Calculus 1B - Supervised Self Study exercises, week 4

1. Find the derivative of each of the following functions:

   1.1 $f(x) = (1 + x)^5$

   1.2 $g(x) = (1 + 2x)^5$

   1.3 $h(x) = (1 + 2\sin x)^5$

   1.4 $k(x) = (1 + 2\sin x^2)^5$

   1.5 $p(x) = \frac{1}{\ln x}$

   1.6 $q(x) = \frac{x^2}{\ln x}$

   1.7 $r(x) = e^{3x^7}$

   1.8 $s(x) = e^{\cos^2 x}$

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2. Find an anti-derivative for each of the following functions:

   2.1 $f(x) = (1+x)^5$

   2.2 $g(x) = (1+2x)^5$

   2.3 $h(x) = \cos 7x$

   2.4 $p(x) = 3e^{24x}$

   2.5 $q(x) = 7e^{\frac{x}{7}} + \sin 4x$

   2.6 $r(x) = \frac{1}{2x+1}$

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3. Read the subsections ‘Separable Differential Equations’ on pp. 428–430 and ‘Slope Fields: Viewing Solution Curves’ on pp. 516-517.

   Now consider the following differential equation:

$$y' = x - xy\,. \tag{1}$$

   Also study the figure on the next page.

   3.1 Why are the red arrows that intersect the y-axis all horizontal?
   3.2 Show that the constant function $y = 1$ is a solution to Equation (1). Do you also see this in the figure?
   3.3 Find the general solution $y = y(x)$ to Equation (1) using the ‘separation of variables’ technique*.
   3.4 Find the particular solution $y = y(x)$ to Equation (1) satisfying the initial condition $y(1) = 0$.
   (In other words, find the function whose graph corresponds to the blue line in the figure on the next page.)

*As in Examples 1 and 2 on pp. 429–430.

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3. (Continued.)

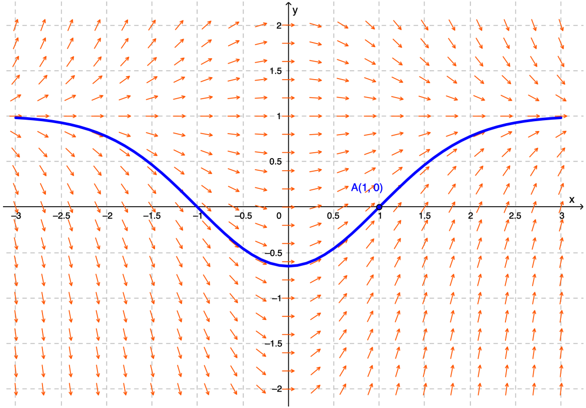

Figure: Slope field for $y' = x - xy$, with the particular solution curve through the point $A(0, 1)$ in blue.

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4. Consider the following differential equation:

$$y' = \frac{x - y}{x + 1}, \qquad x > -1. \tag{2}$$

4.1 Is this a separable differential equation? Recall (see p. 429) that this is only the case if Equation (2) can be written in the form

$$y' = g(x)H(y).$$

4.2 Is this a linear differential equation? Recall (see p. 522) that this is only the case if Equation (2) can be written in the form[†]

$$y' + P(x)y = Q(x).$$

4.3 Solve Equation (2).

[†] the so-called *standard form* of the linear equation

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Advanced exercise

5. Consider the following differential equation:

$$y' = 4y^2 + y \,. \tag{3}$$

Note that this is not a linear differential equation.

5.1 Show that by defining

$$z(x) = \frac{1}{y(x)} \,,$$

Equation (3) can be written as a linear differential equation in the function z.

5.2 Use your solution to 5.1 to solve Equation (3) using the 'integrating factor' technique.

5.3 Solve Equation (3) using separation of variables.