PROGRAMMING AND DATA STRUCTURES

HASHING

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OUTLINE

- Hashing and Hash Tables
- Collisions in Hash Tables
- Solutions to the collision problem
    - Open Addressing and Separate Chaining
- Performance of Hash Tables
- Implementing a Hash Table (class HashMap)

STUDENT LEARNING OUTCOMES

At the end of this chapter, your should be able to:

- Describe how hashing and hash tables work
- Apply different solutions to handle collisions
- Implement and test a hash table data structure (hash map)
- Evaluate the time complexity of the operations on a Hash Table

Search operation

- Array List: `O(n)`
- Linked List: `O(n)`
- BST: `O(log n)` (when close to balanced)
- Can we perform search in less time?

Hash tables

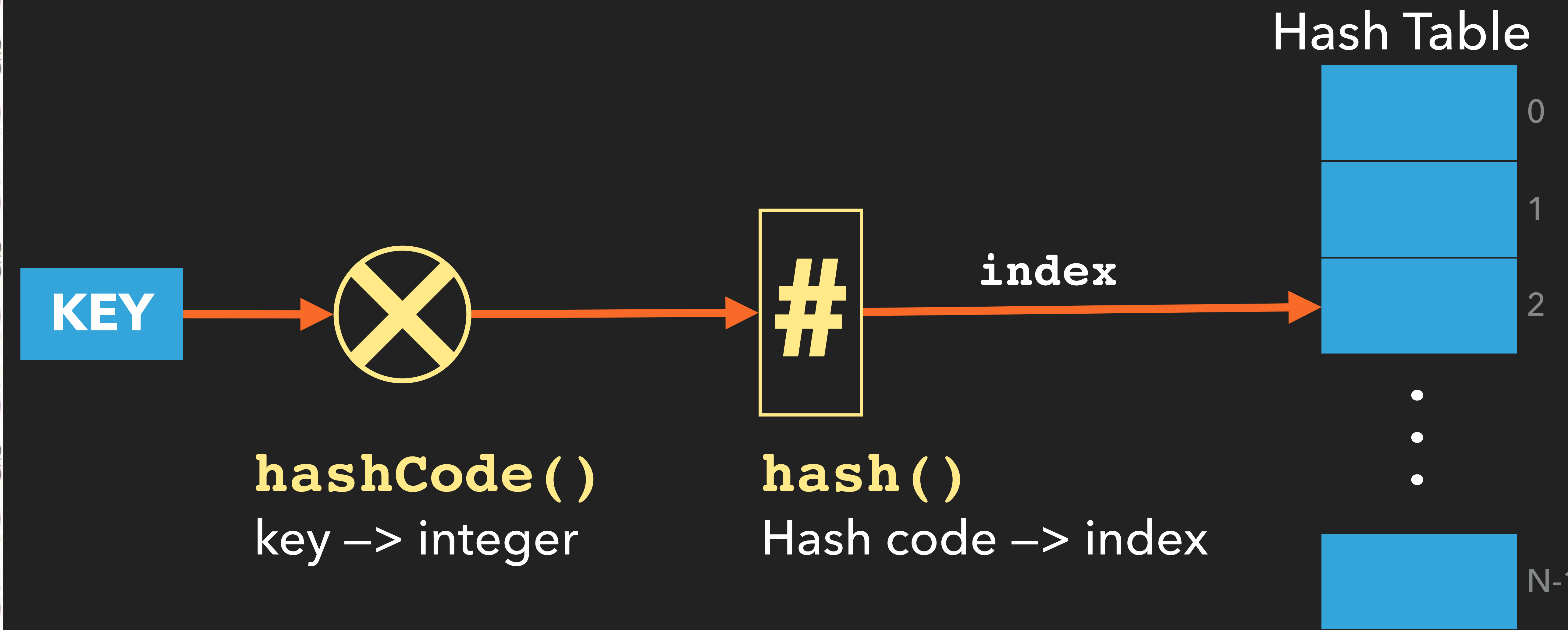
- Hash tables allow to perform search operation in `O(1)`
- Hash tables use associative access to data
- Associative memory: access data using the data itself instead of an address (index)

Hash tables

- Store the data in an array - Hash Table (**`HT`**)

- Access the elements in **`HT`** using a hash function **`h()`** that returns an index in **`HT`**

Hash tables


Hash Table
0
1
2
N-1
KEY
index
hashCode()
key –> integer
hash()
Hash code –> index

Hash tables

- If the size of **`HT`** is **`N`**, then

  `0 ≤ hash()≤ N-1` (valid index)

- Searching for a value **`v`** is performed using one comparison with

  `HT[hash(hashCode(v))]`

- How data is added/found in **`HT`** using **`hash()`**?

- How **`hashCode()`** **and** **`hash()`** are defined?

Hash tables

- Adding data to the table
  - Apply `hash()` to the data to determine the index where the data should be added
- Retrieving data from the table
  - Apply `hash()` to the data to find the index where it is in the table

Hash tables

Example

- Values `{11, 34, 57, 60, 72, 85, 91, 93}` to store in a `HT` of size 11
- Each value `v` is stored at an index `i` calculated by `hash(v)= v%(size of HT) [i = v % 11]`
- Searching for a value `v`: compare `v` to `HT[hash(v)]`

Hash tables

- Adding data to **HT**

```
hash(11) = 11 % 11 = 0
hash(34) = 34 % 11 = 1
hash(57) = 57 % 11 = 2
hash(60) = 60 % 11 = 4
hash(72) = 72 % 11 = 6
hash(85) = 85 % 11 = 8
hash(91) = 91 % 11 = 3
hash(93) = 93 % 11 = 5
```

HT

HT	
11	0
34	1
57	2
91	3
60	4
93	5
72	6
-1	7
85	8
-1	9
-1	10

Hash tables

- Retrieving data from **HT**
  - `key = 91`

    `hash(key) = 91 % 11 = 3`

    key found

  - `key = 75`

    `hash(75) = 75 % 11 = 9`

    key not found

  - Search operation: **O(1)**

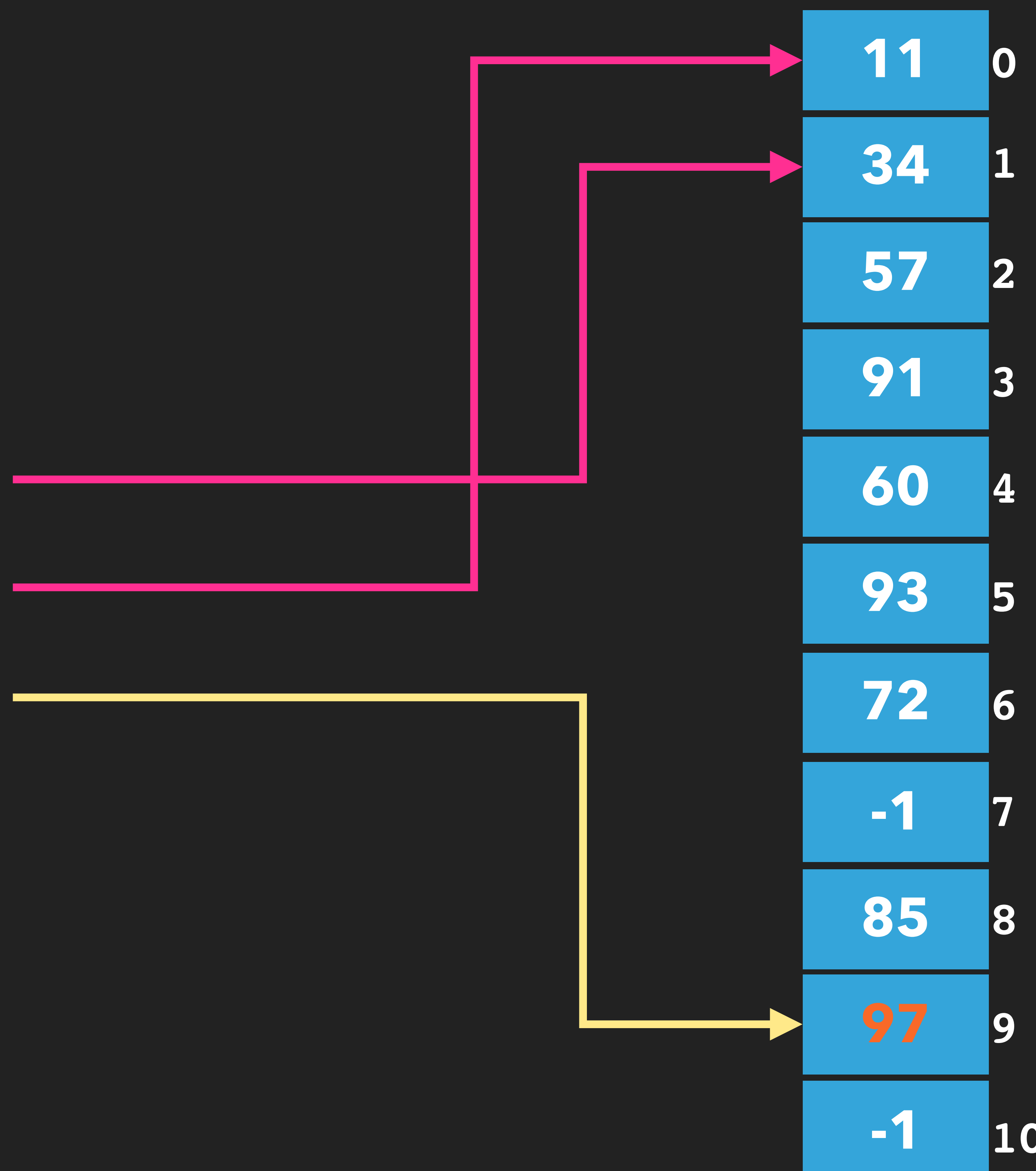
HT	
11	0
34	1
57	2
91	3
60	4
93	5
72	6
-1	7
85	8
-1	9
-1	10

