Lecture 20: Red-Black Trees II

2023/10/11

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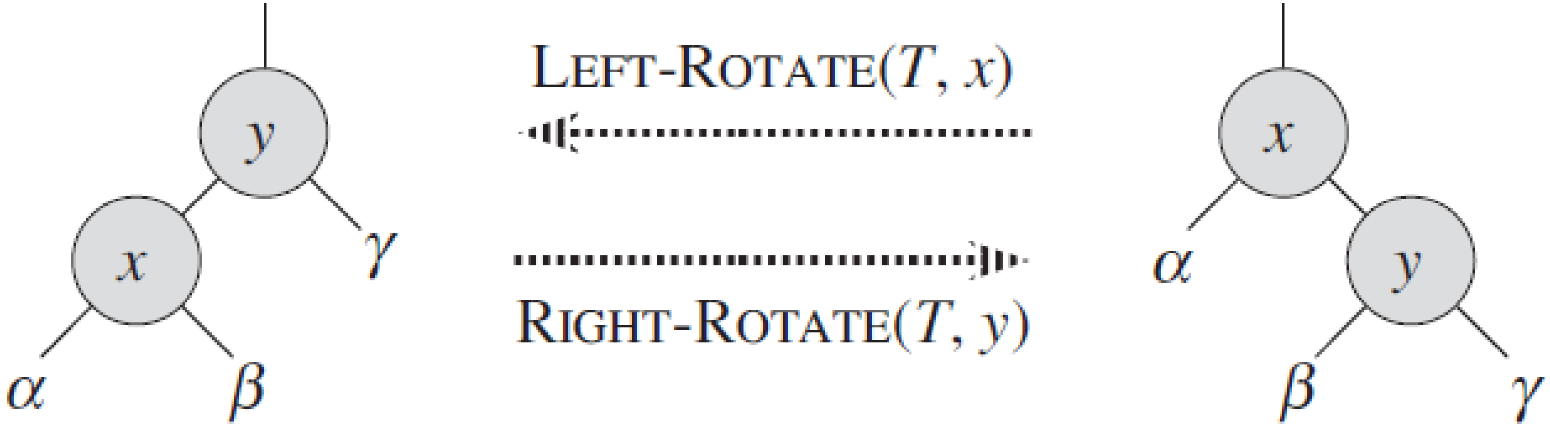
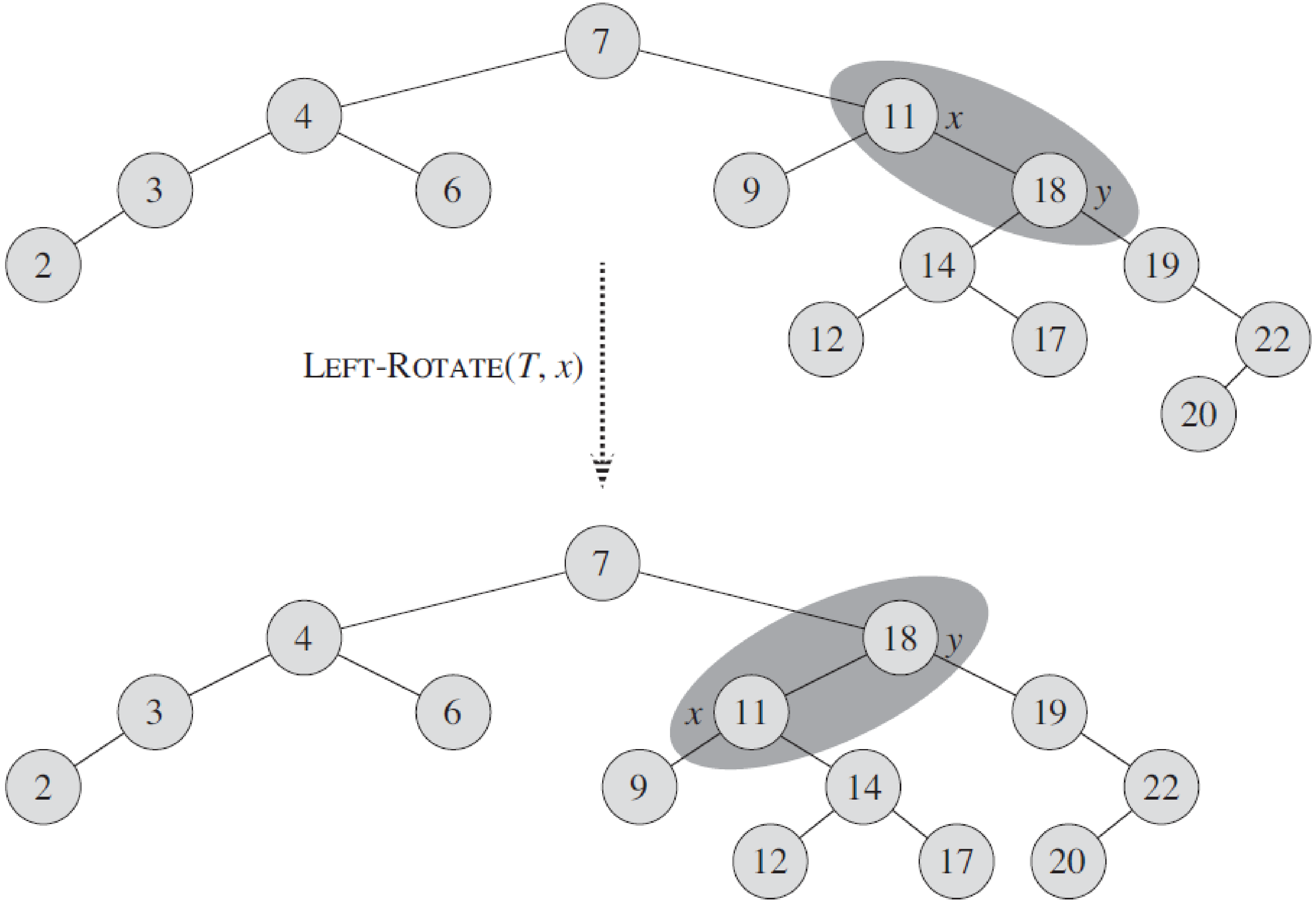
Rotations



