

Pearl 110 - Intelligent Interaction

Solution guidelines to tutorial Assignment 7.2 and 7.3



Assignment 7.2

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Objective: compute the probability that 4 out of 10 cars are blue.

Being blue is the event of interest!

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Compute the probability that out of 10 cars, 4 are blue and 6 are not blue

$$P(b = 4, \neg b = 6) = ?$$

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It is similar to a coin toss event sequence: you flip a coin 10 times - what is the probability of having 4 tails (and 6 non-tails)?

Can be formulated as a Bernoulli trial experiment.

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Probability of finding a blue car

Probability of NOT finding a blue car (find other color cars)

Look into available information

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We use this information to estimate the probability (as frequency) of the different classes:

Probability of a green car

$$P(g) = \frac{30}{100} = 0.3$$

Probability of a red car

$$P(r) = \frac{50}{100} = 0.5$$

Probability of a blue car

$$P(b) = \frac{20}{100} = 0.2$$

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The probability of the opposite event (not having a blue car) is important for the problem solution.

(It's also the sum of the probability of the other classes.)


$$P(\neg b) = 1 - P(b) = 0.8$$

Solution

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$$P(b = 4, \neg b = 6) = \binom{10}{4} p^4 (1 - p)^6 = \frac{10!}{4!6!} (0.2)^4 (0.8)^6$$

Assignment 7.3

Given the following data set, containing data from two classes C_1 and C_2 :

$$\mathbf{C}_1 = \{ 3; 4.56; -2; 0; 1.25; -3.45; 2.11; 2.45 \}$$

$$\mathbf{C}_2 = \{ 4.8; 4.1; 3.1; 2.7; 2.2; 2; 4.4; 4.5; 2; 8.8; 7.3; 9.6 \}$$

Assuming that they are drawn from two normal distributions, answer the following problems:

- What are the values of the mean μ_1 and standard deviation σ_1 for class \mathbf{C}_1 ?
- What are the values of the mean μ_2 and standard deviation σ_2 for class \mathbf{C}_2 ?
- Determine the analytical form of a decision criterion (i.e. a decision boundary for 1-dimensional data) for the given classes.
- According to the decision criterion that you defined, in which classes the new samples $x_1 = 3.2$, $x_2 = 2.3$, $x_3 = -5$, $x_4 = 1$ are classified?

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a) and **b)**: mean and standard deviation

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We use the **maximum likelihood estimation** of the mean and standard deviation:

mean:
$$\hat{\mu} = \frac{1}{n} \sum_{k=1}^n x_k$$

variance:
$$\hat{\sigma}^2 = \frac{1}{n} \sum_{k=1}^n (x_k - \hat{\mu})^2$$

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$$\hat{\sigma} = \sqrt{\frac{1}{n} \sum_{k=1}^n (x_k - \hat{\mu})^2}$$

Reminder: the standard deviation is the square root of the variance

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Mean: $\hat{\mu} = \frac{1}{n} \sum_{k=1}^n x_k$

$$\mu_1 = \frac{3+4.56-2+0+1.25-3.45+2.11+2.45}{8} = 0.99$$

$$\mu_2 = \frac{4.8+4.1+3.1+2.7+2.2+2+4.4+4.5+2+8.8+7.3+9.6}{12} = 4.625$$

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$$\hat{\sigma}_2 = \sqrt{\frac{1}{n_2} \sum_{k=1}^{n_2} (x_k - \hat{\mu}_2)^2} = \sqrt{\frac{1}{12} \sum_{k=1}^{12} (x_k - 4.625)^2} = 2.5$$

Approximated to the
1st decimal digit (as
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The two classes have
the same standard
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c) decision criterion

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Class-conditional probability
(class is C)

evidence

priori probability
for class C

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We need to find the mathematical formulation of the two posterior probabilities

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8 samples in \mathbf{C}_1 (cardinality $|\mathbf{C}_1| = \mathbf{8}$) and **12** in \mathbf{C}_2 (cardinality $|\mathbf{C}_2| = \mathbf{12}$): we have a total of **20** samples

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We can compute the prior probability as the frequency:

$$P(C_1) = \frac{|\mathbf{C}_1|}{|\mathbf{C}_1| + |\mathbf{C}_2|} = \frac{8}{8+12} = 0.4$$

$$P(C_2) = \frac{|\mathbf{C}_2|}{|\mathbf{C}_1| + |\mathbf{C}_2|} = \frac{12}{8+12} = 0.6$$