Collisions

- Issue with the hash method
  - Set of values may be hashed to the same index (23, 34, 56, 78, 89 all have the hash value = 1) for size=11
  - Collision: two or more values have the same hash function value

Collisions

```
h(45)=45%11
h(55)=55%11
h(97)=97%11
```



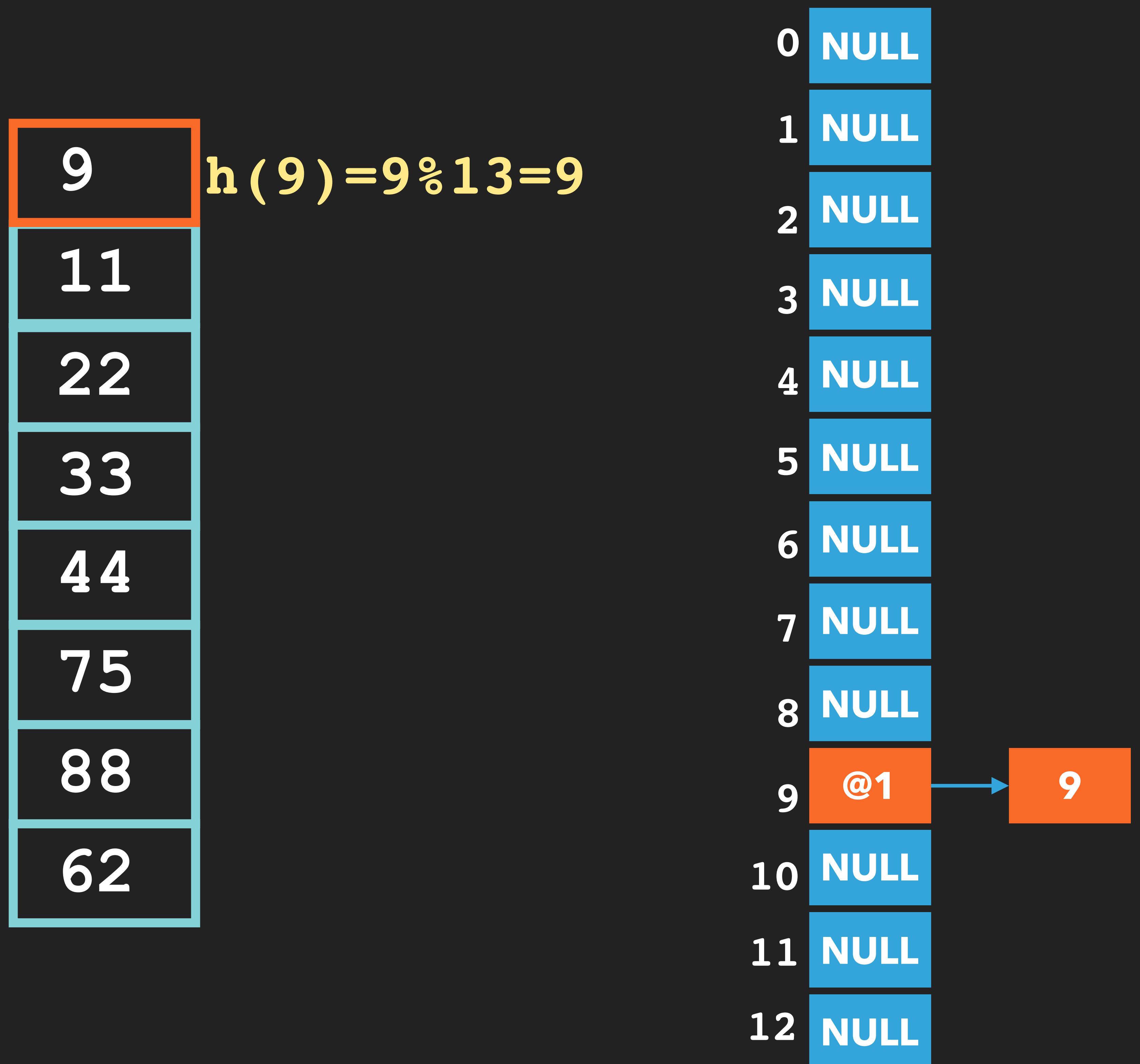
Collisions

- Collisions - two or more values have the same hash function value
- Two solutions for the collision problem
  - Separate Chaining (open hashing)
  - Open Addressing (closed hashing)

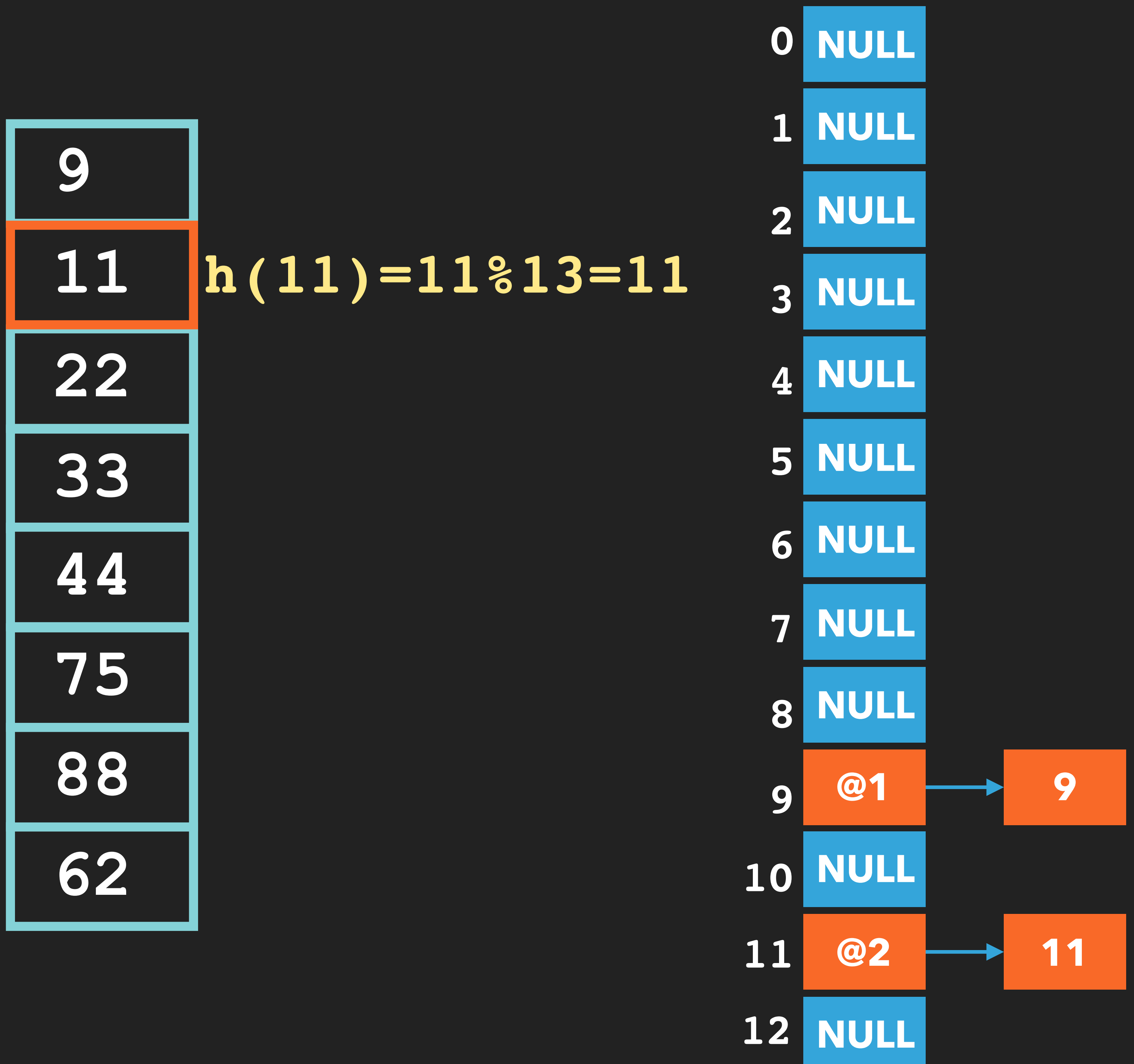
Collisions

- Solutions for the collision problem
    - Separate Chaining - collisions are stored outside the hash table in a list (array list or linked list, or even a tree)
    - Open Addressing - collisions are stored in the hash table itself at another index

