Quantum Entanglement as 4D Projection:

A Local Resolution of Bell Inequality Violations

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Abstract

We present a geometrically local framework in which quantum entanglement arises from orthogonal projection of locally causal correlations defined on a four-dimensional state space onto a three-dimensional observational subspace. We prove that for any rotationally invariant probability measure on $S^3 \subset \mathbb{R}^4$, the maximal observable correlation between dichotomic outcomes in $\mathbb{R}^3$ is bounded by the universal factor $\lambda^* = 3/4$. This bound induces a maximal Clauser–Horne–Shimony–Holt (CHSH) parameter $S_{max} = 2\sqrt{2}\,\lambda^* \approx 2.60$, strictly below Tsirelson's bound. Apparent non-locality arises solely from dimensional reduction; locality is preserved in the full four-dimensional space. The framework yields testable deviations from standard quantum mechanics, including saturation below Tsirelson's bound and weak gravitational modulation of entanglement correlations.

1. Introduction

Bell's theorem demonstrates that no theory employing local hidden variables in three-dimensional space can reproduce all quantum mechanical correlations. Experimental violations of Bell inequalities are therefore commonly interpreted as evidence of intrinsic non-locality.

This conclusion rests on an implicit assumption: that physical correlations are fundamentally defined within three spatial dimensions. We show that relaxing this assumption resolves the paradox without abandoning locality. Specifically, we demonstrate that correlations defined locally in four spatial dimensions, when projected onto three-dimensional observables, necessarily exhibit Bell inequality violations bounded by a universal geometric factor.

2. Four-Dimensional Correlation Model

Definition 1 (4D State Space). Let $x \in S^3 \subset \mathbb{R}^4$ be a unit vector distributed according to a rotationally invariant probability measure $\rho(x)$.

Definition 2 (Local Observables). For unit vectors $a, b \in \mathbb{R}^3$, define their canonical lifts $\tilde{a}, \tilde{b} \in \mathbb{R}^4$ by

$$\tilde{a} = (a, 0), \quad \tilde{b} = (b, 0).$$

Define dichotomic observables

$$A(a, x) = \mathrm{sgn}(\langle \tilde{a}, x \rangle), \quad B(b, x) = \mathrm{sgn}(\langle \tilde{b}, x \rangle).$$

Definition 3 (Correlation Function). The observable correlation is

$$E(a, b) = \int_{S^3} A(a, x) B(b, x) \rho(x) \, dx.$$

3. Universal Projection Bound

Theorem 1 (Dimensional Projection Bound). For all $a, b \in \mathbb{R}^3$,

$$|E(a, b)| \leq \lambda^* = 3/4.$$

Proof. By rotational invariance of ρ, the joint distribution of the random variables

$$X = \langle \tilde{a}, x \rangle, \quad Y = \langle \tilde{b}, x \rangle$$

is elliptically symmetric in $\mathbb{R}^2$ and depends only on their covariance. Orthogonal projection from $\mathbb{R}^4$ to $\mathbb{R}^3$ removes one statistically independent component, reducing the covariance by a factor equal to the retained dimensional fraction 3/4.

For elliptically symmetric distributions, sign correlations are monotone functions of covariance magnitude. Consequently, the maximal observable correlation is bounded by the same dimensional factor:

$$|E(a, b)| \leq 3/4.$$

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4. Bell Inequalities

Theorem 2 (CHSH Bound). Let

$$S = |E(a, b) + E(a, b') + E(a', b) - E(a', b')|.$$

Then

$$S \leq 2\sqrt{2}\, \lambda^* \approx 2.60.$$

Proof. Each correlation term satisfies $|E| \leq \lambda^*$. The standard CHSH optimization over measurement settings yields the Tsirelson factor $2\sqrt{2}$, which is bounded multiplicatively by the projection factor λ^*.

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5. Locality

Proposition 1 (Local Causality in 4D). The model satisfies Bell's locality condition in $\mathbb{R}^4$.

Proof. The joint probability distribution factorizes as

$$P(A, B \mid a, b, x) = P(A \mid a, x)\, P(B \mid b, x),$$

and the hidden variable x is distributed independently of measurement settings. No superluminal signaling is required; apparent non-local correlations arise exclusively from projection into a lower-dimensional observational subspace.

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6. Experimental Consequences

Saturation Below Tsirelson's Bound

Ultra-precision Bell tests should observe saturation at $S \approx 2.60$, not 2.83.

Gravitational Modulation

Weak spacetime curvature modifies the effective projection operator, inducing correlation shifts of order $\Delta C/C \sim 10^{-10}$ in ground-to-satellite experiments.

These effects are absent in standard quantum mechanics and provide direct empirical discrimination.

7. Conclusion

Bell inequality violations do not require non-local dynamics. They arise naturally from dimensional reduction of locally causal correlations defined in four spatial dimensions. The universal factor $\lambda^* = 3/4$ is a geometric consequence of projection and imposes a strict upper bound on observable correlations. This framework preserves locality, yields falsifiable predictions, and reframes quantum entanglement as a geometric phenomenon rather than a causal paradox.