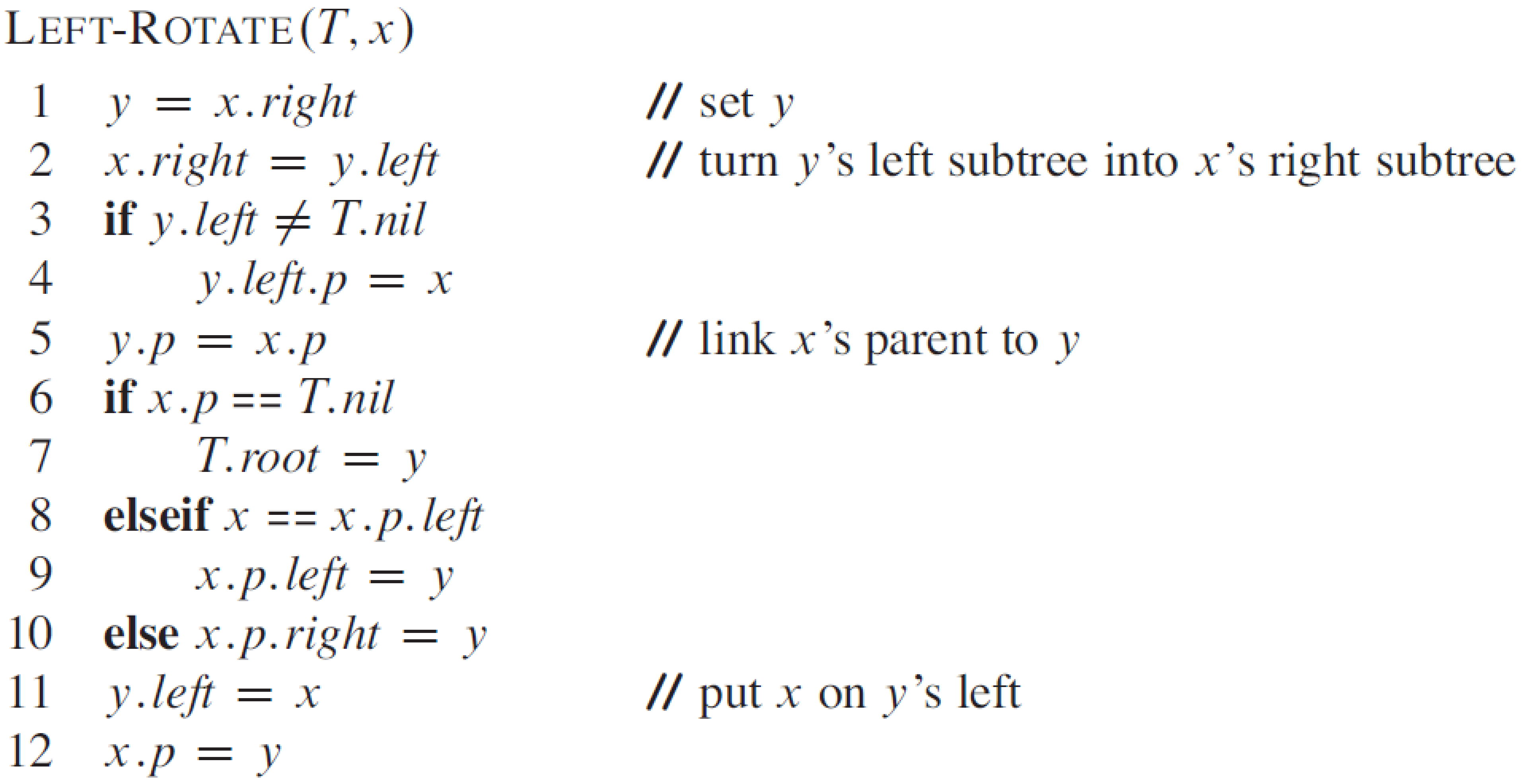
Note:

- Invariants of binary search trees are maintained.
- What is really maintained is the **inorder traversal** of the tree.

