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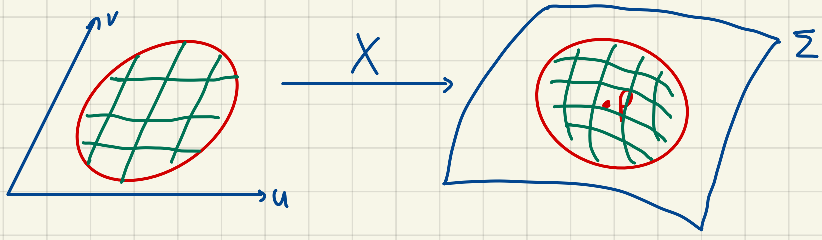
RECALL

For X: open $\Omega \subseteq \mathbb{R}^2 \longrightarrow \Sigma \subseteq \mathbb{R}^3$, it is a local parametrization of surface Σ if:

- C^k immersion: $\partial_u X \times \partial_v X \neq 0$
- $X: \Omega \to X(\Omega) \ni p$ is a homeomorphism

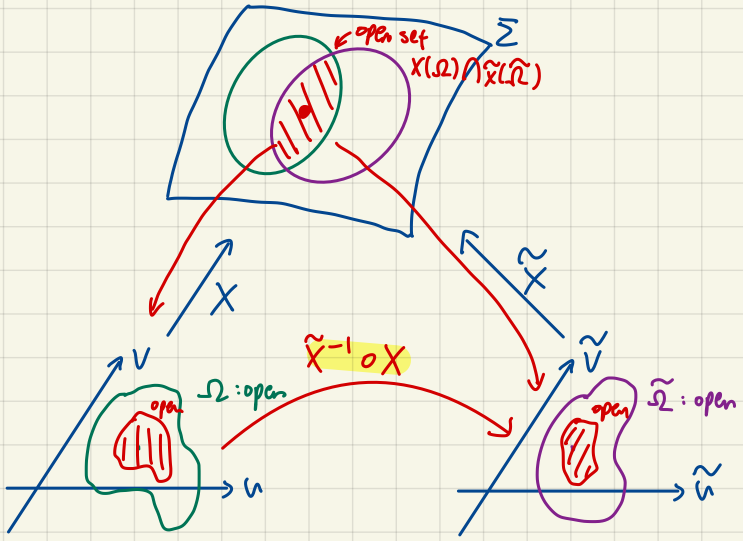
  └ Relatively open set in Σ: coordinate neighborhood of p

Coordinate neighborhood:



CHANGE IN PARAMETERS (TRANSITION MAPS)

Consider the intersection of coordinate neighborhoods used as change in parameters:



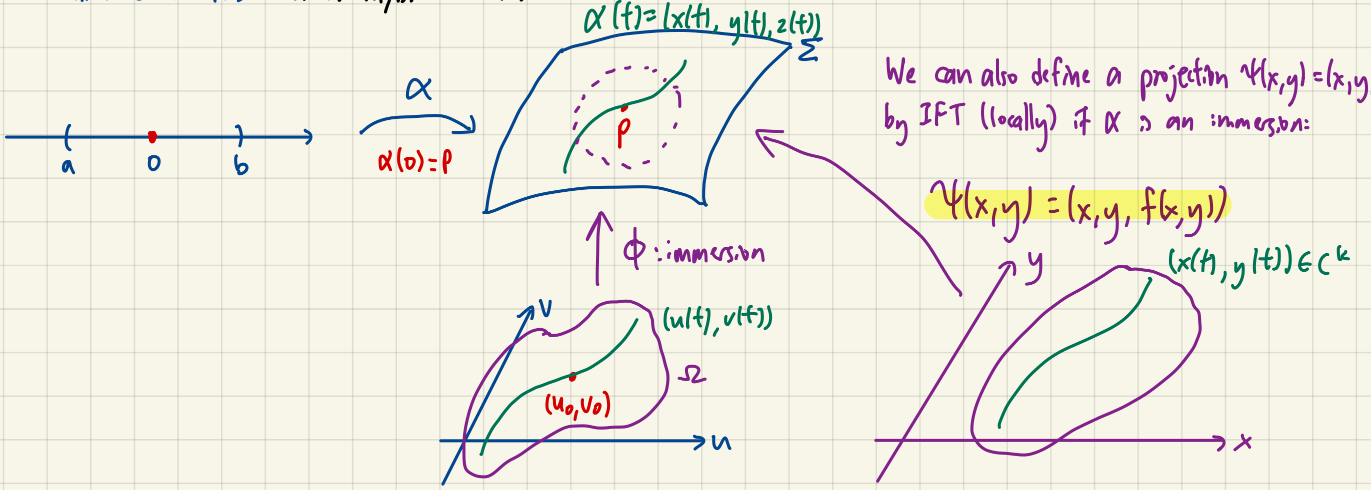
Formally, $\tilde{X}^{-1} \circ X : X^{-1}(X(\Omega) \cap \tilde{X}(\tilde{\Omega})) \longrightarrow \tilde{X}^{-1}(X(\Omega) \cap \tilde{X}(\tilde{\Omega}))$ is a diffeomorphism

