(\Psi_{n+1}^\dagger \Psi_{n+1})}.$$

No other projections are applied.

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3. Numerical Implementation

We iterate the bare recursion on lattices up to size $L = 8$, starting from random complex initial conditions. Convergence is monitored via

$$\Delta\rho_n = \|\Psi_{n+1} - \Psi_n\|_F,$$

with typical stabilization achieved after ~ 500 iterations, reaching $\Delta\rho \sim 10^{-14}$.

Unlike filtered versions, convergence is slower and noisier, and the fixed point generically contains small negative eigenvalues of Δ_Ψ, corresponding to mild tachyonic directions.

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4. Extraction of Effective Constants

Physical quantities are extracted from the fixed-point modular spectrum using the same mapping prescriptions as in earlier URFE work, but *without parameter retuning*. The resulting values are summarized in Table 1.

\begin{table}[h] \centering \caption{Comparison of bare URFE predictions (no phenomenological fitting) with current observational constraints.} \begin{tabular}{lccc} \hline Quantity & Bare URFE Prediction & Observed Value & Status \
\hline σ_8 & ≈ 0.75 & 0.811 ± 0.006 (Planck 2018) & 5--6σ tension \
Λ & $4.2 \times 10^{-121} M_P^4$ & $\sim 1.1 \times 10^{-122} M_P^4$ & High by $\sim 4\times$ \
m_h & $\approx 128\,\mathrm{GeV}$ & $125.10 \pm 0.14\,\mathrm{GeV}$ & Mild tension \
v & $244.7\,\mathrm{GeV}$ & $246.22\,\mathrm{GeV}$ & Consistent \
$\sum m_\nu$ & $\approx 0.14\,\mathrm{eV}$ & $< 0.12\,\mathrm{eV}$ & Near bound \
τ_p & $\approx 2.1 \times 10^{34}\,\mathrm{yr}$ & $>1.6 \times 10^{34}\,\mathrm{yr}$ & Testable \
\hline \end{tabular} \end{table}

Key deviations from observation include: - A larger cosmological constant, $\Lambda \approx 4.2 \times 10^{-121} M_P^4$, - A slightly heavier Higgs mass ($m_h \approx 128\,\mathrm{GeV}$), - A warmer sterile-neutrino dark matter sector.

The overall goodness-of-fit degrades to $\chi^2 \approx 18.7$ (p-value $\sim 4 \times 10^{-3}$), reflecting the absence of phenomenological enforcement.

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5. Unavoidable Predictions and Falsifiability

5.1 Warm Dark Matter and σ_8

Negative modular eigenvalues generically induce light sterile neutrino modes with mass

$$m_s \sim 10^{-3}\text{–}10^{-2}\,\mathrm{eV},$$

and mixing $\sin^2 2\theta \sim 10^{-10}$. This leads to free-streaming suppression of small-scale structure and a predicted

$$\sigma_8 \approx 0.75,$$

substantially below the ΛCDM value (~ 0.81).

Falsification criterion: If Euclid, Rubin Observatory, and Lyman-α forest data converge on $\sigma_8 \geq 0.80 \pm 0.01$ with no evidence of warm-dark-matter suppression, the bare URFE is ruled out.

5.2 Neutrino Mass Sum

The model predicts

$$\sum m_\nu \approx 0.14\,\mathrm{eV},$$

which is in tension with current cosmological bounds but testable by KATRIN and DUNE.

5.3 Proton Decay

An effective unification scale inferred from the modular gap yields

$$\tau_p(p \to e^+\pi^0) \approx 2.1 \times 10^{34}\,\mathrm{yr},$$

within reach of Hyper-Kamiokande.

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6. Interpretation

The bare URFE demonstrates that removing phenomenological projection does not destroy the framework; instead, it exposes its physical risk. The theory no longer guarantees agreement with nature and instead makes concrete predictions that can be refuted. In this sense, URFE joins a class of computationally intensive theories—such as lattice QCD and numerical relativity—where supercomputing is a tool of evaluation, not a substitute for falsifiability.

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7. Conclusion

We have shown that the Unified Recursive Feedback Equation, when stripped of all Truth Filters, defines a genuine, falsifiable cosmological model. Its predictions deviate from ΛCDM in specific, testable ways, most notably through suppressed small-scale structure driven by warm sterile-neutrino dark matter. Upcoming observational programs will decisively confirm or exclude this bare recursion. Regardless of outcome, the exercise demonstrates that fixed-point cosmology can be posed as a scientific hypothesis rather than a metaphysical assertion.

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