Rotations in Practice



Rotation: implementation



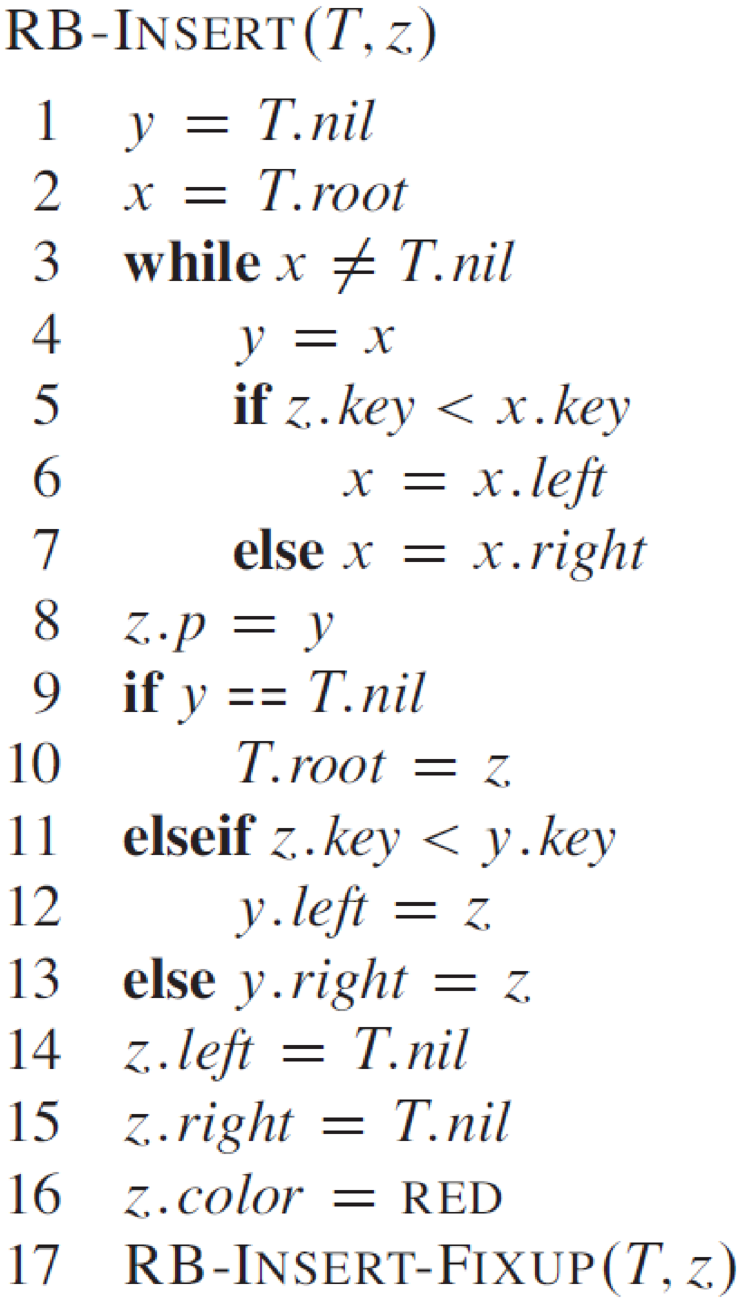
```
LEFT-ROTATE($T, x$)
$y = x.right$                  // set $y$
$x.right = y.left$             // turn $y$'s left subtree into $x$'s right subtree
if $y.left \neq T.nil$
    $y.left.p = x$
$y.p = x.p$                    // link $x$'s parent to $y$
if $x.p == T.nil$
    $T.root = y$
elseif $x == x.p.left$
    $x.p.left = y$
else $x.p.right = y$
$y.left = x$                   // put $x$ on $y$'s left
$x.p = y$
```

Red-Black Trees: insertion

- Initial steps same as insertion for usual BST.
- The newly inserted node is colored **red**.
- Then, fixup is performed to recover the invariant.

Implementation

- Line 1-13: same as insertion in usual BST.
- Line 14-16: set fields of z.
- Line 17: fixup operations (to be expanded).



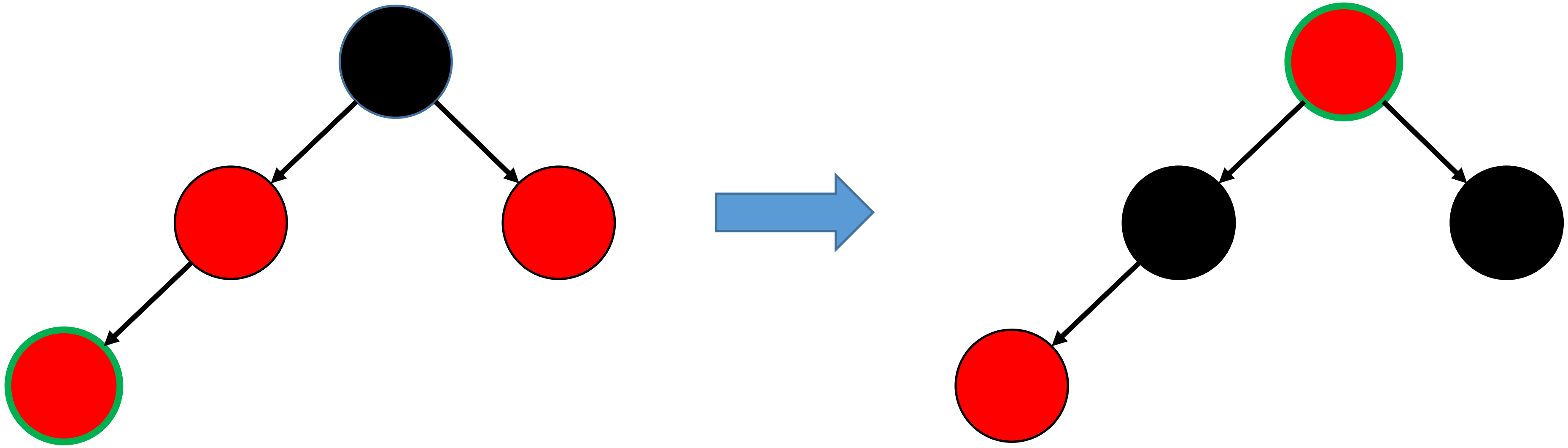
```
RB-INSERT(T, z)
 1  y = T.nil
 2  x = T.root
 3  while x ≠ T.nil
 4      y = x
 5      if z.key < x.key
 6          x = x.left
 7      else x = x.right
 8  z.p = y
 9  if y == T.nil
10      T.root = z
11  elseif z.key < y.key
12      y.left = z
13  else y.right = z
14  z.left = T.nil
15  z.right = T.nil
16  z.color = RED
17  RB-INSERT-FIXUP(T, z)
```

