Calculus 1B - Supervised Self Study exercises, week 3

1. Consider the basic power series

$$\sum_{n=0}^{\infty} x^n ,$$

which is divergent for $|x| \geq 1$ and convergent for $-1 < x < 1$ (with sum $\frac{1}{1-x}$).

For the power series below, find the series' radius and interval of convergence; in case of convergence, find the sum.

1.1 $\sum_{n=0}^{\infty} (x+5)^n$

1.2 $\sum_{n=0}^{\infty} (2x)^n$

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2. The series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots$$

converges to $\sin x$ for all x. Find a series that converges to $\cos 2x$ for all x.

(Hint: Use the above series for $\sin x$ and the Term-by-Term Differentiation Theorem on page 598 of Thomas' Calculus.)

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3. Recall the definition of the Taylor polynomial on page 604. Then find the Taylor polynomials of orders 0, 1, 2, and 3 generated by $f(x) = \frac{1}{x+2}$ at $x = 0$.

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4. Find the Maclaurin series for the function $\sinh x = \frac{e^x - e^{-x}}{2}$.

   (Hint: Instead of using the definition of Maclaurin series on page 603, you may also use the well-known Maclaurin series for e^x on page 604 to find the Maclaurin series for $\sinh x$.)

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5. Study the frequently used Taylor series of the functions $\frac{1}{1-x}$, e^x, $\sin x$, and $\cos x$ in Table 10.1 on page 620 of Thomas' Calculus*. Then find the sum of each of the following series:

5.1 $$\sum_{n=0}^{\infty} \frac{x^{4n}}{n!}$$

5.2 $$\sum_{n=0}^{\infty} \frac{(-1)^n}{5^{2n+1}(2n+1)!}$$

5.3 $$\sum_{n=0}^{\infty} \frac{(-1)^n}{5^n \cdot n!}$$

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*Note that you are expected to know these four Taylor/Maclaurin series by heart, or to be able to very quickly calculate them (see the Calculus 1B Study Guide on Canvas)