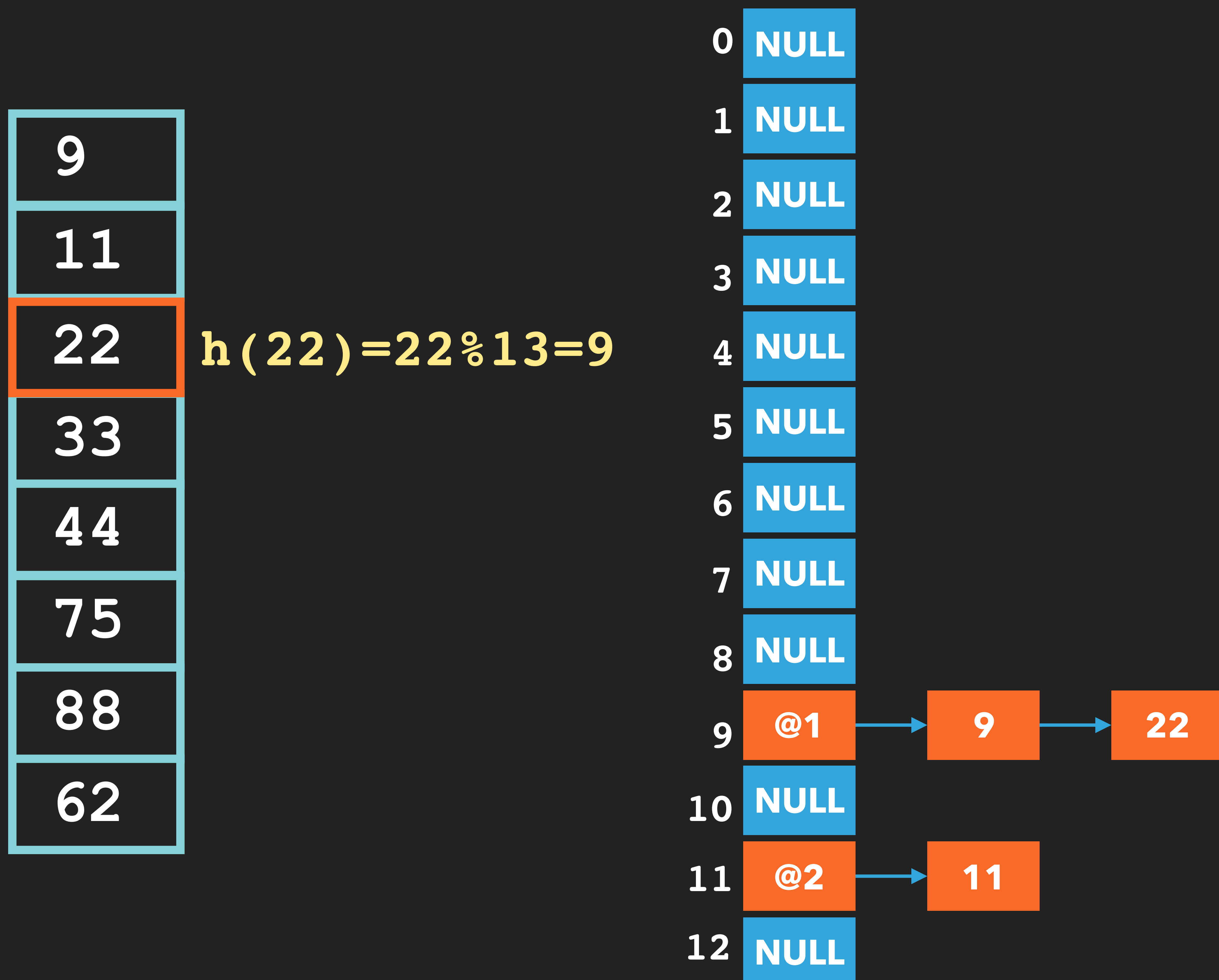
Collisions - Separate Chaining



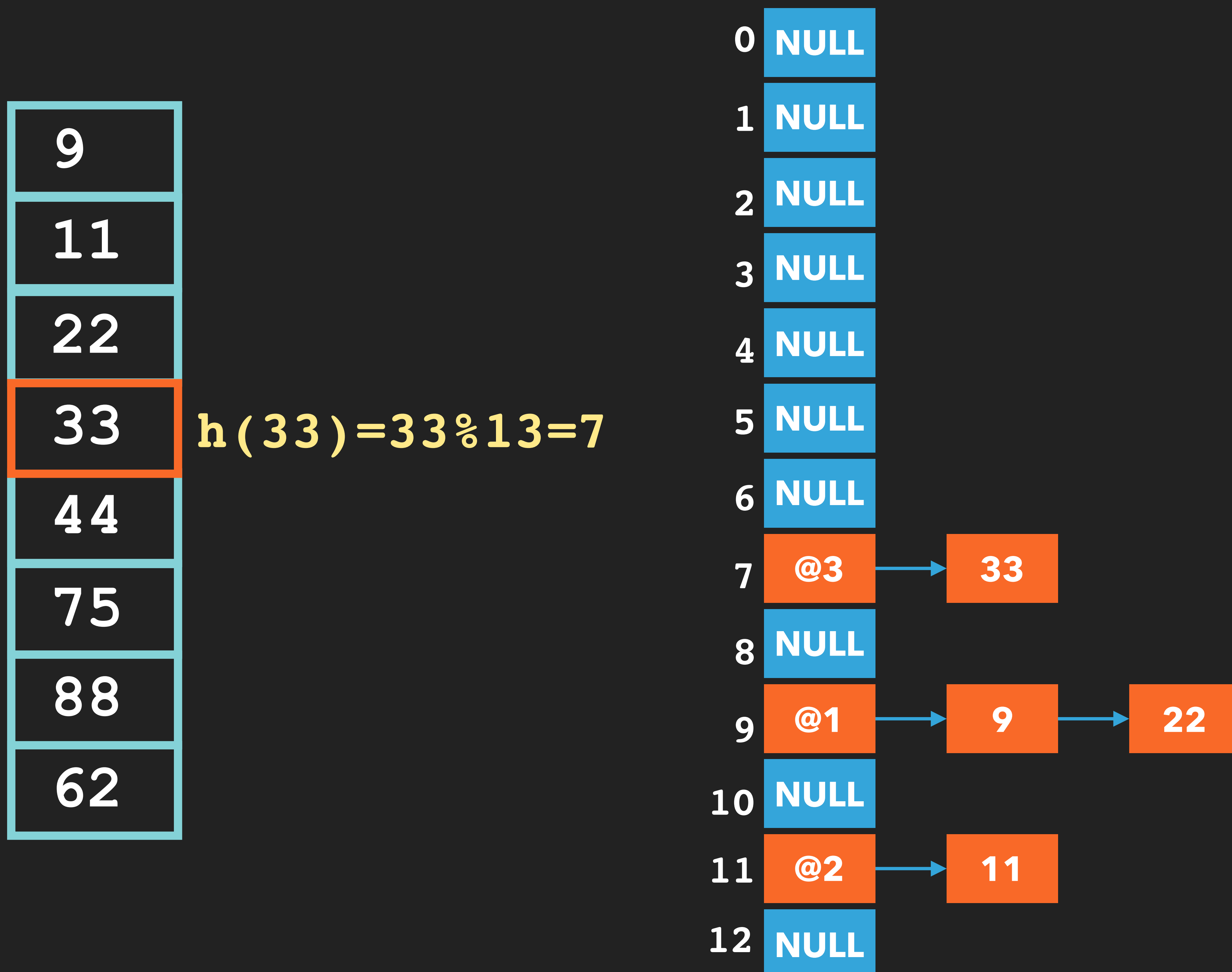
Collisions - Separate Chaining



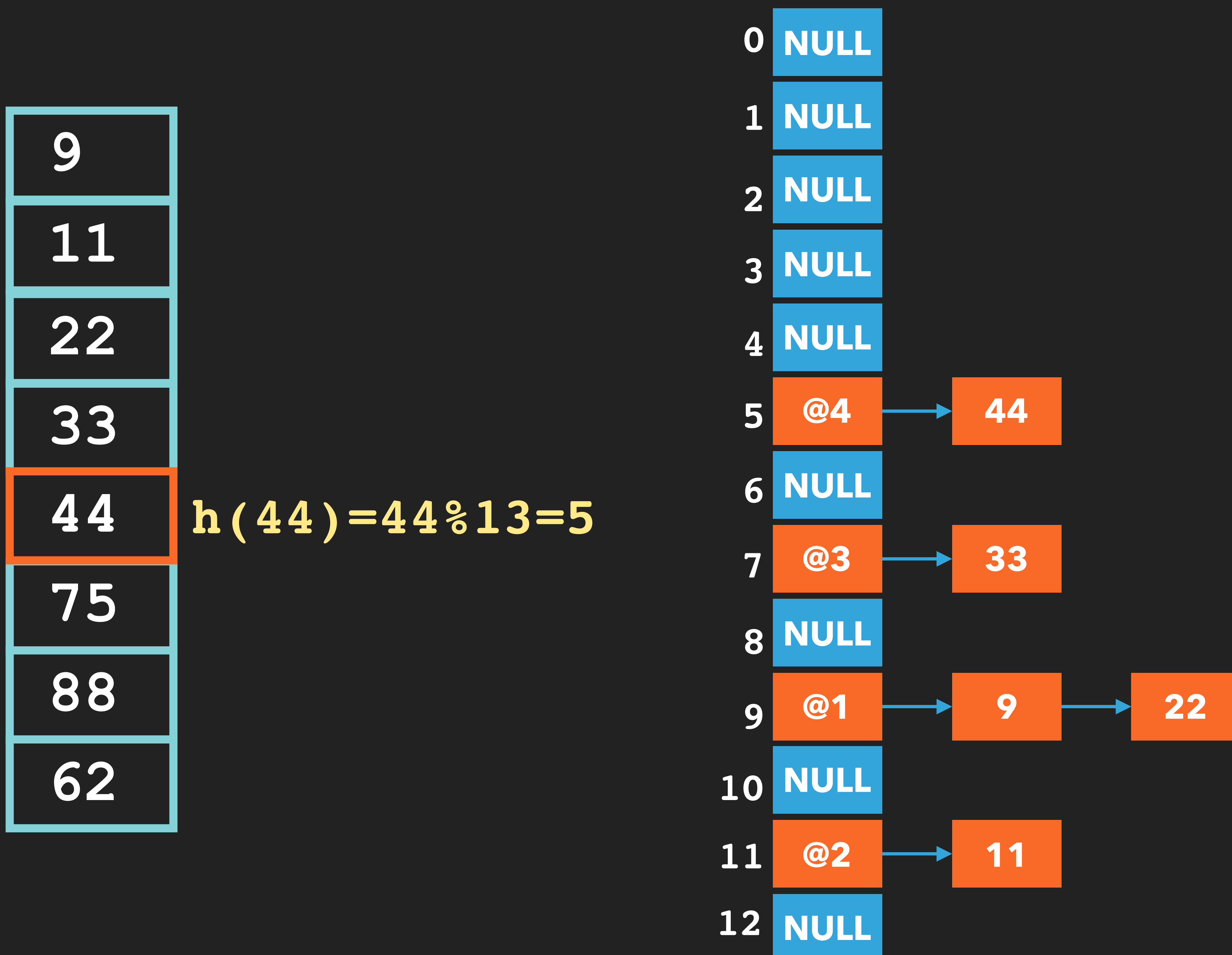
Collisions - Separate Chaining



Collisions - Separate Chaining



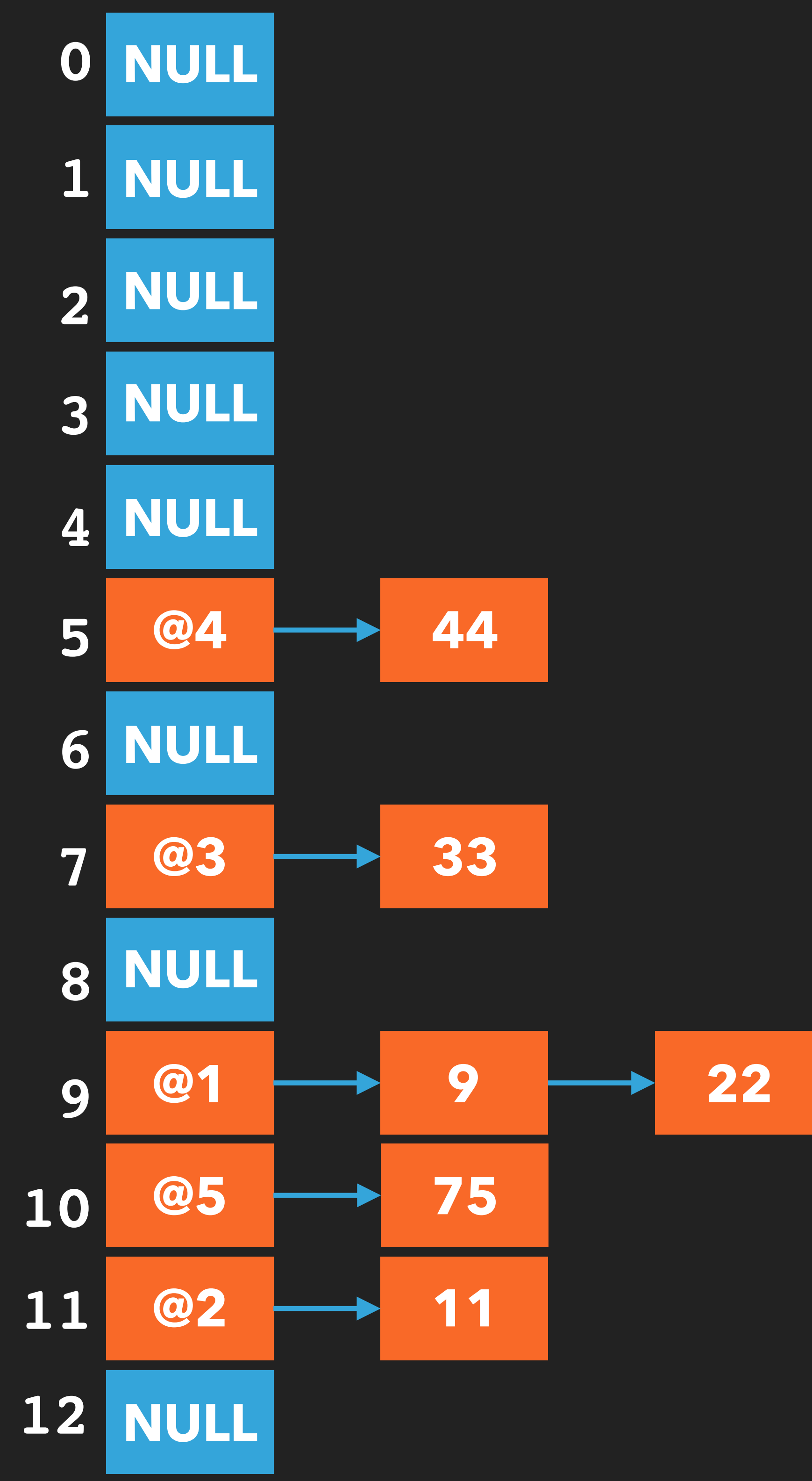
Collisions - Separate Chaining