Fixup operation

- Starting from the bottom, where the new node is added.
- Proceed as long as there are two consecutive red nodes.
- Perform fixup so all invariant of the tree is maintained:
    1. No two consecutive reds.
    2. All paths have same number of black nodes.
    3. Order in binary search tree.
- Finally, if the root becomes red, recolor it black.

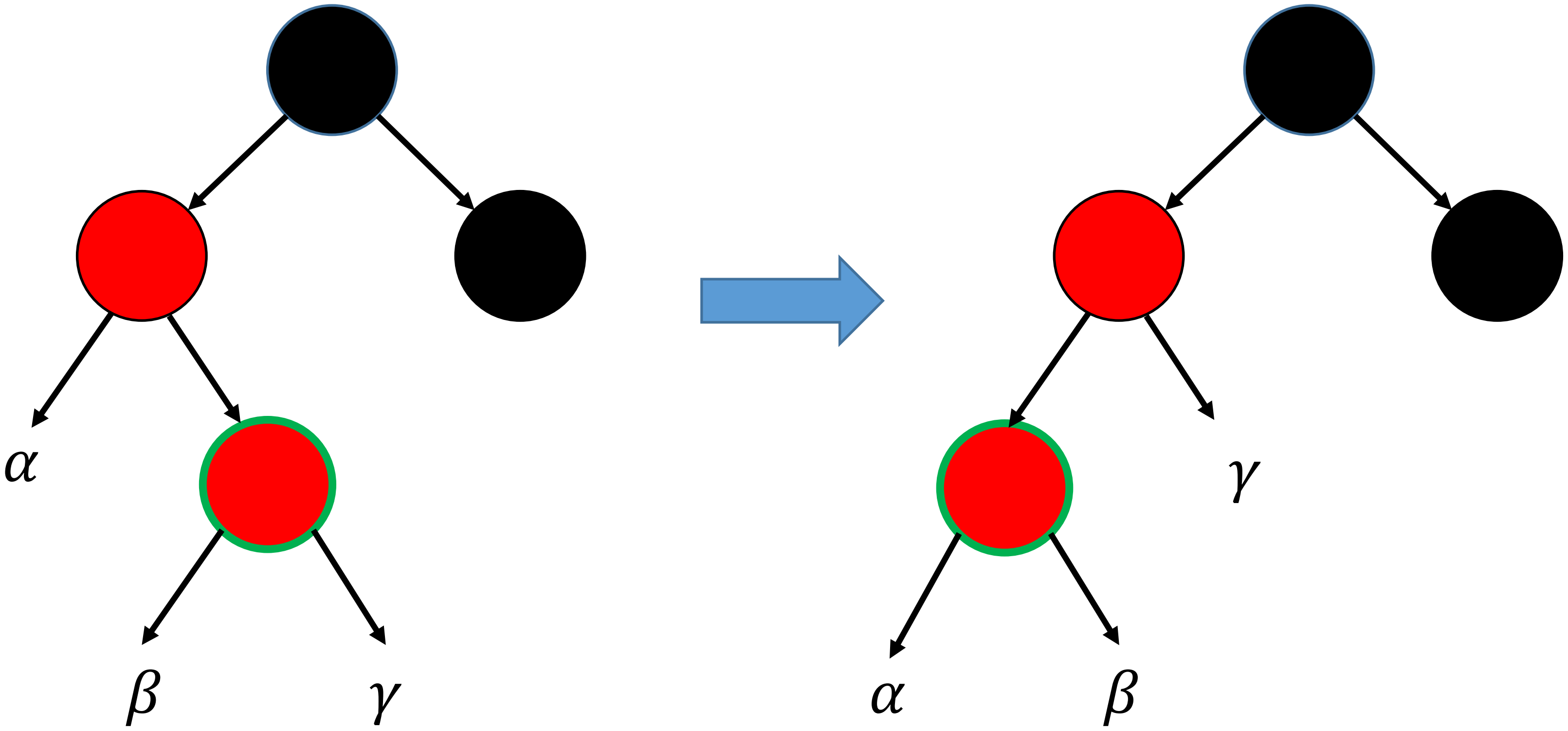
Fixup: case 1

- The uncle of the current node is also red.
  - Change parent and uncle to black, grandparent to red.



