Calculus 1B - Supervised Self Study exercises, week 2

1. Evaluate the following indefinite integrals:

1.1 $\displaystyle\int \frac{4x^3}{(x^4+1)^2}\,dx$ (substitute $u = x^4 + 1$)

1.2 $\displaystyle\int \frac{1}{5s+4}\,ds$ (substitute $u = \ldots$)

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2. See Thomas' Calculus, page 344, for the substitution formula in definite integrals (the limits of integration change by substitution!). Use this formula in evaluating the following definite integrals:

   2.1 $\displaystyle\int_0^1 r\sqrt{1-r^2}\,dr$ (substitute $u = 1 - r^2$)

   2.2 $\displaystyle\int_0^{\frac{\pi}{3}} \frac{4\sin\theta}{1-4\cos\theta}\,d\theta$ (substitute $u = \ldots$)

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3. Use the integration by parts formula

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

on page 454 to evaluate the following integrals:

3.1 $\int x \ln x \, dx$ (take $f(x) = \ln x$ and $g'(x) = x$)

3.2 $\int xe^{4x}\, dx$

3.3 $\int x^2 e^{4x}\, dx$

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4. See Thomas' Calculus, page 458, for the integration by parts formula for definite integrals. Use that formula to calculate:

$$\int_0^{\pi} t \sin t \, dt$$

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5. Study the following parts of Section 8.7 in Thomas' Calculus:
   - the definition of improper integrals of Type I on page 496
   - the definition of improper integrals of Type II on page 499
   - the examples of all types of improper integrals on page 504

   Then consider the following integral:

$$\int_0^4 \frac{dx}{\sqrt{4-x}}$$

   5.1 What type* of improper integral is this?

   5.2 Evaluate the integral. (Don't forget to use limit notation!)

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*Type I.1, I.2 or I.3 in the definition on p. 496?
Or Type II.1, II.2 or II.3 in the definition on p. 499?

Calculus 1B - Answers to SSS exercises, week 2

6. Recall the definition of *even* and *odd* functions on page 6 of Thomas' Calculus. Note that $\cos x$ is an even function, since $\cos(-x) = \cos(x)$ for every $x \in \mathbb{R}$. Similarly, $\sin x$ is an odd function, since $\sin(-x) = -\sin(x)$ for every $x \in \mathbb{R}$.

   6.1 Show that $\sin x \cos x$ is an odd function.

   6.2 Show that $\sin^2 x$ is an even function.

   Now study Theorem 8 on page 346, which deals with definite integrals of symmetric functions.

   6.3 Use Theorem 8, together with the statements in 6.1 and 6.2 above, to evaluate the following two integrals:

$$\int_{-\pi}^{\pi} \sin x \cos x \, dx \quad \text{and} \quad \int_{-\pi}^{\pi} \sin^2 x \, dx$$

   6.4 **(Advanced exercise)** Prove Theorem 8(b) on page 346†.

† Hint: Look at the proof of Theorem 8(a) on page 346 and use similar ideas.