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We estimated these values already!

$$P(x|C_2) = \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}$$

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$$P(x|C_2) = \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}} = \frac{1}{2.5\sqrt{2\pi}} e^{-\frac{(x-4.625)^2}{2(2.5)^2}}$$

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1. $p(x)$ is on both side and we can simplify it
2. We computed already all the remaining values

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$$\frac{1}{2.5\sqrt{2\pi}} e^{-\frac{(x-0.99)^2}{2(2.5)^2}} \cdot 0.4 = \frac{1}{2.5\sqrt{2\pi}} e^{-\frac{(x-4.625)^2}{2(2.5)^2}} \cdot 0.6$$

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The coefficient at the beginning of the left and right part is the same. We can remove it.

$$e^{-\frac{(x-0.99)^2}{2(2.5)^2}} \cdot 0.4 = e^{-\frac{(x-4.625)^2}{2(2.5)^2}} \cdot 0.6$$

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$$e^{-\frac{(x-0.99)^2}{2(2.5)^2}} \cdot 0.4 = e^{-\frac{(x-4.625)^2}{2(2.5)^2}} \cdot 0.6$$

To simplify the mathematical form of the exponential, we take the natural logarithm on both side of the equation

$$\ln 0.4 - \frac{(x-0.99)^2}{2(2.5)^2} = \ln 0.6 - \frac{(x-4.625)^2}{2(2.5)^2}$$

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We have to solve the 2nd order equation.
Note: use the calculator to compute the value of the logarithms

$$-5.07 - x^2 + 1.98x - 0.98 = -x^2 + 9.25x - 21.39$$

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$$x = 2.11$$

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**Decision
criterion!**

$$x^* = 2.11$$

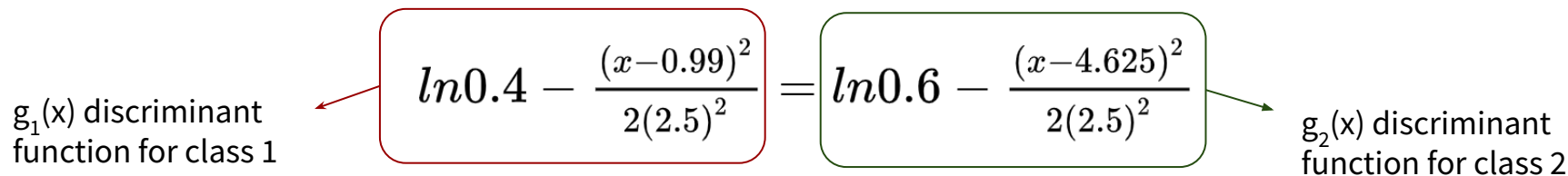
c) decision criterion: decision regions

The decision criterion (more generally, decision boundary) is the 'locus of points' (set of points that satisfy a condition) that satisfy:

$$g_1(x) \text{ discriminant function for class 1} \quad \leftarrow \ln 0.4 - \frac{(x-0.99)^2}{2(2.5)^2} = \ln 0.6 - \frac{(x-4.625)^2}{2(2.5)^2} \rightarrow g_2(x) \text{ discriminant function for class 2}$$

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The diagram shows the equation $\ln 0.4 - \frac{(x-0.99)^2}{2(2.5)^2} = \ln 0.6 - \frac{(x-4.625)^2}{2(2.5)^2}$. A red arrow points from the left side of the equation to the text " $g_1(x)$ discriminant function for class 1". A green arrow points from the right side of the equation to the text " $g_2(x)$ discriminant function for class 2".

