

Calculus 1B - Answers to SSS exercises, week 5

1. 1.1 $\overline{z_1} = 3 - 4i$

1.2 $|z_1| = 5$

1.3 $z_1 + 3z_2 = -18 + 37i$

1.4 $z_1 - i \cdot z_2 = 14 + 11i$

1.5 $z_1 z_2 = -65 + 5i$

1.6 $\frac{z_2}{z_1} = \frac{23}{25} + \frac{61}{25}i$

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2. -

(Hint: Use the fact that cosine and sine are even and odd functions, respectively; see SSS exercise 6 of week 2.)

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3. 3.1 $\theta = k \cdot 2\pi$, $k \in \mathbb{Z}$ (that is, θ is any integer multiple of 2π)

3.2 $\theta = \frac{5\pi}{4} + k \cdot 2\pi$, $k \in \mathbb{Z}$ (or $\theta = -\frac{3\pi}{4} + k \cdot 2\pi$, $k \in \mathbb{Z}$)

Note: The answer $\theta = \frac{\pi}{4} + k \cdot 2\pi$, $k \in \mathbb{Z}$ is **not** correct!*

*Remember that $\cos \theta = -\frac{\sqrt{2}}{2}$ and $\sin \theta = -\frac{\sqrt{2}}{2}$ imply that $\tan \theta = 1$, but this does **not** imply that $\theta = \arctan(1) = \frac{\pi}{4}$!!

If both $\cos \theta$ and $\sin \theta$ are negative, we know that we're looking at the third quadrant, so $\pi \leq \theta \leq \frac{3\pi}{2}$ (or, equivalently, $-\pi \leq \theta \leq -\frac{\pi}{2}$). As we explain in Exercise 1 in the second lecture video for week 3 ('Representation of complex numbers'): draw an Argand diagram to avoid mistakes!

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4. -

(Hint: Apply the same approach as in Example 4 on p. 1072 or AP-32 of Thomas' Calculus, but with $n = 2$ instead of $n = 3$.)

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5. 5.1 $z = -1 - 3i$ or $z = -1 + 3i$

5.2 $z_0 = \sqrt{2}e^{i\frac{\pi}{12}}, z_1 = \sqrt{2}e^{i\frac{7\pi}{12}}, z_2 = \sqrt{2}e^{i\frac{13\pi}{12}}, z_3 = \sqrt{2}e^{i\frac{19\pi}{12}}$