The renormalization group equation for the metric has been given:

$$g^{\mu}_{ab} = g^{\Lambda}_{ab} + \frac{\epsilon}{8\pi^2}\{\mathrm{Tr}(\mathrm{Hess}(V)R_{acbd}) - \Lambda^2 R_{ab})\} \tag{1}$$

Since

$$\mathrm{Tr}(\mathrm{Hess}(V)R_{acbd}) = \frac{1}{k}\begin{bmatrix} -M_2 & m \\ m & -M_1 \end{bmatrix} \tag{2}$$

$$R_{ab} = -\frac{1}{\varphi_2^2}\mathbb{1} \tag{3}$$

the metric at a scale μ can be written:

$$\tilde{g}_{ab} := g^{\mu}_{ab} = \begin{bmatrix} \frac{k}{\varphi_2^2} - \frac{\epsilon}{8\pi^2}\{\frac{M_2}{k} - (\frac{\Lambda}{\varphi_2})^2\} & \frac{\epsilon}{8\pi^2}\frac{m}{k} \\ \frac{\epsilon}{8\pi^2}\frac{m}{k} & \frac{k}{\varphi_2^2} - \frac{\epsilon}{8\pi^2}\{\frac{M_1}{k} - (\frac{\Lambda}{\varphi_2})^2\} \end{bmatrix} \tag{4}$$

Then inverse metric is:

$$\tilde{g}^{ab} = \begin{bmatrix} \frac{\varphi_2^2}{k} + \frac{\epsilon}{8\pi^2}(\frac{\varphi_2^2}{k})^2(\frac{M_2}{k} - (\frac{\Lambda}{\varphi_2})^2) & -\frac{\epsilon}{8\pi^2}\frac{m}{k}(\frac{\varphi_2^2}{k})^2 \\ -\frac{\epsilon}{8\pi^2}\frac{m}{k}(\frac{\varphi_2^2}{k})^2 & \frac{\varphi_2^2}{k} + \frac{\epsilon}{8\pi^2}(\frac{\varphi_2^2}{k})^2(\frac{M_1}{k} - (\frac{\Lambda}{\varphi_2})^2) \end{bmatrix} \tag{5}$$

and one may compute:

$$\tilde{\Gamma}^1_{11} = -\frac{\epsilon}{8\pi^2}\frac{m}{k^2}\varphi_2 \tag{6}$$

$$\tilde{\Gamma}^1_{12} = -\frac{1}{\varphi_2} - \frac{\epsilon}{8\pi^2}\frac{M_2}{k^2}\varphi_2 \tag{7}$$

$$\tilde{\Gamma}^1_{22} = \frac{\epsilon}{8\pi^2}\frac{m}{k^2}\varphi_2 \tag{8}$$

$$\tilde{\Gamma}^2_{11} = \frac{1}{\varphi_2} + \frac{\epsilon}{8\pi^2}\frac{M_1}{k^2}\varphi_2 \tag{9}$$

$$\tilde{\Gamma}^2_{12} = \frac{\epsilon}{8\pi^2}\frac{m}{k^2}\varphi_2 \tag{10}$$

$$\tilde{\Gamma}^2_{22} = -\frac{1}{\varphi_2} - \frac{\epsilon}{8\pi^2}\frac{M_1}{k^2}\varphi_2 \tag{11}$$

Therefore, the new geodesic equations are:

$$\begin{cases} \ddot{\varphi}_1 - \frac{2}{\varphi_2}\dot{\varphi}_1\dot{\varphi}_2 = \frac{\epsilon}{8\pi^2}\frac{\varphi_2}{k^2}(m\dot{\varphi}_1^2 + 2M_2\dot{\varphi}_1\dot{\varphi}_2 - m\dot{\varphi}_2^2) & (12) \\ \ddot{\varphi}_2 + \frac{1}{\varphi_2}(\dot{\varphi}_1^2 - \dot{\varphi}_2^2) = \frac{\epsilon}{8\pi^2}\frac{\varphi_2}{k^2}(-M_1\dot{\varphi}_1^2 - 2m\dot{\varphi}_1\dot{\varphi}_2 + M_1\dot{\varphi}_2^2) & (13) \end{cases}$$

Again, from (12), it can be seen that the vertical lines are no longer the geodesics, and there need some correction terms if $m \neq 0$. Restricting corrected geodesics with the initial conditions that the initial values and initial derivatives are equal to the ones for uncorrected geodesics, which is written as $(\varphi_1, \varphi_2) = (\varphi_1^0, \varphi_2^0 e^{\pm\frac{s}{\sqrt{k}}})$, find the new geodesics. What I did tried was as following: First, as I did, pluged the original solution $\varphi_2 = \varphi_2^0 e^{\pm\frac{s}{\sqrt{k}}}$ into eq. (13) and solved for φ_1. This ended up with:

$$\varphi_1 = \varphi_1^0 - \frac{\epsilon}{24\pi^2}\frac{m}{k}(\varphi_2^3 - (\varphi_2^0)^3) \tag{14}$$

And then from this, plug $\dot{\varphi}_1 = -\frac{\epsilon}{8\pi^2}\frac{m}{k}\varphi_2^2\dot{\varphi}_2$ into eq.(12) back to back.This gives:

$$\begin{aligned} &\varphi_2\{-\frac{\epsilon}{8\pi^2}\frac{m}{k}(2\varphi_2\dot{\varphi}_2^2 + \varphi_2^2\ddot{\varphi}_2)\} + 2\frac{\epsilon}{8\pi^2}\frac{m}{k}\varphi_2^2\dot{\varphi}_2^2 = \frac{\epsilon}{8\pi^2}\frac{\varphi_2^2}{k^2}\{2M_2(-\frac{\epsilon}{8\pi^2}\frac{m}{k})\varphi_2^2\dot{\varphi}_2^2 - m\dot{\varphi}_2^2\} \\ \Rightarrow &\varphi_2\ddot{\varphi}_2 = \frac{1}{k}(\dot{\varphi}_2^2 + \frac{\epsilon}{8\pi^2}\frac{2M_2}{k^2}\varphi_2^2\dot{\varphi}_2^2) \end{aligned} \tag{15}$$

However, by solving this differential equation, I stucked with:

$$\int \exp\left(-\frac{\ln\varphi_2}{k} - \frac{\epsilon}{8\pi^2}\frac{M_2}{k^3}\varphi_2^2\right)\mathrm{d}\varphi_2 = \int C\mathrm{d}s \tag{16}$$

here C is a constant which can be determined with initial conditions.

So I tried another way. Instead of $\varphi_2 = \varphi_2^0 e^{\pm\frac{s}{\sqrt{k}}}$, plugging $\varphi_2 = \varphi_2^0 e^{\pm\frac{s}{\sqrt{k}}} + \epsilon\delta\varphi_2$ with the constraints stated above into eq. (13), then solve for φ_1, but it didn't work well too.

I had been suspicious if I did my calculation correctly and so checked from the beginning, but it seemed fine to me. So I suppose I have had mistakes and/or misunderstandings with computations of geodesic equations with corrections.