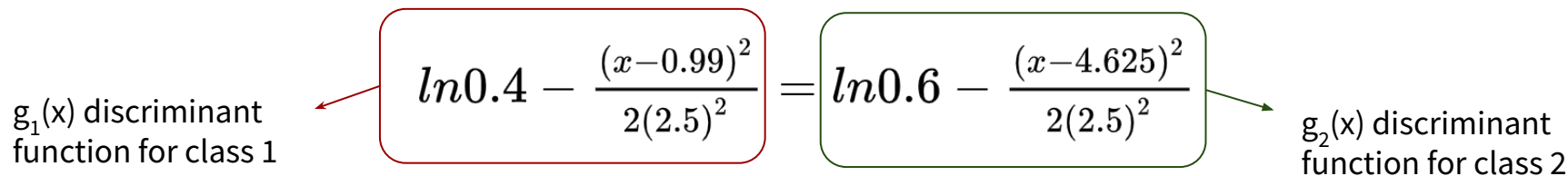
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We classify x in class 1, if $g_1(x) > g_2(x)$ [see lecture slides]

$$-5.07 - x^2 + 1.98x - 0.98 > -x^2 + 9.25x - 21.39$$

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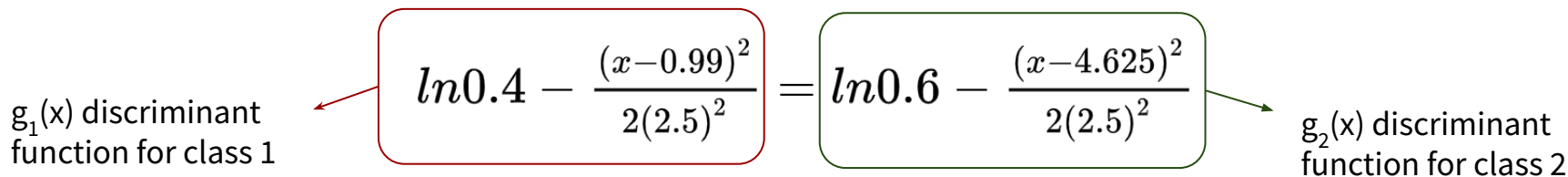
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$$-6.05 + 1.98x > 9.25x - 21.39$$

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We classify x in class 1, if $g_1(x) > g_2(x)$ [see lecture slides]

$$-5.06 - x^2 - 0.98 + 1.98x > -x^2 + 9.25x - 21.39$$

$$-6.05 + 1.98x > 9.25x - 21.39$$

$$15.34 > 7.27x$$

c) decision criterion: decision regions

The decision criterion (more generally, decision boundary) is the 'locus of points' (set of points that satisfy a condition) that satisfy:

$$g_1(x) \text{ discriminant function for class 1} \quad \boxed{\ln 0.4 - \frac{(x-0.99)^2}{2(2.5)^2}} = \boxed{\ln 0.6 - \frac{(x-4.625)^2}{2(2.5)^2}} \quad g_2(x) \text{ discriminant function for class 2}$$

We **classify x in class 1, if $g_1(x) > g_2(x)$** [see lecture slides]

$$-5.06 - x^2 - 0.98 + 1.98x > -x^2 + 9.25x - 21.39$$

$$-6.04 + 1.98x > 9.25x - 21.39$$

$$15.34 > 7.27x$$

$$2.11 > x$$

We **classify in class 1 if $x < 2.11$**
(decision region)

Decision criterion!

$$x^* = 2.11$$

d) classifications

According to the decision criterion that you defined, in which classes the new samples $x_1 = 3.2$, $x_2 = 2.3$, $x_3 = -5$, $x_4 = 1$ are classified?

$$x^* = 2.11$$

If $x < x^*$, classify in C_1
If $x > x^*$, classify in C_2

d) classifications

According to the decision criterion that you defined, in which classes the new samples $x_1 = 3.2$, $x_2 = 2.3$, $x_3 = -5$, $x_4 = 1$ are classified?

$$x^* = 2.11$$

If $x < x^*$, classify in C_1
If $x > x^*$, classify in C_2

$x_1 = 3.2$ is labeled as C_2

$x_2 = 2.3$ is labeled as C_2

$x_3 = -5$ is labeled as C_1

$x_4 = 1$ is labeled as C_1

d) classifications

According to the decision criterion that you defined, in which classes the new samples $x_1 = 3.2$, $x_2 = 2.3$, $x_3 = -5$, $x_4 = 1$ are classified?

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If $x < x^*$, classify in C_1
If $x > x^*$, classify in C_2

$x_1 = 3.2$ is labeled as C_2

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$x_3 = -5$ is labeled as C_1

$x_4 = 1$ is labeled as C_1

to double check you can compute the probability using the discriminant functions estimated earlier