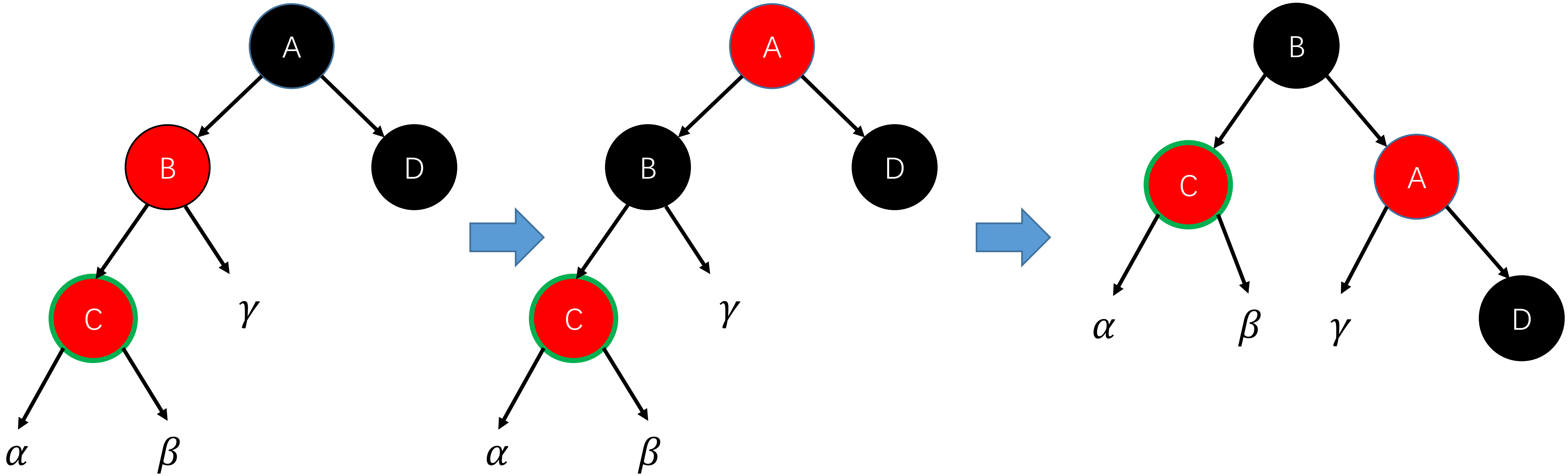
Fixup: case 2

- Uncle of current node is black. Current node is right child.
  - Perform left rotation on parent.



