Pearl 110 - Intelligent Interaction

Solution guidelines to tutorial Assignment 7.2 and 7.3

Assignment 7.2

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Objective: compute the probability that 4 out of 10 cars are blue.

Being blue is the event of interest!

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Compute the probability that out of 10 cars, 4 are blue and 6 are not blue

$$P(b = 4, \neg b = 6) = ?$$

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It is similar to a coin toss event sequence: you flip a coin 10 times - what is the probability of having 4 tails (and 6 non-tails)?
Can be formulated as a Bernoulli trial experiment.

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Probability of finding a blue car

Probability of NOT finding a blue car (find other color cars)

Look into available information

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We use this information to estimate the probability (as frequency) of the different classes:

Probability of a green car

$$P(g) = \frac{30}{100} = 0.3$$

Probability of a red car

$$P(r) = \frac{50}{100} = 0.5$$

Probability of a blue car

$$P(b) = \frac{20}{100} = 0.2$$

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Probability of a blue car

$$P(b) = \frac{20}{100} = 0.2$$

The probability of the opposite event (not having a blue car) is important for the problem solution.

(It's also the sum of the probability of the other classes.)

$$P(\neg b) = 1 - P(b) = 0.8$$

Solution

We know that in a specific city, out of 100 cars, 30 are green, 50 are red and 20 are blue. In the parking lot of the municipality building there are 10 cars. What is the probability that four of them are blue?

$$P(b=4, \neg b=6) = \binom{10}{4} p^4 (1-p)^6 = \frac{10!}{4!6!}(0.2)^4(0.8)^6$$

Assignment 7.3

Given the following data set, containing data from two classes C_1 and C_2:

$\mathbf{C_1}$ = { 3; 4.56; −2; 0; 1.25; −3.45; 2.11; 2.45 }

$\mathbf{C_2}$ = { 4.8; 4.1; 3.1; 2.7; 2.2; 2; 4.4; 4.5; 2; 8.8; 7.3; 9.6 }

Assuming that they are drawn from two normal distributions, answer the following problems:

a. What are the values of the mean $\boldsymbol{\mu_1}$ and standard deviation $\boldsymbol{\sigma_1}$ for class $\mathbf{C_1}$?
b. What are the values of the mean $\boldsymbol{\mu_2}$ and standard deviation $\boldsymbol{\sigma_2}$ for class $\mathbf{C_2}$?
c. Determine the analytical form of a decision criterion (i.e. a decision boundary for 1-dimensional data) for the given classes.
d. According to the decision criterion that you defined, in which classes the new samples $x_1 = 3.2$, $x_2 = 2.3$, $x_3 = -5$, $x_4 = 1$ are classified?

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We use the **maximum likelihood estimation** of the mean and standard deviation:

mean: $\hat{\mu} = \frac{1}{n} \sum_{k=1}^{n} x_k$

variance: $\hat{\sigma}^2 = \frac{1}{n} \sum_{k=1}^{n} (x_k - \hat{\mu})^2$

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$$\hat{\sigma} = \sqrt{\frac{1}{n} \Sigma_{k=1}^{n} (x_k - \hat{\mu})^2}$$

Reminder: the standard deviation is the square root of the variance

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Mean: $\hat{\mu} = \frac{1}{n}\sum_{k=1}^{n} x_k$

$$\mu_1 = \frac{3+4.56-2+0+1.25-3.45+2.11+2.45}{8} = 0.99$$

$$\mu_2 = \frac{4.8+4.1+3.1+2.7+2.2+2+4.4+4.5+2+8.8+7.3+9.6}{12} = 4.625$$

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Approximated to the 1st decimal digit (as per assignment)

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Standard deviation: $$\hat{\sigma} = \sqrt{\frac{1}{n}\Sigma_{k=1}^{n}(x_k - \hat{\mu})^2}$$

The two classes have the same standard deviation.

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c) decision criterion

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priori probability
for class C

Class-conditional probability
(class is **C**)

evidence

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We need to find the mathematical formulation of the two posterior probabilities

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8 samples in $\mathbf{C_1}$ (cardinality $\mathbf{|C_1| = 8}$) and **12** in $\mathbf{C_2}$ (cardinality $\mathbf{|C_2| = 12}$): we have a total of **20** samples

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We can compute the prior probability as the frequency:

$$P(C_1) = \frac{|C_1|}{|C_1|+|C_2|} = \frac{8}{8+12} = 0.4$$

$$P(C_2) = \frac{|C_2|}{|C_1|+|C_2|} = \frac{12}{8+12} = 0.6$$

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We estimated these values already!