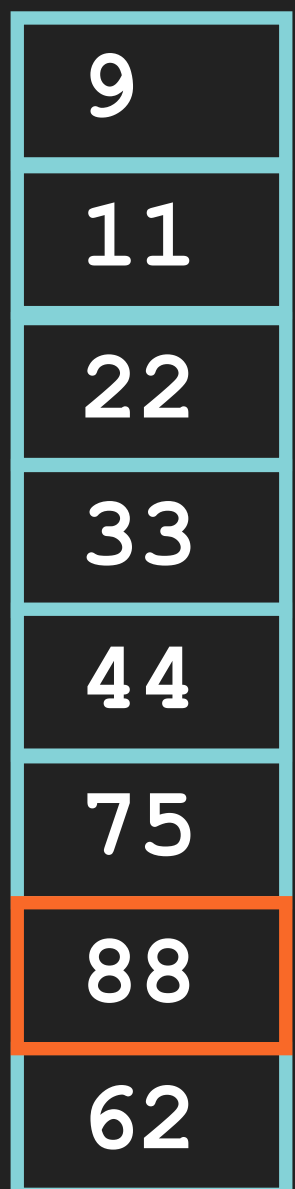
Collisions - Separate Chaining

9
11
22
33
44
75
88
62

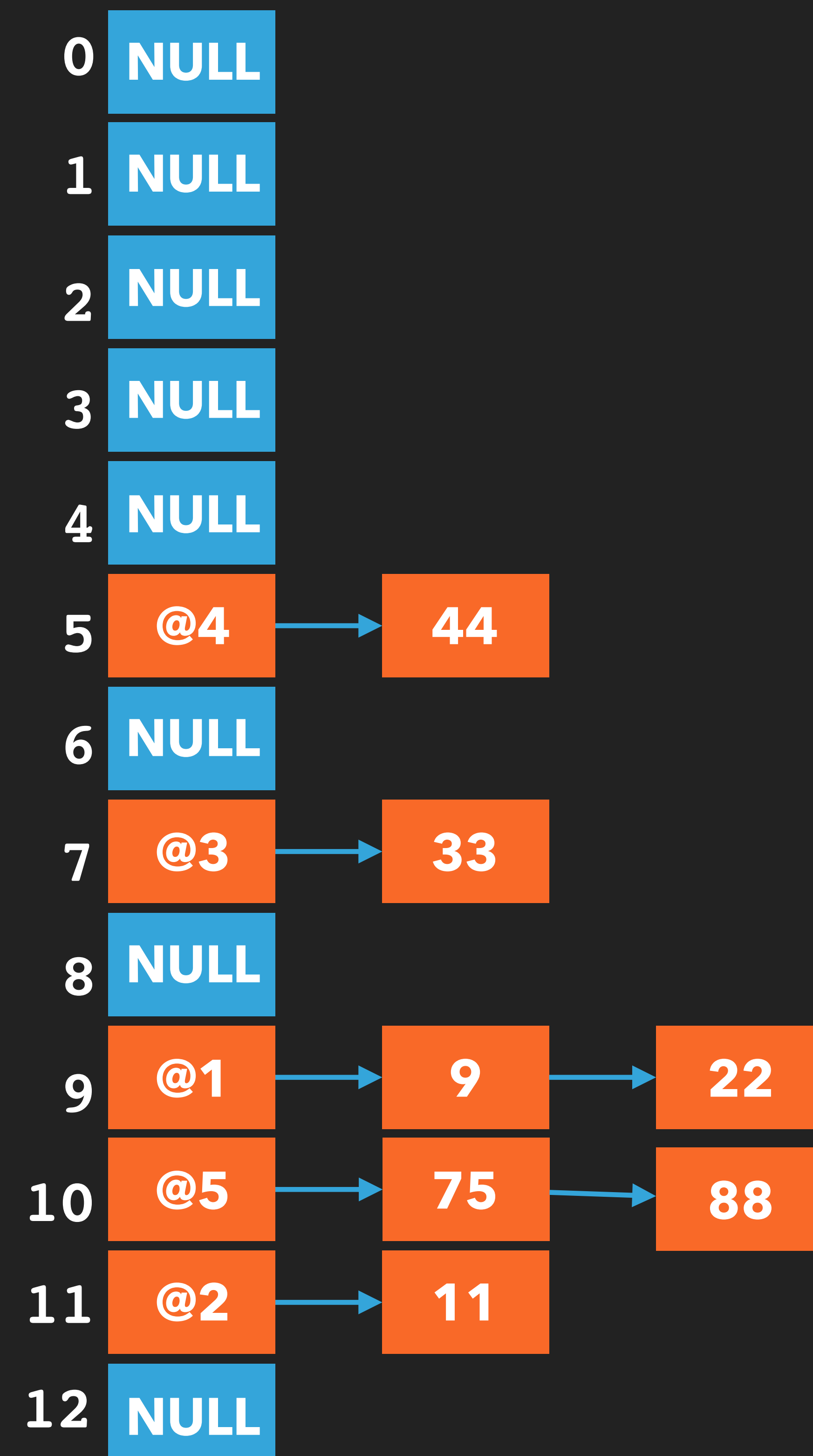
h(75)=75%13=10



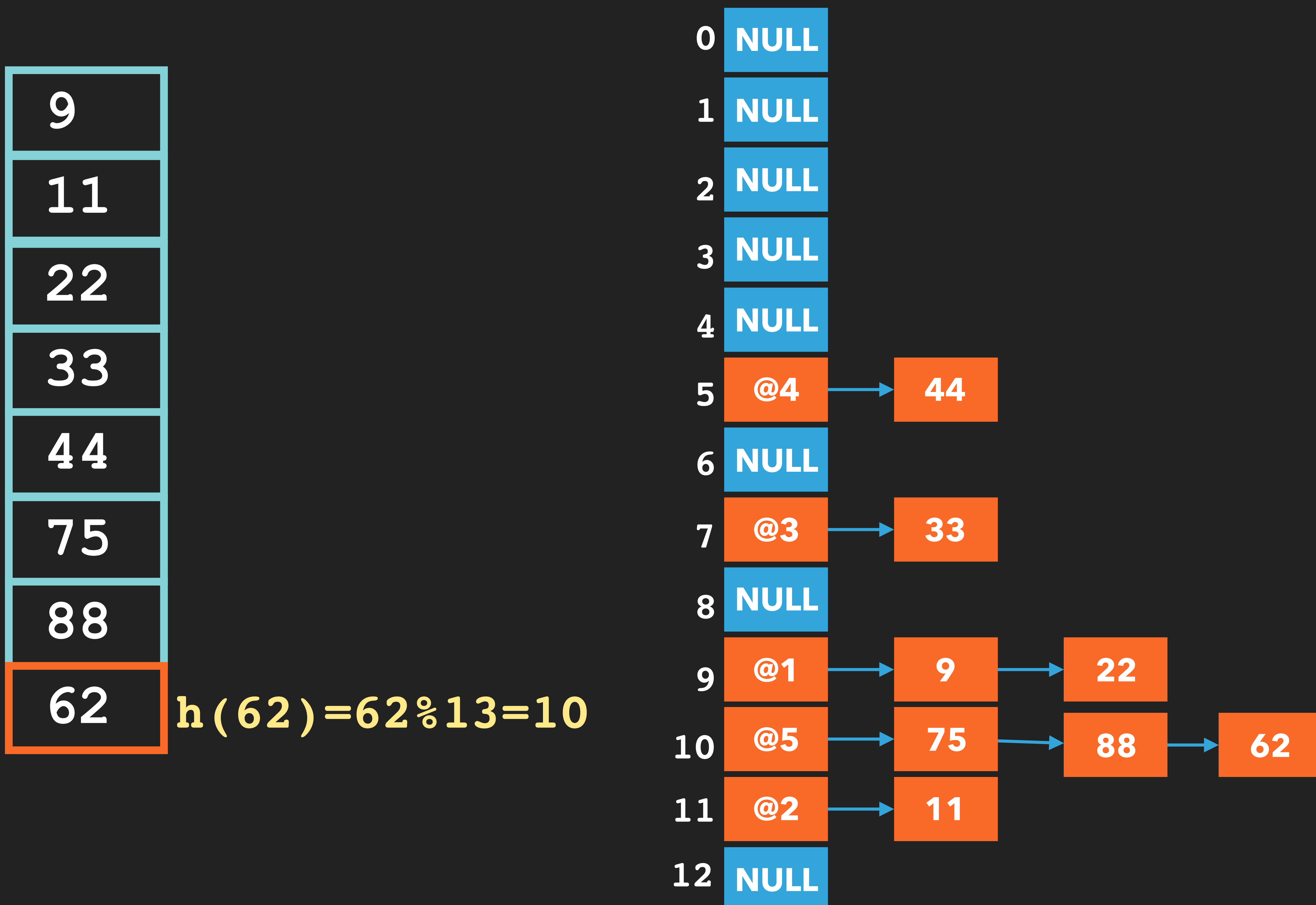
Collisions - Separate Chaining



`h(88)=88%13=10`



Collisions - Separate Chaining



Collisions - Open Addressing

- **Linear probing** - look for the next available slot using a linear function (`index+j` until an empty slot is found)
- **Quadratic probing** - look for the next available slot using a quadratic function ($index+j^2$ until an empty slot is found)
- **Double hashing** - look for the next available slot at an index using another hash function (`index + hash2(v)`)