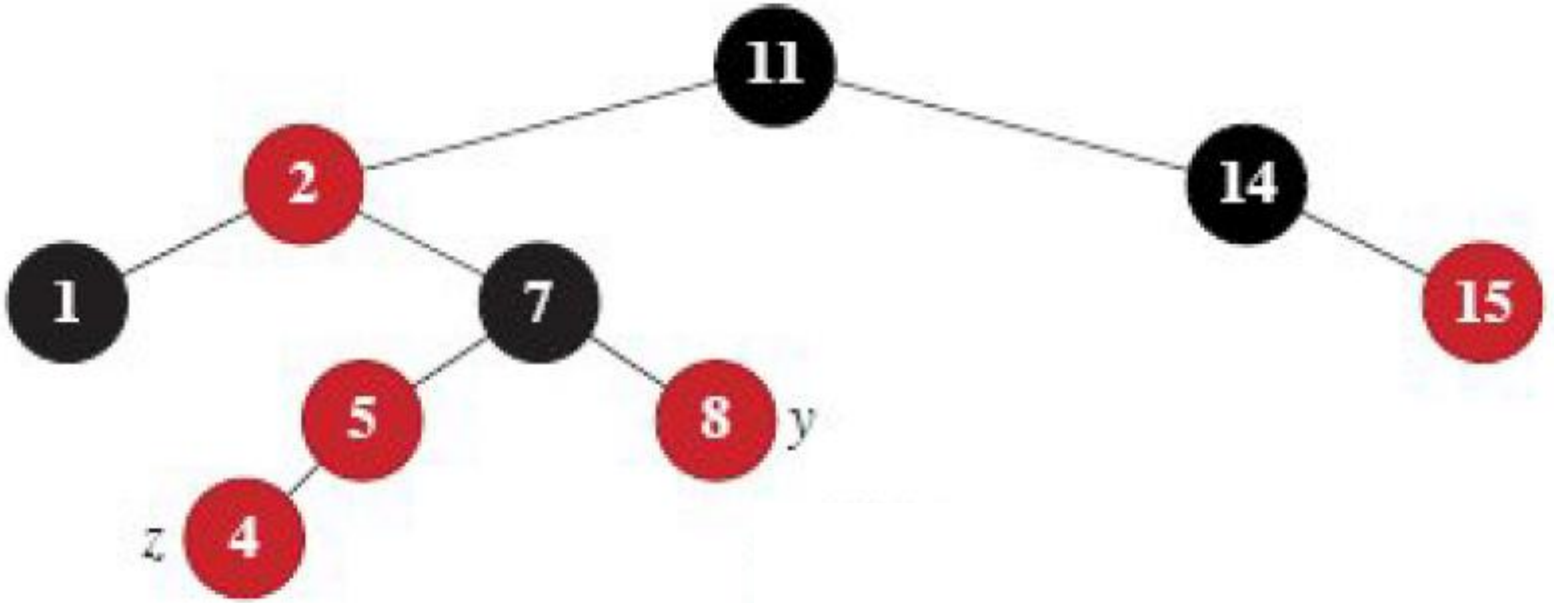
Fixup: case 3

- Uncle of current node is black. Current node is left child.
  - Recolor parent and grandparent.
  - Perform right rotation on grandparent.



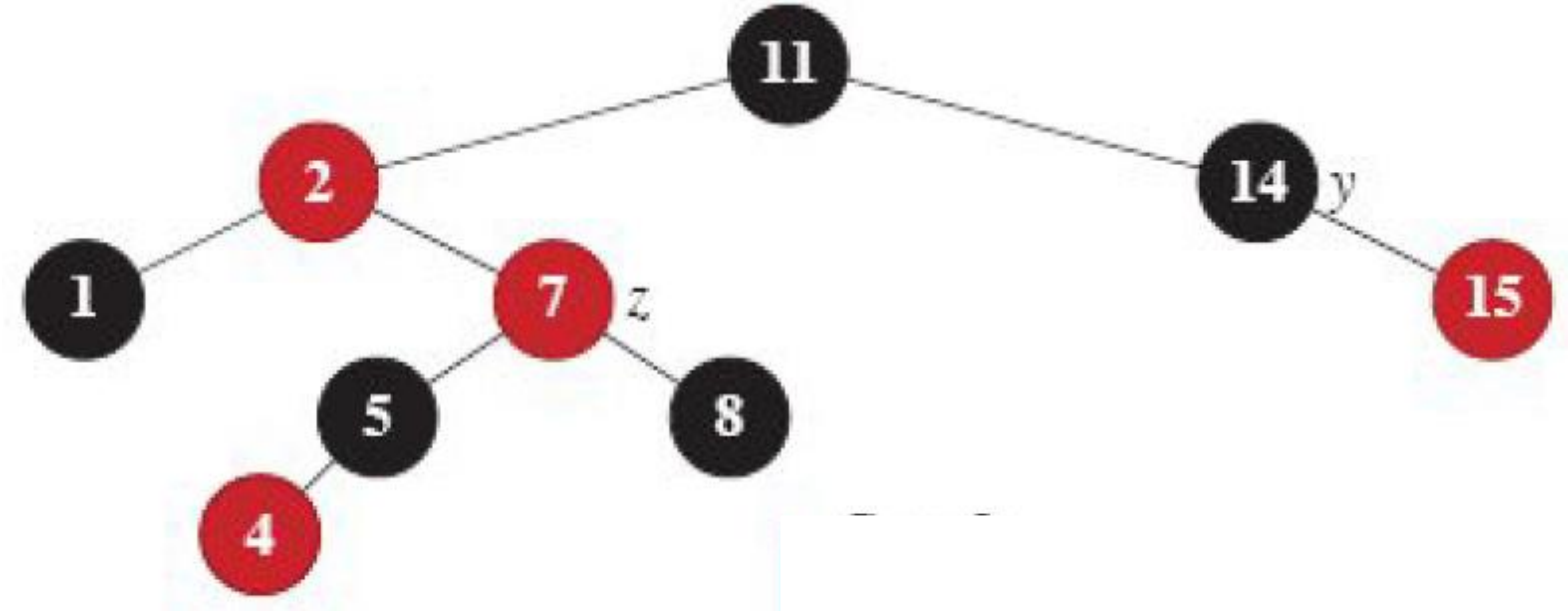
Concrete example

- Current node is z, its uncle y is red, so we are in case 1.
  - Recolor nodes 5, 7, and 8.



