Taras Shevchenko National University of Kyiv

Physics Department

Course description

Mathematical Analysis

Level:	**Language:**	**Duration:**	**Occurrence:**
Bachelor	Ukrainian	3 semesters	1^{st} – 3^{rd} semesters
Credits:	**Total Hours:**	**Contact Hours:**	**Self-study Hours:**
15	480	240	240

Description of Course Work and Examinations

Module-rating system, results are evaluated on a 100-point scale. Each semester contains 3 module tests 20 pts/90 min each, an exam on higher complexity problems 10 pts/180 min, and a final exam 30 pts/120 min.

Prerequisites

None

Syllabus

Introduction to calculus*:* basic concepts and notations of set theory, types of relations and their properties (reflexivity, transitivity, symmetry, antisymmetry, equivalence relations, order relations), real numbers, complex numbers, functions (surjection, injection, bijection, inverse, composition, parametric function), graphs of common functions, mathematical induction.

Limits and continuity: sequences of real and complex numbers, properties of convergent sequences, limit theorems and criteria, limits of a function, asymptotic notations (Landau symbols), continuity of a function at a point and on a set, types of discontinuities, local and global properties of continuous functions, uniform continuity.

Real single-variable differential calculus: derivatives and differentials (also of higher orders), differentiation rules for basic elementary functions, differentiation rules for combined functions (sum, product, quotient, and chain rules), differentiation rules for parametric and inverse functions, mean value theorems, Taylor's and Maclaurin's formulas, L'Hôpital's rules, applications of derivatives (finding local and global extrema, inflection points, asymptotes, building graphs of functions and curves), derivatives of vector-valued functions.

Real single-variable integral calculus: antiderivative, definition and properties of the indefinite integral, methods of finding indefinite integrals (integration by substitution, integration by parts), methods of indefinite integration for irrational and trigonometric functions, Riemann's and Darboux's definitions of the definite integral, classes of integrable functions (continuous functions, monotonous function, Lebesgue's criterion), properties of the definite integral, mean value theorems, methods of computing definite integrals (integration by substitution, integration by parts), the fundamental theorem of calculus (Newton–Leibniz formula), applications of definite integrals to geometry and mechanics, improper integrals (definition, properties, evaluation, Cauchy principal value).

Real multivariable differential calculus: general concepts (definitions of metric space, normed vector space, Euclidean space, examples of maps between metric spaces, contraction mapping, Banach fixed-point theorem), limits and continuity, partial derivatives and differentials (also of higher orders), differentiability condition, Schwarz's theorem of the equality of mixed partial derivatives, differentiation rules, directional derivative, gradient, implicit function theorem, Taylor's formula, Jacobian matrix of a vector-valued function of several real variables, local extrema, local conditional extrema (method of Lagrange multipliers), global extrema, applications to geometry.
Real multivariable integral calculus: definition and properties of the multiple integral, iterated integrals, Fubini's theorems of computing double and triple integrals, substitution rule in multiple integrals, applications to geometry and mechanics.
Integrals on manifolds*:* definition of the path and surface integrals over scalar and vector fields, Green's theorem, Gauss–Ostrogradsky theorem, Stokes' theorem, elements of vector calculus (gradient, divergence, curl, del operator).
Infinite series: definition and basic properties of infinite series of real numbers, convergence tests for series with non-negative terms, absolute and conditional convergence, convergence at a point and uniform convergence of a family of functions, properties of the limit function (limit, continuity, differentiability, integrability), convergence at a point and uniform convergence of a series of functions, property of the sum function (limit, continuity, differentiability, integrability), power series, Taylor series.
Integrals with parameters*:* proper integrals with a parameter (limit, continuity, differentiability, integrability, evaluation methods), improper integrals with a parameter (uniform convergence, limit, continuity, differentiability, integrability, evaluation methods), special improper integrals (Frullani's integral, Laplace's integral, Gauss integral, Fresnel integrals, Dirichlet integral), beta function (Euler integral of the first kind), gamma function (Euler integral of the second kind).
Fourier analysis: periodic functions, Fourier series (convergence at a point and uniform convergence), Fourier series for even and odd functions, Fourier series in the Euclidean space, Bessel's inequality, Parseval's identity, Fourier integrals, basic rules of the Fourier transform.

Literature

1. I. I. Lyashko, A. K. Boyarchuk, Y. G. Ghuy, G. P. Golovach. *Mathematical Analysis*. Vol. 1: *Introduction to Analysis, Derivative, Integral*. M.: Editorial, 2001. 360 p. ISBN 5-354-00018-1.
2. I. I. Lyashko, A. K. Boyarchuk, Y. G. Ghuy, G. P. Golovach. *Mathematical Analysis*. Vol. 2: *Series, Differential Calculus of Multivariable Functions*. M.: Editorial, 2003. 224 p. ISBN 5-354-00272-9.
3. I. I. Lyashko, A. K. Boyarchuk, Y. G. Ghuy, G. P. Golovach. *Mathematical Analysis*. Vol. 3: *Integrals with Parameters, Integral Calculus of Multivariable Functions*. M.: Editorial, 2001. 224 p. ISBN 5-354-00019-X.

Instructors

Associate Professor Nataliya V. Mayko.