Collisions - Linear Probing

9
11
22
33
44
75
88
62

HT

0	Null
1	null
2	null
3	null
4	NULL
5	NULL
6	NULL
7	NULL
8	NULL
9	NULL
10	NULL
11	NULL
12	NULL

Collisions - Linear Probing

- 9
- 11
- 22
- 33
- 44
- 75
- 88
- 62

`h(9)= 9 % 13 = 9`

0	null	null
1	null	null
2	null	null
3	null	null
4	null	null
5	null	null
6	null	null
7	null	null
8	null	null
9	null	9
10	null	null
11	null	null
12	null	null

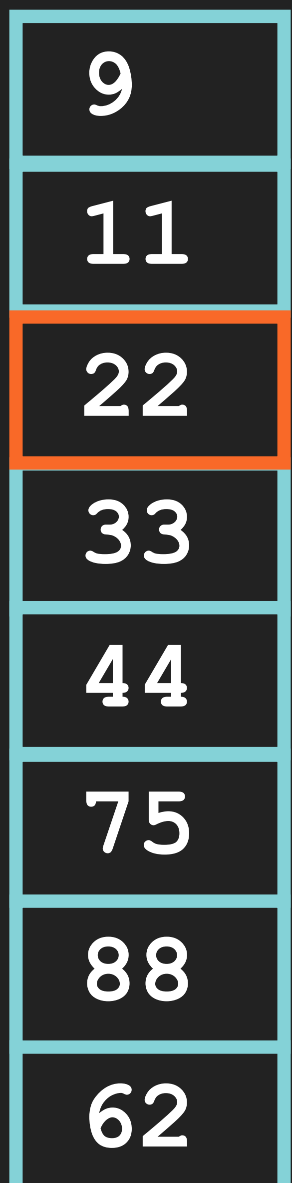
Collisions - Linear Probing

9
11
22
33
44
75
88
62

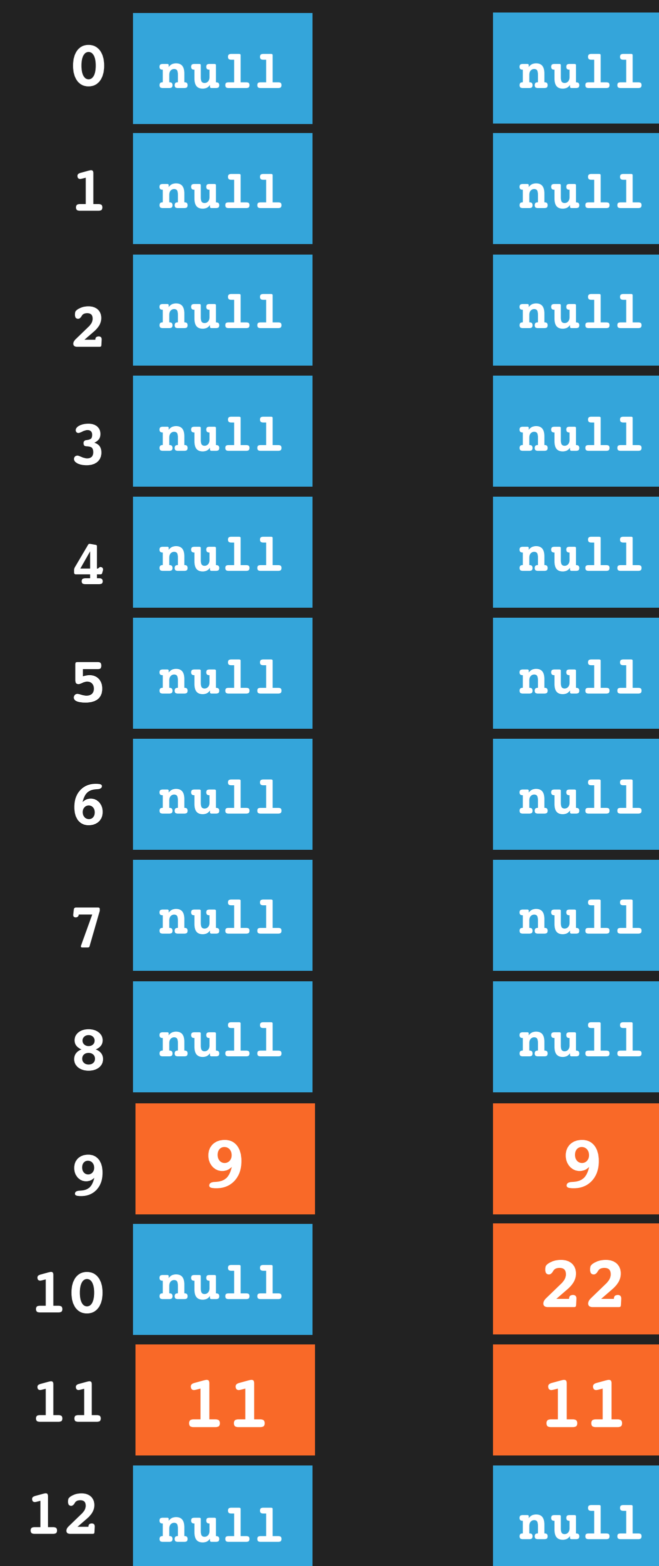
`h(11)= 11 % 13 = 11`

0	null	null
1	null	null
2	null	null
3	null	null
4	null	null
5	null	null
6	null	null
7	null	null
8	null	null
9	9	9
10	null	null
11	null	11
12	null	null

Collisions - Linear Probing



h(22)=(9 + 1) % 13 10



Collisions - Linear Probing

9
11
22
33
44
75
88
62

h(33) = 33 % 13 = 7

0	null	null
1	null	null
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	NULL	NULL
6	NULL	NULL
7	NULL	33
8	NULL	NULL
9	9	9
10	22	22
11	11	11
12	NULL	NULL

Collisions - Linear Probing

9
11
22
33
44
75
88
62

`h(44) = 44 % 13 = 5`

0	NULL	NULL
1	NULL	NULL
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	NULL	44
6	NULL	NULL
7	33	33
8	NULL	NULL
9	9	9
10	22	22
11	11	11
12	NULL	NULL

Collisions - Linear Probing

9
11
22
33
44
75
88
62

`h(75)=(10 + 2) % 13 = 12`

0	NULL	NULL
1	NULL	NULL
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	44	44
6	NULL	NULL
7	33	33
8	NULL	NULL
9	9	9
10	22	22
11	11	11
12	NULL	75

Collisions - Linear Probing

9
11
22
33
44
75
88
62

`h(88) =(10 + 3) % 13 = 0`

0	NULL	88
1	NULL	NULL
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	44	44
6	NULL	NULL
7	33	33
8	NULL	NULL
9	9	9
10	22	22
11	11	11
12	75	75

Collisions - Linear Probing

9
11
22
33
44
75
88
62

`h(62) = (10 + 4) % 13 = 1`

0	88	88
1	NULL	62
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	44	44
6	NULL	NULL
7	33	33
8	NULL	NULL
9	9	9
10	22	22
11	11	11
12	75	75

Collisions - Quadratic Probing

9
11
22
33
44
75
88
62

HT

0	NULL
1	NULL
2	NULL
3	NULL
4	NULL
5	NULL
6	NULL
7	NULL
8	NULL
9	NULL
10	NULL
11	NULL
12	NULL

Collisions - Quadratic Probing

9
11
22
33
44
75
88
62

`h(9) = 9 % 13 = 9`

0	NULL	NULL
1	NULL	NULL
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	NULL	NULL
6	NULL	NULL
7	NULL	NULL
8	NULL	NULL
9	NULL	9
10	NULL	NULL
11	NULL	NULL
12	NULL	NULL

Collisions - Quadratic Probing

9
11
22
33
44
75
88
62

`h(11) = 11 % 13 = 11`

0	NULL	NULL
1	NULL	NULL
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	NULL	NULL
6	NULL	NULL
7	NULL	NULL
8	NULL	NULL
9	9	9
10	NULL	NULL
11	NULL	11
12	NULL	NULL

