KHARKIV PHYSICS & MATHEMATICS LYCEUM 27 OF KHARKIV MUNICIPAL COUNCIL IN KHARKIV REGION
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Introduction to Calculus

Scholar subject

Duration: 5 semesters
Occurrence: 9th Spring - 11th Spring

Total Hours: 174
Difficulty: High

Course Format and Grading

The whole course is split into modules. Each module has several (1-4) midterm tests and a final written exam (45 or 90 min). Each semester contains 1-3 modules, the semester grade is calculated as the average of the module grades. All grades are calculated on a 12-point scale. Two oral theoretical exams (90 min) based on all of the previous material are held at the end of the 10th grade and at the end of the course. Theoretical materials include careful proofs of all mentioned facts and theorems.

Prerequisites

Scholar algebra and geometry.

Syllabus

1. Limit of a Sequence (27 hrs)
Occurrence: 9th Spring
Content: axioms of real numbers, Cantor's intersection theorem, limits of sequences, arithmetic operations on limits, squeezing theorem, monotone convergence theorem, definition of the *e* constant, function continuity, logarithmic function, subsequences, Bolzano-Weierstrass theorem, Cauchy sequences (Cauchy test), introduction to numeric series.
Exams: 2 midterms (1 theoretical and 1 practical) 45 min each, final exam 90 min.

2. Limit of a Function (18 hrs)
Occurrence: 10th Fall
Content: Cauchy and Heine definitions and their equivalence, arithmetic operations on limits, limit of composition of functions, Cauchy test, left and right limits, fundamental limits ($\lim_{x\to 0}\sin(x)/x$, $\lim_{x\to\pm\infty}(1+1/x)^x$), limits of common functions, asymptotes and asymptotic behavior of a function, O-notation.
Exams: 2 practical midterms 45 min each, final exam 45 min.

3. Continuous Functions (16 hrs)
Occurrence: 10th Fall
Content: Definition of a continuous function, arithmetic operations and continuity, continuity of a composition of functions, types of singularities in $\mathbb{R}^1$, boundedness theorem, extreme value theorem, intermediate value theorem, numerical solution of equations using bisection, continuity of common functions, continuity of a function inverse, uniform continuity, Heine-Cantor's theorem.
Exams: 2 practical midterms 45 min each, final exam 45 min.

4. Derivative-1 (13 hrs)
Occurrence: 10th Fall
Content: difference and differential, definition of the derivative, geometrical and physical meanings, equivalence of differentiability and existence of derivative, left and right derivative, derivatives of common functions, arithmetic operations and differentiation, derivative of a composition of functions, derivative of a function inverse, technics of differentiation.
Exams: 2 practical midterms 45 min each, final exam 45 min.

4. Derivative-2 (14 hrs)
Occurrence: 10th Spring
Content: equation of the tangent line to a function, decreasing and increasing of a function in a point, interior extremum, Fermat's theorem, Darboux's theorem, Rolle's theorem, Cauchy's mean value theorem, Lagrange's mean value theorem, solving inequalities using the mean value theorem.
Exams: 2 practical midterms 45 min each, final exam 45 min.

5. Derivative-3 (25 hrs)
Occurrence: 10th Spring
Content: conditions of monotony and constancy of a function, locating interior extrema, sufficient condition of existence of a local extremum based on the sign of the 2nd derivative, extreme values on an interval, solving equations and inequalities using monotony and extrema, geometrical extreme value problems, monotony and extrema of a parametric function, parametric equations and inequalities, concavity and convexity, conditions of concavity and convexity, locating inflection points, graphing functions using derivative analysis.
Exams: 4 practical midterms 45 min each, final exam 45 min.

6. Indefinite Integral (16 hrs)
Occurrence: 11th Fall
Content: antiderivative of a function, physical meaning of the antiderivative and the integration constant, integrals of common functions, properties of integrals, method of substitutions, common substitutions (linear, polynomial, exponential, trigonometric, hyperbolic), integrals of rational functions, integration by parts, recursive reduction, integrals with quadratic polynomials.
Exams: 2 practical midterms 45 min each, final exam 45 min.

7. Differential Equations (23 hrs)
Occurrence: 11th Fall
Content: equations of form $y^{(n)} = f(x)$, equations with separable variables, equations of form $y^{(n)} = f(ax + by)$, homogeneous equations, equations of form $y' = f\left(\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}\right)$, method of undetermined parameters, linear ODEs of the 1st order, Bernoulli equations, reduction to known types of equations using substitution, general theory for linear ODEs, Wronskian, Liouville's theorem, homogeneous and nonhomogeneous linear ODEs with constant parameters, characteristic equation, finding general solution, finding partial solutions of nonhomogeneous ODEs with special right-hand sides, damped and driven damped oscillations, variation of parameters.
Exams: 2 practical midterms 45 min each, final exam 90 min.

7. Definite Integral (21 hrs)
Occurrence: 11th Spring
Content: area of a curvilinear trapezoid, Riemann sums, definition of the Riemann integral, physical and geometrical meaning, Darboux sums and integrals, theorems of integrability, mean-value theorem, integral as a function of the upper limit, Newton-Leibniz formula, integration by parts, substitution method, calculating limits of sequences and areas using definite integrals, parametric problems.
Exams: 3 practical midterms 45 min each, final exam 45 min.

Learning methods

Lectures, practical seminars, home assignments, additional problems.

Literature

1. S.M. Nikolsky. *Course of Mathematical Analysis*, 2001 Moscow, 592 p, ISBN 5-9221-0160-9.
2. G.M. Fichtenholz. *Course of Differential and Integral Calculus*, Vol. 1 and 2, 2003 Moscow, 680 p and 864 p, ISBN 5-9221-0156-0 and 5-9221-0157-9.
3. B.P. Demidovich. *Problems in Mathematical Analysis*, 1997 Moscow, 624 p, ISBN 5-211-03645-X.