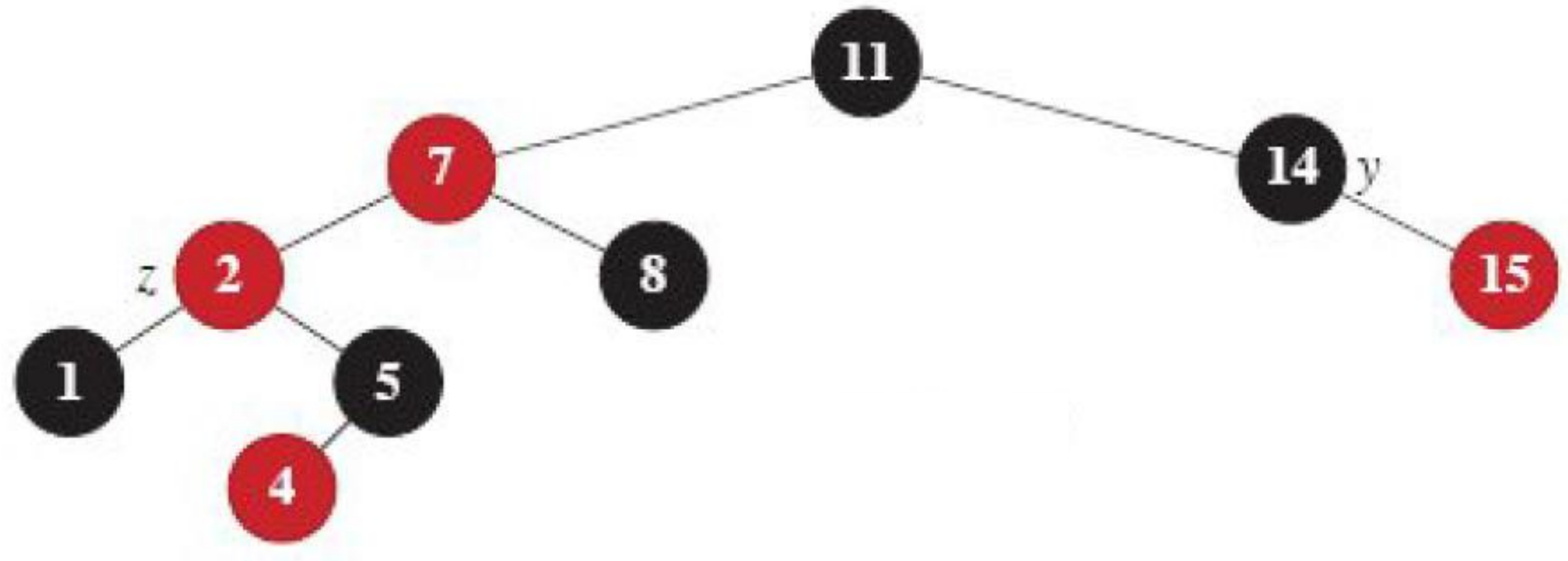
Concrete example

- Current node is z, its uncle y is black. z is right child, so case 2.
  - Perform left rotation.



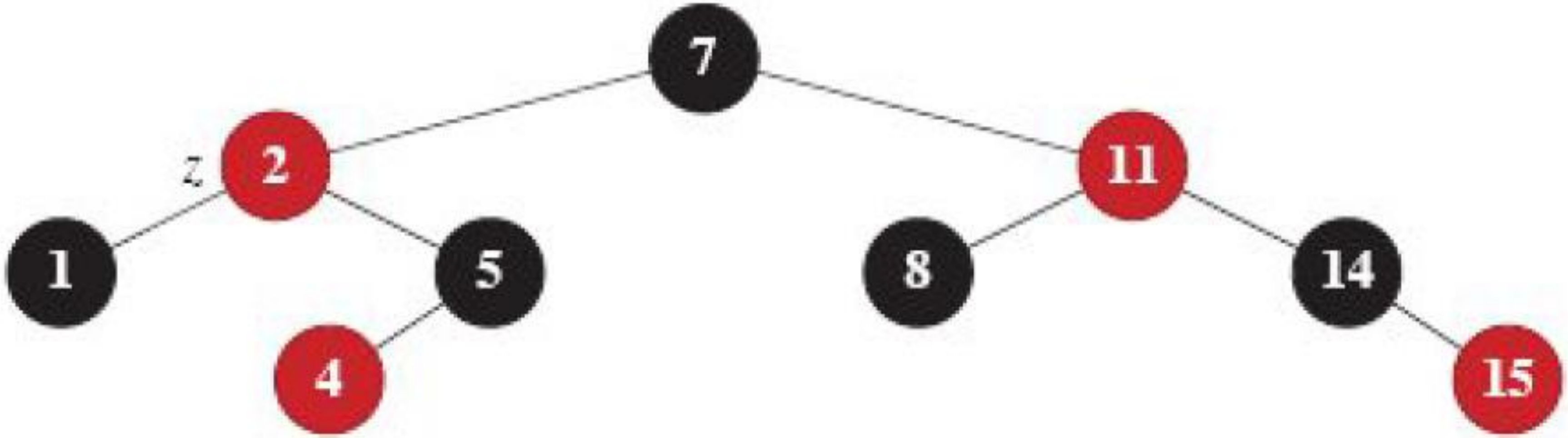
Concrete example

- Current node is z, its uncle y is black. z is left child, so case 3.
  - Recolor nodes 7 and 11, then perform right rotation.