Collisions - Quadratic Probing

9
11
22
33
44
75
88
62

`h(22) = (9 + 1) % 13 = 10`

0	NULL	NULL
1	NULL	NULL
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	NULL	NULL
6	NULL	NULL
7	NULL	NULL
8	NULL	NULL
9	9	9
10	NULL	22
11	11	11
12	NULL	NULL

Collisions - Quadratic Probing

9
11
22
33
44
75
88
62

`h(33) = 7 % 13 = 7`

0	NULL	NULL
1	NULL	NULL
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	NULL	NULL
6	NULL	NULL
7	NULL	33
8	NULL	NULL
9	9	9
10	22	22
11	11	11
12	NULL	NULL

Collisions - Quadratic Probing

9
11
22
33
44
75
88
62

`h(44) = 44 % 13 = 5`

0	NULL	NULL
1	NULL	NULL
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	NULL	44
6	NULL	NULL
7	33	33
8	NULL	NULL
9	9	9
10	22	22
11	11	11
12	NULL	NULL

Collisions - Quadratic Probing

9
11
22
33
44
75
88
62

`h(75) = (10 + 4) % 13 1`

0	NULL	NULL
1	NULL	75
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	44	44
6	NULL	NULL
7	33	33
8	NULL	NULL
9	9	9
10	22	22
11	11	11
12	NULL	NULL

Collisions - Quadratic Probing

9
11
22
33
44
75
88
62

`h(88) = (10 + 9) % 13 = 6`

0	NULL	NULL
1	75	75
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	44	44
6	NULL	88
7	33	33
8	NULL	NULL
9	9	9
10	22	22
11	11	11
12	NULL	NULL

Collisions - Quadratic Probing

9
11
22
33
44
75
88
62

`h(62) = (10 + 16) % 13 = 0`

0	NULL	62
1	75	75
2	NULL	NULL
3	NULL	NULL
4	NULL	NULL
5	44	44
6	88	88
7	33	33
8	NULL	NULL
9	9	9
10	22	22
11	11	11
12	NULL	NULL

Collisions

- The number of collisions may affect the performance of the search operation
- could result in linear search if the number of collisions is high

Practice

- We want to add the following data to a hash table of 15 elements

  2, 8, 31, 20, 19, 52, 27, 30, 33, 37, 42, 39, 51, 24

- The hash function is `hash(x) = x % 15`

- Show the content of the hash table after adding the data using separate chaining, linear probing (+j), and quadratic probing (+j^2) to resolve the collisions

Collisions

- Linear Probing
  - Clusters are formed
  - Clusters can grow in size and slow down the (search, add, and remove) operations
  - Linear probing guarantees to find an empty element if the table is not full

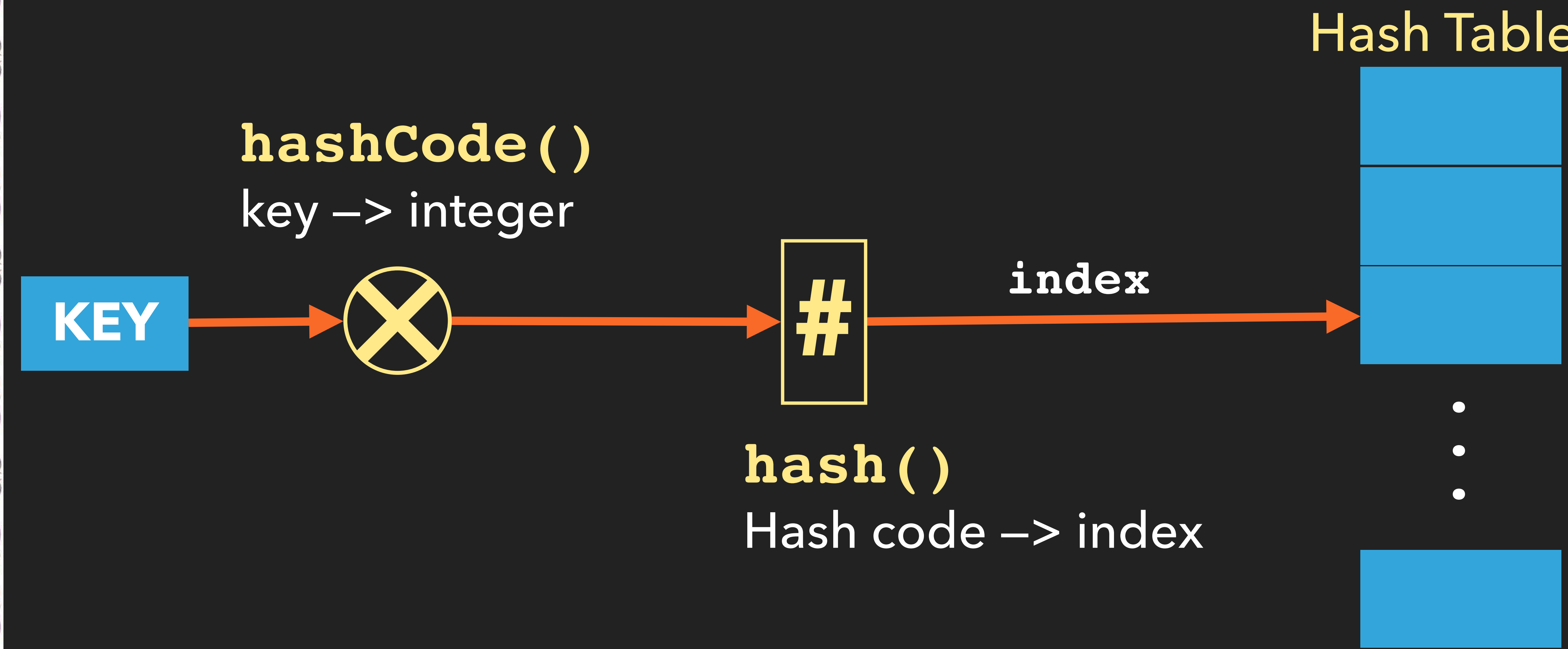
Collisions

- Quadratic Probing
  - Avoids clustering as in linear probing
  - Has its own clustering problem (secondary clustering: keys that collide with an occupied element use the same probing sequence)
  - does not guarantee finding an empty element

Hashing Performance

- Search, Add, Remove - `O(1)`
- Three factors may cause more collisions - affect the constant time performance
  - HashCode function
  - Hash function
  - Size of the hash table

Hash Code function



Hash Code function

- `hashCode()` is simple for integers - return the integer itself
- `hashCode()`? for double, strings, etc.
- Class **`Object`** has a method `hashCode()` that returns the reference to the object
- Override `hashCode()` to generate a hash code for the type you are using
- Wrapper classes and class **`String`** override **`hashCode()`**

Hash Code function

- General guidelines for `hashCode()`
  - You should override `hashCode()` whenever you override `equals()` to ensure that two equal objects return the same hash code
  - Multiple calls to `hashCode` return the same integer provided that the object's data did not change
  - Two unequal objects may have the same hash code, but you should implement `hashCode()` to minimize such cases

Hash Code function

- `hashCode` for primitive types
  - **`byte`** (8 bits), **`short`** (16 bits), **`char`** (16 bits) - cast to **`int`** (32 bits)
  - **`float`** (32 bits)- covert to **`int`** using `Float.floatToIntBits(float f)` - returns an `integer` that corresponds to the bit representation of **`f`**

`f` = 15.25 = 1111.01 = 1.11101 x 2^3 , m=11101, e=3

0 00000011 11101000000000000000000

`floatToIntBits(f)` returns $2^{18} + 2^{20}+2^{21}+2^{22}+2^{23}+2^{24}$

Hash Code function

- `hashCode` for primitive types
  - **`long`** - fold 64 bits into 32 bits -

```
(int) (number^(number>>32))
```

```
x =        0111001010111010100000011001100111110011010101010101010101011101
x>>32=     0000000000000000000000000000000001110010101110101000000110011001
hashCode(x) =                                      10000001111011111101010011000100
```

  - **`double`** - convert to **`long`** using `doubleToLongBits()` and fold into 32 bits

Hash Code function

✦ `hashCode` for strings

- Search keys are often strings - good `hashCode` method for strings
- Intuitive approach
  - Sum of the Unicode of all the characters in the string
  - Does not work well if two strings have the same letters

```
hashCode("tod") = hashCode("dot") = 0x0064 + 0x006F + 0x0074
```

- Better approach
  - Take the position of the character into consideration
  - Polynomial hashCode

$$hashCode(s) = s_0 . b^{n-1} + s_1 . b^{n-2} + ... + s_{n-1}, s_i = s . charAt(i)$$