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$$P(x|C_2) = \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}} = \frac{1}{2.5\sqrt{2\pi}} e^{-\frac{(x-4.625)^2}{2(2.5)^2}}$$

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1. **p(x) is on both side and we can simplify it**
2. **We computed already all the remaining values**

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$$\frac{1}{2.5\sqrt{2\pi}} e^{-\frac{(x-0.99)^2}{2(2.5)^2}} \cdot 0.4 = \frac{1}{2.5\sqrt{2\pi}} e^{-\frac{(x-4.625)^2}{2(2.5)^2}} \cdot 0.6$$

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The coefficient at the beginning of the left and right part is the same. We can remove it.

$$e^{-\frac{(x-0.99)^2}{2(2.5)^2}} \cdot 0.4 = e^{-\frac{(x-4.625)^2}{2(2.5)^2}} \cdot 0.6$$

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$$e^{-\frac{(x-0.99)^2}{2(2.5)^2}} \cdot 0.4 = e^{-\frac{(x-4.625)^2}{2(2.5)^2}} \cdot 0.6$$

To simplify the mathematical form of the exponential, we take the natural logarithm on both side of the equation

$$ln0.4 - \frac{(x-0.99)^2}{2(2.5)^2} = ln0.6 - \frac{(x-4.625)^2}{2(2.5)^2}$$

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We have to solve the 2nd order equation.
Note: use the calculator to compute the value of the logarithms

$$-5.07 - x^2 + 1.98x - 0.98 = -x^2 + 9.25x - 21.39$$

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$$x = 2.11$$

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Decision criterion!

$$\boxed{x^* = 2.11}$$

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$g_1(x)$ discriminant function for class 1

$g_2(x)$ discriminant function for class 2

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$g_1(x)$ discriminant function for class 1

$$\boxed{ln0.4 - \frac{(x-0.99)^2}{2(2.5)^2}} = \boxed{ln0.6 - \frac{(x-4.625)^2}{2(2.5)^2}}$$

$g_2(x)$ discriminant function for class 2

We classify x in class 1, if $g_1(x) > g_2(x)$ [see lecture slides]

$$-5.07 - x^2 + 1.98x - 0.98 > -x^2 + 9.25x - 21.39$$

c) decision criterion: decision regions

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$g_2(x)$ discriminant function for class 2

We classify x in class 1, if $g_1(x) > g_2(x)$ [see lecture slides]

$$-5.07 - x^2 + 1.98x - 0.98 > -x^2 + 9.25x - 21.39$$

$$-6.05 + 1.98x > 9.25x - 21.39$$

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$g_2(x)$ discriminant function for class 2

We classify x in class 1, if $g_1(x) > g_2(x)$ [see lecture slides]

$$-5.06 - x^2 - 0.98 + 1.98x > -x^2 + 9.25x - 21.39$$

$$-6.05 + 1.98x > 9.25x - 21.39$$

$$15.34 > 7.27x$$

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$g_2(x)$ discriminant function for class 2

We **classify x in class 1, if $g_1(x) > g_2(x)$** [see lecture slides]

$$-5.06 - x^2 - 0.98 + 1.98x > -x^2 + 9.25x - 21.39$$

$$-6.04 + 1.98x > 9.25x - 21.39$$

$$15.34 > 7.27x$$

$$2.11 > x$$

We classify in class 1 if x<2.11 (decision region)

Decision criterion!

$$\boxed{x^* = 2.11}$$

d) classifications

According to the decision criterion that you defined, in which classes the new samples $x_1 = 3.2$, $x_2 = 2.3$, $x_3 = -5$, $x_4 = 1$ are classified?

$$x^* = 2.11$$

If $x < x^*$, classify in C_1
If $x > x^*$, classify in C_2

d) classifications

According to the decision criterion that you defined, in which classes the new samples $x_1 = 3.2$, $x_2 = 2.3$, $x_3 = -5$, $x_4 = 1$ are classified?

$$x^* = 2.11$$

If $x < x^*$, classify in C_1
If $x > x^*$, classify in C_2

$x_1 = 3.2$ is labeled as C_2

$x_2 = 2.3$ is labeled as C_2

$x_3 = -5$ is labeled as C_1

$x_4 = 1$ is labeled as C_1

d) classifications

According to the decision criterion that you defined, in which classes the new samples $x_1 = 3.2$, $x_2 = 2.3$, $x_3 = -5$, $x_4 = 1$ are classified?

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If $x < x^*$, classify in C_1
If $x > x^*$, classify in C_2

$x_1 = 3.2$ is labeled as C_2

$x_2 = 2.3$ is labeled as C_2

$x_3 = -5$ is labeled as C_1

$x_4 = 1$ is labeled as C_1

to double check you can compute the probability using the discriminant functions estimated earlier