Calculus 1B - Answers to SSS exercises, week 4

1. 1.1 $f'(x) = 5(1+x)^4$

   1.2 $g'(x) = 10(1+2x)^4$

   1.3 $h'(x) = 10\cos x(1+2\sin x)^4$

   1.4 $k'(x) = 20x\cos x^2(1+2\sin x^2)^4$

   1.5 $p'(x) = \frac{-1}{x\ln^2 x}$

   1.6 $q'(x) = \frac{2x}{\ln x} - \frac{x}{\ln^2 x}$

   1.7 $r'(x) = 21x^6 e^{3x^7}$

   1.8 $s'(x) = -2\sin x\cos x\, e^{\cos^2 x}$

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2. For each of the answers below, adding an arbitrary constant will yield another correct anti-derivative.

    2.1 $F(x) = \frac{1}{6}(1+x)^6$

    2.2 $G(x) = \frac{1}{12}(1+2x)^6$

    2.3 $H(x) = \frac{1}{7}\sin 7x$

    2.4 $P(x) = \frac{1}{8}e^{24x}$

    2.5 $Q(x) = 49e^{\frac{x}{7}} - \frac{1}{4}\cos 4x$

    2.6 $R(x) = \frac{1}{2}\ln|2x+1|$

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3. 3.1 In each point (x, y) on the y-axis, we have that $x = 0$. Substituting $x = 0$ in the equation $y' = x - xy$ yields $y' = 0$, corresponding to a slope of 0 (and therefore a horizontal red arrow) in each point on the y-axis.

   3.2 If $y = 1$, then $y' = 0$ (which is why the red arrows on the line $y = 1$ in the slope field are all horizontal). Clearly, the equation $y' = x - xy$ holds for all x if $y' = 0$ and $y = 1$, so $y = 1$ is indeed a solution to the equation $y' = x - xy$.

   3.3 $y(x) = 1 - Ce^{-\frac{1}{2}x^2}$

   3.4 $y(x) = 1 - e^{\frac{1}{2}(1-x^2)}$

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4. 4.1 No.

 4.2 Yes. The standard form of differential equation

$$y' = \frac{x-y}{x+1}, \qquad x > -1$$

is

$$y' + \frac{1}{x+1}y = \frac{x}{x+1}, \qquad x > -1.$$

In other words,

$$P(x) = \frac{1}{x+1} \quad \text{and} \quad Q(x) = \frac{x}{x+1}.$$

 4.3 $y(x) = \dfrac{x^2 + C}{2(x+1)}$

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Advanced exercise

5. 5.1 If we define

$$z(x) = \frac{1}{y(x)}\,,$$

then

$$y(x) = \frac{1}{z(x)} \quad \text{and} \quad y'(x) = \frac{-z'(x)}{(z(x))^2}$$

and the equation $y' = 4y^2 + 4$ can be written as $z' + z = -4$.

5.2 $y(x) = \dfrac{e^x}{C - 4e^x}$

5.3 Using the fact* that $\frac{1}{4y^2+y} = \frac{1}{y} - \frac{4}{4y+1}$, we find that $y(x) = \dfrac{Ce^x}{1-4Ce^x}$ or, choosing $C' = \frac{1}{C}$, $y(x) = \dfrac{e^x}{C' - 4e^x}$

(Choosing a different technique to solve the differential equation obviously doesn't change the solution.)

*See Section 8.4; this is **not** part of the course!