From previous arguments, we have the flow equations of a metric for two scalars φ_1 and φ_2 with the metric:

$$ds^2 = \frac{1}{\varphi_2^2}(d\varphi_1^2 + d\varphi_2^2) \tag{1}$$

with the homogeneous quadratic function of φ_1 and φ_2 as a potential:

$$V = \frac{1}{2}\sum_{i,j} M_{ij}\varphi^i\varphi^j \tag{2}$$

which are given by:

$$\mu\frac{\partial}{\partial\mu}g_{AB} = \frac{1}{8\pi^2}(\mathcal{R}_{AB} + \mu^2 R_{AB}) \tag{3}$$

$$\mu\frac{\partial}{\partial\mu}M_{ij} = 0 \tag{4}$$

where:

$$R_{AB} = \begin{bmatrix} -\frac{1}{\varphi_2^2} & 0 \\ 0 & -\frac{1}{\varphi_2^2} \end{bmatrix} \tag{5}$$

$$\mathcal{R}_{AB} = -\mathrm{Tr}(H_{CD}R_{CADB}) = -\frac{1}{\varphi_2^2}\begin{bmatrix} M_{22} & -M_{12} \\ -M_{12} & M_{11} \end{bmatrix} \tag{6}$$

Thus, for a variation of the RG scale, as the metric flows, there would be modifications of geodesics and distances. The original geodesics are obtained from the geodesics equation:

$$\frac{d^2\varphi^k}{ds^2} + \Gamma^k_{ij}\frac{d\varphi^i}{ds}\frac{d\varphi^j}{ds} = 0 \tag{7}$$

This leads:

$$\begin{cases} \ddot{\varphi}_1 - \frac{2}{\varphi_2}\dot{\varphi}_1\dot{\varphi}_2 = 0 \\ \ddot{\varphi}_2 + \frac{1}{\varphi_2}(\dot{\varphi}_1)^2 - \frac{1}{\varphi_2}(\dot{\varphi}_2)^2 = 0 \end{cases} \tag{8}$$

Suppose $\dot{\varphi_1} = 0$, then:

$$\begin{aligned} &\varphi_2\ddot{\varphi}_2 - \dot{\varphi_2}^2 = 0 \\ &\Leftrightarrow \frac{d}{ds}(\frac{\dot{\varphi}_2}{\varphi_2}) = \frac{\varphi_2\ddot{\varphi}_2 - \dot{\varphi_2}^2}{\varphi_2^2} = 0 \\ &\Leftrightarrow \frac{\dot{\varphi}_2}{\varphi_2} = c \end{aligned} \tag{9}$$

where c is a real constant. Integrating with respective to s gives $\varphi_2 = e^{c(s-s_0)}$, for some real constant s_0. If s stands for the arc length, there is a constraint about c such as:

$$1 = g_{ij}\dot{\varphi}^i\dot{\varphi}^j = \frac{1}{\varphi_2^2}(\dot{\varphi_1}^2 + \dot{\varphi_2}^2) = c^2 \tag{10}$$

Hence, $c = \pm 1$. This gives the simplest geodesic path, which is a vertical line in the (φ_1, φ_2) plane, transversed either up- or downward according to the choice of the sing $\pm$. From this, the distance between two points along this path can be deduced as $|\ln\frac{\varphi_2}{\tilde{\varphi}_2}|$.

Now let us see how these geodesics and distance change as the RG scale varies. If the flow is small enough, the curvature on the right hand side of the flow equation can be replaced with the curvature corresponding to the initial metric. Then, taking $\mu = \Lambda(1-\epsilon)$ with $\epsilon \ll 1$:

$$\begin{aligned} g_{AB}(\mu) &= g_{AB}(\Lambda) + \tilde{g}_{AB} \\ &= \frac{1}{\varphi_2^2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{8\pi^2}(-\frac{1}{\varphi_2^2}\begin{bmatrix} M_{22} & -M_{12} \\ -M_{12} & M_{11} \end{bmatrix}\ln\frac{\mu}{\Lambda} + \frac{1}{2}(\Lambda^2 - \mu^2)\frac{1}{\varphi_2^2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}) \\ &\sim \frac{1}{\varphi_2^2}\begin{bmatrix} 1 + \frac{1}{8\pi^2}(M_{22}\epsilon + \epsilon\Lambda^2) & -\frac{1}{8\pi^2}M_{12}\epsilon \\ -\frac{1}{8\pi^2}M_{12}\epsilon & 1 + \frac{1}{8\pi^2}(M_{11}\epsilon + \epsilon\Lambda^2) \end{bmatrix} \end{aligned} \tag{11}$$

Thus, the Christoffel symbols are: