Calculus 1B - Supervised Self Study exercises, week 5

1. Calculate for $z_1 = 3 + 4i$ and $z_2 = -7 + 11i$ the expressions given below. (Write them in the form $a + ib$, with $a, b \in \mathbb{R}$.)

   1.1 $\overline{z_1}$

   1.2 $|z_1|$

   1.3 $z_1 + 3z_2$

   1.4 $z_1 - i \cdot z_2$

   1.5 $z_1 z_2$

   1.6 $\dfrac{z_2}{z_1}$

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2. Euler's formula* states that, for every $\theta \in \mathbb{R}$,

$$e^{i\theta} = \cos\theta + i\sin\theta\,.$$

Use this formula to prove that, for every $\theta \in \mathbb{R}$:

2.1 $\overline{e^{i\theta}} = e^{-i\theta}$

2.2 $|e^{i\theta}| = 1$

2.3 $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$

2.4 $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

*See p. 1070 or AP-30 of Thomas' Calculus

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3. In SSS exercise 2.2, you proved that for every $\theta \in \mathbb{R}$,

$$|e^{i\theta}| = 1\,.$$

3.1 Determine all $\theta \in \mathbb{R}$ for which the following equality holds:

$$e^{i\theta} = 1\,.$$

3.2 Determine all $\theta \in \mathbb{R}$ for which the following equality holds:

$$e^{i\theta} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\,i\,.$$

4. Recall the well-known double angle formulas[†]:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$

Use De Moivre's Theorem to prove these two formulas.

(Hint: First study Example 4 on p. 1072 or AP-32.)

[†]See, for example, p. 26 of Thomas' Calculus

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5. Solve the following equations:

   5.1 $z^2 + 2z + 10 = 0$

   5.2 $z^4 = 2 + i \cdot 2\sqrt{3}$

   (That is, find the 4th roots of the complex number $2 + i \cdot 2\sqrt{3}$.)