Hash Code function

- `hashCode` for strings
  - Efficient polynomial hashCode evaluation:

$$hashCode(s) = s_{n-1} + b(s_{n-1} + b(\ldots + b(s_2 + b(s_1 + bs_0)))\ldots.)))$$

  - The weight `b` should be selected to minimize similar values for different strings. Experiments show that using `b` as `31`, `33`, `37`, `39`, and `41` minimize similar values
  - Class `String` overrides `hashCode()` method to use a polynomial hash code with `b=31`

Hash Function

- **`hash()`** method
  - Transform return value of `hashCode()` (can be a large integer) into a valid index in the hash table
  - For a hash table of size N, the most common hash function is **`index = hashCode % N (0 to N-1)`**
  - **`N`** should be prime to spread the indices evenly - time consuming to find a large prime number

Hash Function

- In practice, the size of the hash table is set to an integer power of 2 to simplify the hash function operation

- Java API `java.util.HashMap` sets the size of the hash table to a power of 2

```
hash(key) = hashCode(key) & (N-1)
```

- Bit-level operations **>>**, **^**, **&** are faster than *****, **/**, and **%**

Size of the Hash Table

- Choosing the size of the table
  - A prime number larger than the size of the data set - may take time to find such number
  - Bigger table - less collisions - waste of memory space
  - Tradeoff - space vs. time - use power of 2 to simplify calculations

Size of the Hash Table

- Load Factor - How full is the hash table?
  - Load Factor = # of added elements / size of the HT
  - High load factor results in more collisions - requires rehashing

Size of the Hash Table

- Rehashing - Increase the size of the table and rehash all the data to add it to the new table

- **`0.5 < load factor < 0.9`** (0.5 for probing and 0.9 for chaining)

Implementation

- Hash Table with chaining
- Array of pointers to linked lists

Implementation

```
HashMapEntry<K, V>
-key: K
-value: V
+HashMapEntry(K k, V v)
+getKey(): K
+getValue():V
+setKey(K k): void
+setValue(V v): void
+toString(): String
```

```
HashMap<K, V>
-hashTable: LinkedList<HashMapEntry<K,V>>[]
-loadFactor: double
-size: int
+HashMap()
+HashMap(int capacity)
+HashMap(int capacity, double loadFactor)
-trimToPowerOf2(int capacity): int
-hash(int hashCode): int
-rehash(): void
+get(K key): V
+put(K key, V value): V
+remove(K key): void
+containsKey(K key): boolean
+size(): int
+isEmpty(): boolean
+clear(): void
+toString(): String
+toList(): ArrayList<HashMapEntry<K,V>>
```

Implementation

class HashMapEntry

```
public class HashMapEntry<K, V> {
  private K key;
  private V value;
  public HashMapEntry(K k, V v) {
    key = k;
    value = v;
  }
  public K getKey() { return key; }
  public V getValue() { return value; }
  public void setKey(K k) {
    key = k;
  }
  public void setValue(V v) {
    value=v;
  }
  public String toString() {
    return "(" + key + ", " + value + ")";
  }
```

class HashMap

```
import java.util.ArrayList;
import java.util.LinkedList;
public class HashMap <K, V> {
  private int size;
  private double loadFactor;
  private LinkedList<HashMapEntry<K,V>>[] hashTable;
  // Constructors
  public HashMap() {
    this(100, 0.9);
  }
  public HashMap(int c) {
    this(c, 0.9);
  }
  public HashMap(int c, double lf) {
    hashTable = new LinkedList[trimToPowerOf2(c)];
    loadFactor = lf;
    size = 0;
  }
```

class HashMap

```
// private methods
private int trimToPowerOf2(int c) {
   int capacity = 1;
   while (capacity < c)
      capacity  = capacity << 1; // * 2
   return capacity;
}
private int hash(int hashCode) {
   return hashCode & (hashTable.length-1);
}
private void rehash() {
   ArrayList<HashMapEntry<K,V> list = toList();
   hashTable = new LinkedList[hashTable.length << 1];
   size = 0;
   for(HashMapEntry<K,V> entry: list)
      put(entry.getKey(), entry.getValue());

}
```

class HashMap

```
// public interface
  public int size() {
    return size;
  }
  public void clear() {
    size = 0;
    for(int i=0; i<hashTable.length; i++)
      if(hashTable[i] != null)
        hashTable[i].clear();
  }
  public boolean isEmpty() {
    return (size == 0);
  }
  // search for key - returns true if found
  public boolean containsKey(K key) {
    if(get(key) != null)
      return true;
    return false;
  }
```

class HashMap

```
// returns the value of key if found, null otherwise
  public V get(K key) {
    int HTIndex = hash(key.hashCode());
    if(hashTable[HTIndex] != null) {
      LinkedList<HashMapEntry<K,V>> ll = hashTable[HTIndex];
      for(HashMapEntry<K,V> entry: ll) {
        if(entry.getKey().equals(key))
          return entry.getValue();
      }
    }
    return null;
  }
```

class HashMap

```
// remove a key if found
  public void remove(K key) {
    int HTIndex = hash(key.hashCode());
    if (hashTable[HTIndex]!=null) { //key is in the hash map
      LinkedList<HashMapEntry<K,V>> ll = hashTable[HTIndex];
      for(HashMapEntry<K,V> entry: ll) {
        if(entry.getKey().equals(key)) {
          ll.remove(entry);
          size--;
          break;
        }
      }
    }
  }
```

```java
// adds a new key or modifies an existing key
public V put(K key, V value) {
  if(get(key) != null) { // The key is in the hash map
    int HTIndex = hash(key.hashCode());
    LinkedList<HashMapEntry<K,V>> ll;
    ll = hashTable[HTIndex];
    for(HashMapEntry<K,V> entry: ll) {
      if(entry.getKey().equals(key)) {
        V old = entry.getValue();
        entry.setValue(value);
        return old;
      }
    }
  }
```

class HashMap

```
// key not in the hash map - check load factor
    if(size >= hashTable.length * loadFactor)
      rehash();
    int HTIndex = hash(key.hashCode());
    //create a new LL if empty
    if(hashTable[HTIndex] == null){
      hashTable[HTIndex] = new LinkedList<>();
    }
    hashTable[HTIndex].add(new HashMapEntry<>(key, value));
    size++;
    return value;
}
```

class HashMap

```
// returns the elements of the hash map as a list
   public ArrayList<MapEntry> toList(){
      ArrayList<HashMapEntry<K,V>> list = new ArrayList<>();
      for(int i=0; i< hashTable.length; i++) {
         if(hashTable[i]!= null) {
            LinkedList<HashMapEntry<K,V>> ll = hashTable[i];
            for(HashMapEntry<K,V> entry: ll)
               list.add(entry);
         }
      } return list;
   }
// returns the elements of the hash map as a string
   public String toString() {
      String out = "[";
      for(int i=0; i<hashTable.length; i++) {
         if(hashTable[i]!=null) {
            for(HashMapEntry<K,V> entry: hashTable[i])
               out += entry.toString();
            out += "\n";
         }
      }
      out += "]"; return out;
   }
}
```

class HashMap

class Test

```
public static void main(String[] args) {
     HashMap<String, String> states = new HashMap<>(10);
     states.put("PA", "Pennsylvania");
     states.put("NY", "New York");
     states.put("MA", "Massachusetts");
     states.put("CA", "California");
     states.put("NJ", "New Jersey");
     states.put("OH", "Ohio");
     states.put("NM", "New Mexico");
     states.put("WA", "Washington");

     System.out.println(states);
     System.out.println("Code NJ is for " + states.get("NJ"));
     System.out.println("NY is in the map? " +
                                  states.containsKey("NY"));
     states.remove("MA");
     System.out.println(states);
     states.clear();
     System.out.println(states);
  }
```

Performance of the HashMap

Operation	Complexity	Operation	Complexity
`HashMap()`	**O(log n)**	`isEmpty()`	**O(1)**
`HashMap(int)`	**O(log n)**	`containsKey(K)`	**O(1)**
`HashMap(int, double)`	**O(log n)**	`get(K)`	**O(1)**
`trimToPowerOf2(int)`	**O(log n)**	`put(K, V)`	**O(1)**
`hash(int)`	**O(1)**	`remove(K)`	**O(1)**
`rehash()`	**O(n)**	`toList()`	**O(n)**
`size()`	**O(1)**	`toString()`	**O(n)**

SUMMARY

- Hash Tables
  - Efficient search operation (`O(1)`)
  - Hash Code - Hash function - size of the table
  - Load Factor and Rehashing
- Collisions
  - Chaining
  - Probing (linear, quadratic, double hashing)
- Implementation of the HashMap data structure