9-5-25 (WEEK 1)

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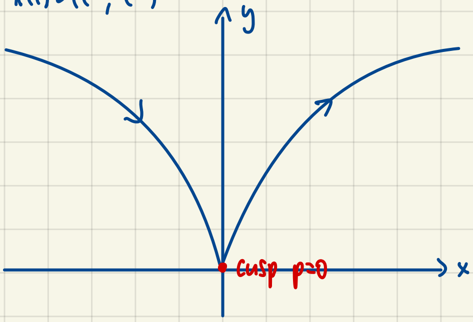
EXAMPLE

$X(t)=(t^3, t^2)$



It's not smooth because $X'(0)=\vec{0}$, even if t^3, t^2 are smooth

DEFAULT ASSUMPTIONS

- $X:(a,b)\to\mathbb{R}^2$ is C^k
- $X'(t)\neq 0\ \forall t$ (immersion) (which implies inj)
- $X:(a,b)\longrightarrow\Gamma\ (=X(a,b))\subseteq\mathbb{R}^2$ is a homeomorphism

we assume embedded unless stated otherwise

Then, $X:(a,b)\to\mathbb{R}^2$ is called an embedding and Γ is an (embedded) curve

EXAMPLE

$X: \underbrace{S^1}_{\text{unit circle}} \longrightarrow \mathbb{R}^2$ is an injective immersion with period 2π

In fact, we say it is a simple closed curve

DEFINITION

$X:(a,b)\to\mathbb{R}^2$ is called a local parametrization of Γ if $X(t)\in C^k$, $X'(t)\neq 0\ \forall t$, $X(t)\in\Gamma$

PROOF OF REPARAMETRIZATION BEING DIFFEOMORPHISM

$X=\phi(t)=(\varphi_1(t), \varphi_2(t))$

$\phi(t_0)=p\in\Gamma$

$\Rightarrow \varphi_2(t)=f(\varphi_1(t))$ for t near t_0

Then, $\phi'(t)=(\varphi_1'(t), f'(\varphi_1(t))\varphi_1'(t)) = \varphi_1'(t)\cdot(1, f'(\varphi_1(t)))$. Hence, it is nonzero.

$\therefore\ 0\neq\varphi_1'(t)$

$\therefore$ By IFT, we have C^k: $t=\varphi_1^{-1}(x)$ □

EXAMPLE

$F(x)=(x, f(x))$ is a local parametrization of Γ

Then, $\phi=F\circ\varphi_1$ is a diffeomorphism

Say $\psi:(c,d)\to\mathbb{R}^2$ is a local parametrization of Γ near p, and $\psi(\tau_0)=p$, $\psi(\tau)=(\psi_1(\tau), \psi_2(\tau)) \Rightarrow \psi=F\circ\psi_1$

Then, we have $\phi\circ\varphi_1^{-1}=F$ and $\psi\circ\psi_1^{-1}=F$, then $\psi=\phi\circ(\varphi_1^{-1}\circ\psi_1)$