Start with the action of the non-linear sigma model (NLSM) on the target space $\mathcal{M}$ defined as:

$$S[X] = \frac{1}{2}\int_{\mathcal{M}} d^dx\sqrt{g}G_{AB}(X)\partial_\mu X^A\partial^\mu X^B \tag{1}$$

As before, one may need to obtain the 1-loop effective action for the observation of the renormalization equation for the theory. To do so, consider writing the field $X(x)$ in terms of a background $\bar{X}(x)$ and the corresponding quantum fluctuation $\xi(x)$. Assume that there is smooth map $X_s(x)$ such that $X_0 = \bar{X}(x)$, $X_1 = X(x)$ with $\dot{X}_0 = \xi$. Consider a curve $X_s$ in $\mathcal{M}$ which represents the geodesic between initial and final points, i.e. $X_0$ and $X_1$,

$$\ddot{X}^A_s(x) + \Gamma^A_{BC}\dot{X}^B_s(x)\dot{X}^C_s(x) = 0 \tag{2}$$

here $\Gamma^A_{BC}$ denotes the Christoffel symbol corresponding to the target space metirc $G_{AB}$. Then the expansion of the action around the background $\bar{X}$ is:

$$\begin{aligned} S[X] &= \sum_{n=0}^{\infty}\frac{1}{n!}\frac{d^n}{ds^n}S[X_s]\bigg|_{s=0} \\ &= \sum_{n=0}^{\infty}\frac{1}{n!}(\nabla_s)^n S[X_s]\bigg|_{s=0} \end{aligned} \tag{3}$$

where $\nabla_s$ is the covariant derivative along the curve $X^A_s$. Using the formulae:

$$\begin{aligned} \nabla_s\partial_\mu X^i = \partial_\mu\frac{dX^i}{ds} + \partial_\mu X^k\Gamma^i_{kj}\frac{dX^j}{ds} = \nabla_\mu\xi^i \ , \ \nabla_s G_{AB} = 0 \\ \nabla_s\frac{dX^i}{ds} = 0 \ , \ [\nabla_s, \nabla_\mu]Z^k = \frac{dX^i}{ds}\partial_\mu X^j R^k_{lij}Z^l \end{aligned} \tag{4}$$

where $Z^i$ is an arbitrary vector, one finds:

$$\begin{aligned} S[X] =& \frac{1}{2}\sqrt{g}\int_{\mathcal{M}} d^dx G_{AB}(\bar{X})\partial_\mu\bar{X}^A\partial^\mu\bar{X}^B + \sqrt{g}\int_{\mathcal{M}} d^dx G_{AB}\partial_\mu\xi^A\partial^\mu\bar{X}^B + \\ &+ \frac{1}{2}\sqrt{g}\int_{\mathcal{M}} d^dx\{G_{AB}\nabla_\mu\xi^A\nabla^\mu\xi^B + R_{ACDB}\xi^C\xi^D\partial_\mu\bar{X}^A\partial^\mu\bar{X}^B\} + \cdots \end{aligned} \tag{5}$$

From the quadratic part:

$$D \equiv -G_{AB}\Box + R_{ACDB}\partial_\mu\bar{X}^C\partial^\mu\bar{X}^D \tag{6}$$

So our interest is to compute:

$$\begin{aligned} \Gamma_{\text{1-loop}} &= \frac{1}{2}\mathrm{Tr}\ln D \\ &= -\frac{1}{2}\int_0^\infty\frac{dt}{t}\mathrm{Tr}e^{-tD} \end{aligned} \tag{7}$$

As before, it can be expanded with the heat kernel coefficients $a_k$ such that:

$$\Gamma_{\text{1-loop}} = -\frac{1}{2}\int\frac{dt}{t}\sum_{k\geq 0}t^{\frac{k-d}{2}}a_k(f, D) \tag{8}$$

where the first few terms are explicitly given in literatures. Introducing the UV and IR cutoff $\Lambda$ and $\mu$ respectively on the proper time integral:

$$\Gamma_{\text{1-loop}} = -\frac{1}{2}\int_{\Lambda^{-2}}^{\mu^{-2}} dt\sum t^{\frac{k-d}{2}-1}a_k(f, D) \tag{9}$$

Again, there are finitely many terms which involve its divergence. For instance, for the case of $d = 2$:

$$
\begin{aligned}
\Gamma^{div}_{\text{1-loop}} &= -\frac{1}{2}\int_{\Lambda^{-2}}^{\mu^{-2}} dt\{t^{-2}a_0(f,D) + t^{-1}a_2(f,D)\} \\
&= -\frac{1}{2}\{(\Lambda^2-\mu^2)a_0 + \ln\frac{\Lambda^2}{\mu^2}a_2\} \\
&= -\frac{1}{2}\{(\Lambda^2-\mu^2)(4\pi)^{-1}\int\sqrt{g}G_{AB} + \ln\frac{\Lambda^2}{\mu^2}(4\pi)^{-1}\int\sqrt{g}R_{DC}\partial_\alpha X^C\partial^\alpha X^D\}
\end{aligned}
\tag{10}
$$

Taking a derivative with respect to $\mu$:

$$
\begin{aligned}
\beta_{AB} &= \mu\frac{\partial}{\partial\mu}G^{\mu}_{AB} \\
&= \mu^2 G_{AB} + R_{AB}
\end{aligned}
\tag{11}
$$

Adding a generic potential term to the action, it becomes:

$$
S[X] = \frac{1}{2}\int_{\mathcal{M}} d^d x\sqrt{g}G_{AB}(X)\partial_\mu X^A\partial^\mu X^B + V(X) \tag{12}
$$

Similarly to the previous computations:

$$
D' = -G_{AB}\Box + R_{ACDB}\partial_\mu\bar{X}^C\partial^\mu\bar{X}^D + V^{(2)} \tag{13}
$$

Thus, in addition to the flow of the metric, there are flows of the couplings in the potential, which can be denoted as in this case:

$$
\mu\frac{\partial}{\partial\mu}V_\mu = V^{(2)} \tag{14}
$$

and each beta function can be deduced.