

期末复习题

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一、时间复杂度

(a) 将以下函数按照增长速度排序($\log n = \log_2 n$):

$$\begin{aligned} f_1(n) &= n^\pi & f_2(n) &= \pi^n & f_3(n) &= \binom{n}{5} & f_4(n) &= \sqrt{2^{\sqrt{n}}} \\ f_5(n) &= \binom{n}{n-4} & f_6(n) &= 2^{\log^4 n} & f_7(n) &= n^{5(\log n)^2} & f_8(n) &= n^4 \binom{n}{4} \end{aligned}$$

$$f_1(n), f_5(n), f_3(n), f_8(n), f_7(n), f_6(n), f_4(n), f_2(n)$$

$$f_1(n) = n^\pi = o(n^4), f_5(n) = \Theta(n^4) \quad f_5(n) = \Theta(n^4), n^4 = o(n^5), f_3(n) = \Theta(n^5)$$

$$f_8(n) = \Theta(n^8), n^8 = o(n^{5 \log^2 n}) \quad f_7(n) = o(n^{\log^3 n}), f_6(n) = 2^{\log^4 n} = n^{\log^3 n}$$

$$f_6(n) = 2^{\log^4 n}, f_4(n) = 2^{n/4}, \log^4 n = o(n/4)$$

$$f_4(n) = 2^{n/4} = o(2^n), 2^n = o(2^{(\log_2 \pi)n}), f_2(n) = \pi^n = 2^{(\log_2 \pi)n}$$

一、时间复杂度

(b) 求解以下分治算法分析产生的递推关系：

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + \Theta(n).$$

$$\begin{aligned} T(n) &= T(n/3) + T(2n/3) + \Theta(n) = \\ &T(n/3^2) + T(2n/3^2) + T(2n/3^2) + T(2^2n/3^2) + \Theta(n) + \Theta(n/3) + \Theta(2n/3) \\ &= T(n/3^2) + T(2n/3^2) + T(2n/3^2) + T(2^2n/3^2) + 2\Theta(n) \end{aligned}$$

...

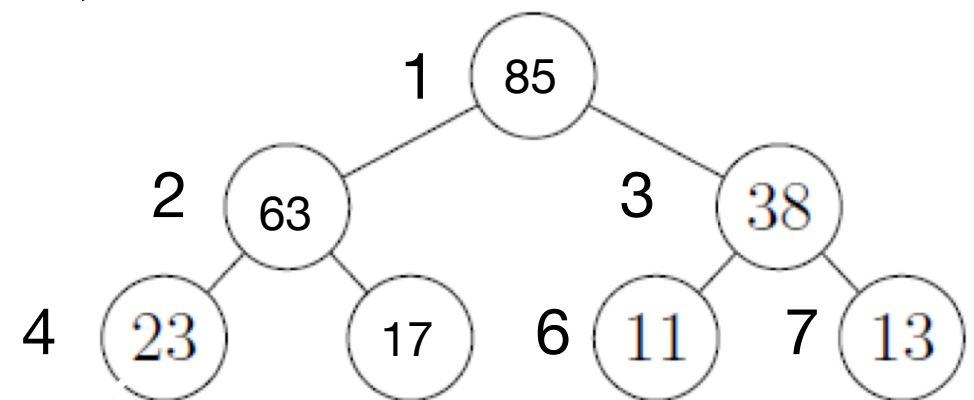
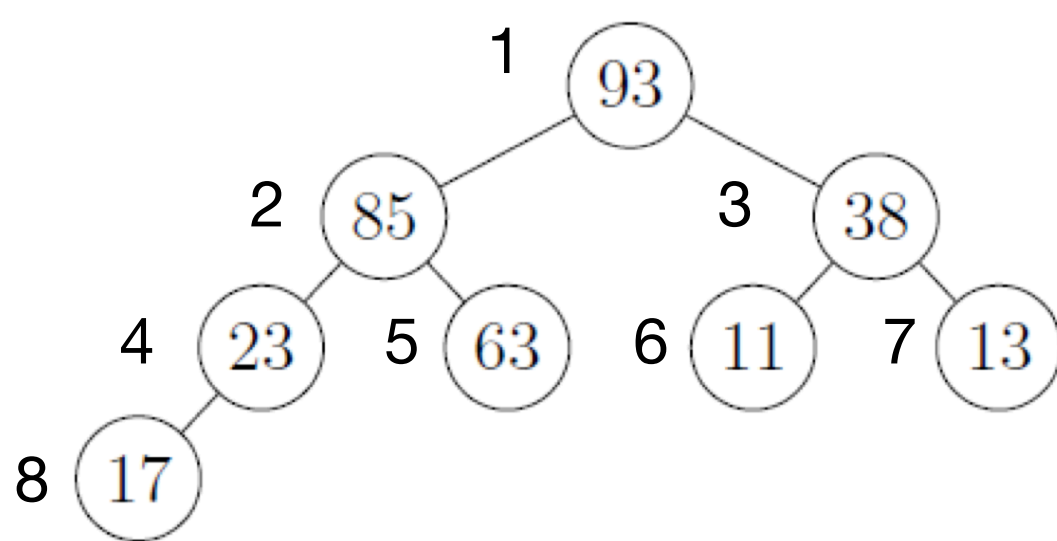
$$= k\Theta(n) + \sum_{j=0}^k C_k^j T(n(1/3)^j(2/3)^{k-j}) =$$

$$\dots = O(n \log_{3/2} n)$$

二、堆排序算法

说明这个堆如何使用数组表达。如果这个堆里的最大元素被删除，剩下的堆的数组表达是什么？

删除93：首先调换93和17，并删除93，然后MAX-HEAPIFY



93	85	38	23	63	11	13	17
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85	63	38	23	17	11	13
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