PARAMETRIZATION

We want C^k immersion $\alpha(t): (a,b) \longrightarrow \Sigma \subseteq \mathbb{R}^3$

We can also define a projection $\Psi(x,y) = (x,y,f(x,y))$ by IFT (locally) if α is an immersion:

$\Psi(x,y) = (x,y,f(x,y))$



Formally, we have $(u(t), v(t)) = \phi^{-1} \circ \alpha(t) = (\phi^{-1} \circ \Psi)(x(t), y(t))$

Notice, $\alpha(t) = \phi(u(t), v(t)) = (x(u(t), v(t)), y(u(t), v(t)), z(u(t), v(t)))$
Denote ϕ as X.

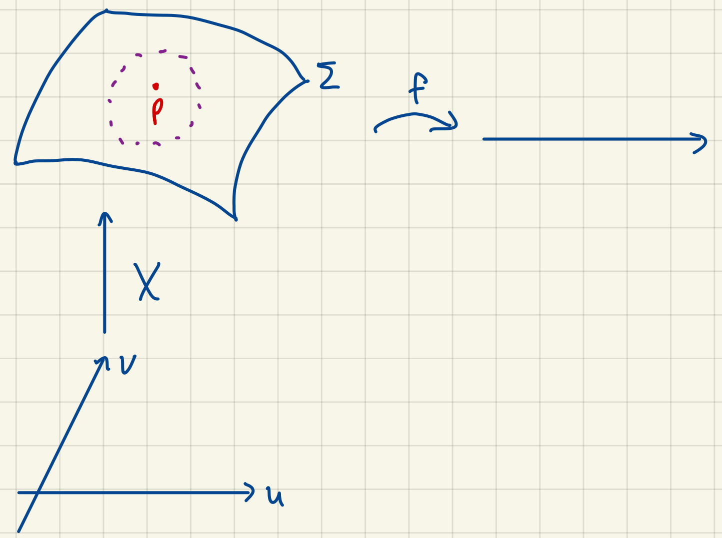
$$\alpha'(t) = \underbrace{\partial_u X \frac{du}{dt} + \partial_v X \frac{dv}{dt}}$$

linearly indep as X is immersion

STATEMENT

$f: \Sigma \subseteq \mathbb{R}^3 \longrightarrow \mathbb{R}$ is continuous near $p \Leftrightarrow f \circ X: \Omega \to \mathbb{R}$ is continuous near $(u_0, v_0) = X^{-1}(p)$ for some local parametrization $X: \Omega \to \Sigma$ near p

Intuition:



DEFINITION

$f: \Sigma \to \mathbb{R}$ is C^k near p if $f \circ X: \Omega \to \mathbb{R}$ is C^k