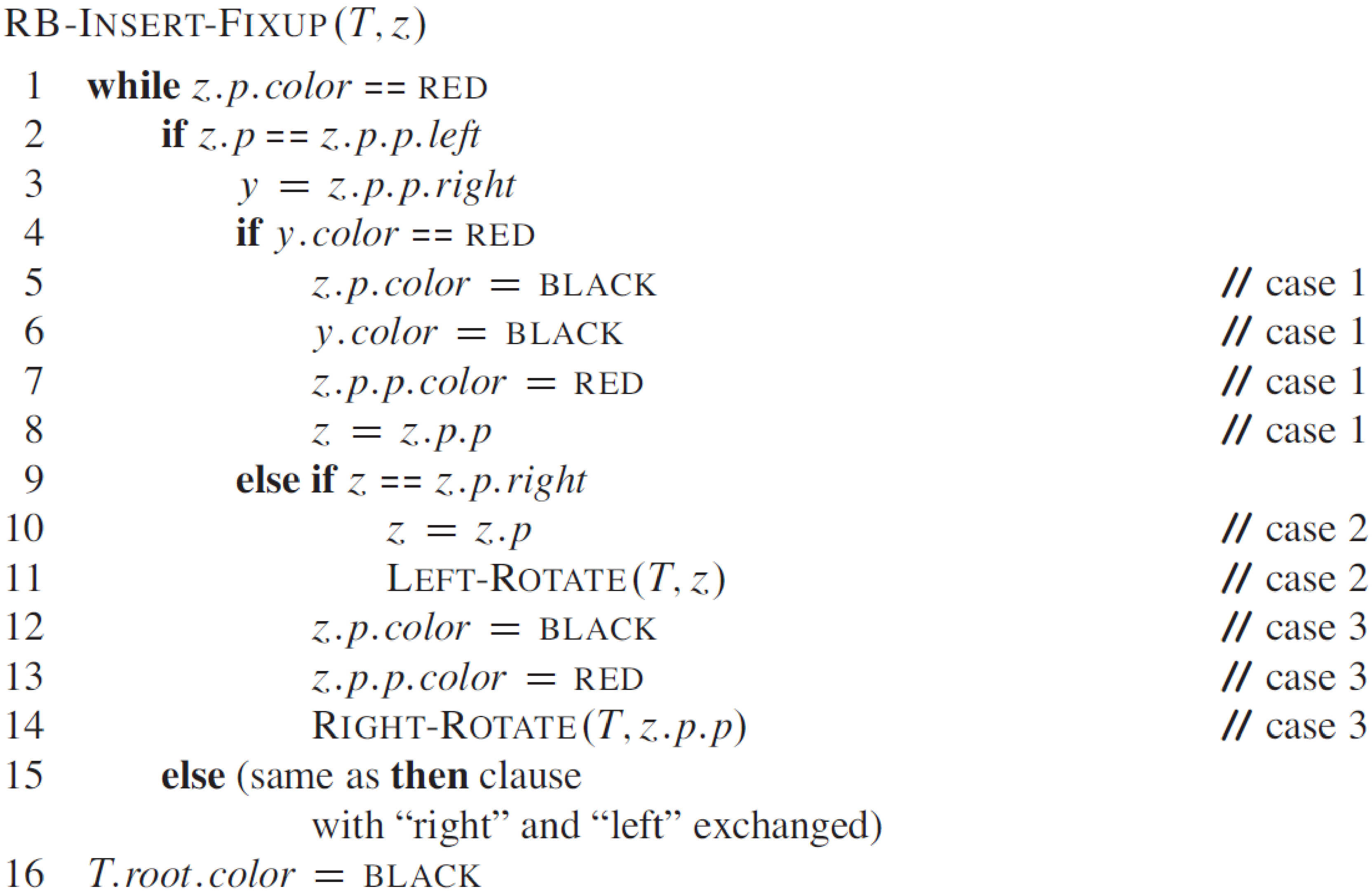


Concrete example

- Final state.



Implementation



```
RB-INSERT-FIXUP($T, z$)
while $z.p.color == \text{RED}$
    if $z.p == z.p.p.left$
        $y = z.p.p.right$
        if $y.color == \text{RED}$
            $z.p.color = \text{BLACK}$                    // case 1
            $y.color = \text{BLACK}$                      // case 1
            $z.p.p.color = \text{RED}$                    // case 1
            $z = z.p.p$                                   // case 1
        else if $z == z.p.right$
                $z = z.p$                                 // case 2
                LEFT-ROTATE($T, z$)                       // case 2
            $z.p.color = \text{BLACK}$                    // case 3
            $z.p.p.color = \text{RED}$                    // case 3
            RIGHT-ROTATE($T, z.p.p$)                      // case 3
    else (same as then clause
            with “right” and “left” exchanged)
$T.root.color = \text{BLACK}$
```

Exercise (previous week)

- Consider inserting the keys 10, 22, 31, 4, 15, 28, 17, 88, 59 into a hash table of length $m = 11$ using open addressing with the auxiliary hash function $h'(k) = k$. Illustrate the result of inserting these keys using linear probing, using quadratic probing with $c_1 = 1$ and $c_2 = 3$, and using double hashing with $h_1(k) = \mathrm{k}$ and $h_2(k) = 1 + (k \bmod (m - 1))$.
- 考虑用开放寻址法将元素10, 22, 31, 4, 15, 28, 17, 88, 59 加入到长度为 $m = 11$ 的散列表中，使用辅助散列函数$h'(k) = k$。分别展示使用线性探查，二次探查（$c_1 = 1$, $c_2 = 3$）和双重散列（$h_1(k) = \mathrm{k}$, $h_2(k) = 1 + (k \bmod (m - 1))$）加入散列表的过程。

Exercise (this week)

- Construct red-black tree with insertion of values:

  1, 2, 3, 4, 5, 6, 7, 8

- Remember:
  1. Nil nodes at leaf position (not shown) are black.
  2. Use symmetric version of rules if necessary.
  3. If root node becomes red, it is recolored black.