UNIVERSITY OF TWENTE.

Tag : Calc1B.22-23.SampleTest2

Course : **Calculus 1B**

Duration : 2 hours

Motivate all your answers.

Simplify all your answers as much as possible.

The use of electronic devices is not allowed.

The answer form

Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored. If you need more space, you can write in the frame provided at the end of the answer form. Clearly refer to this space in the original answer.

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1. [3 pt] *Only write your final answers to (a) and (b) in the frame on the answer form.*

   a. [2pt] Evaluate the integral $\int_{-6}^{0} \sqrt{36 - x^2}\, dx$.

   (Hint: Sketch the integrand and use the relation between area and integral.)

   b. [1pt] Find the average value av(f) of $f(x) = -2\sqrt{36 - x^2}$ on the interval $[-6, 0]$.

2. [2 pt] *Only write your final answer in the frame on the answer form.*

   Determine
   $$\frac{d}{dx} \int_{-x^2}^{0} \frac{1}{1 + \cos^2(\theta)}\, d\theta\,.$$

3. [4 pt] Evaluate
   $$\int 4x^2 e^{2x}\, dx\,.$$

4. [4 pt] Evaluate
   $$\int_{e}^{\infty} \frac{1}{x\,(\ln x)^2}\, dx\,.$$

5. [2 pt] Consider the following series:
   $$\sum_{n=0}^{\infty} \frac{x^{-2n}}{4^n\, n\,!}\,.$$

   Exactly one of the expressions below equals the sum of this series. Which one?

   A) $\sin(4x^2)$

   B) $\cos(4x^2)$

   C) e^{4x^2}

   D) $\dfrac{1}{1 - 4x^2}$

   E) $\sin\left(\frac{1}{4x^2}\right)$

   F) $\cos\left(\frac{1}{4x^2}\right)$

   G) $e^{\frac{1}{4x^2}}$

   H) $\dfrac{1}{1 - \frac{1}{4x^2}}$

Continue on the next page.

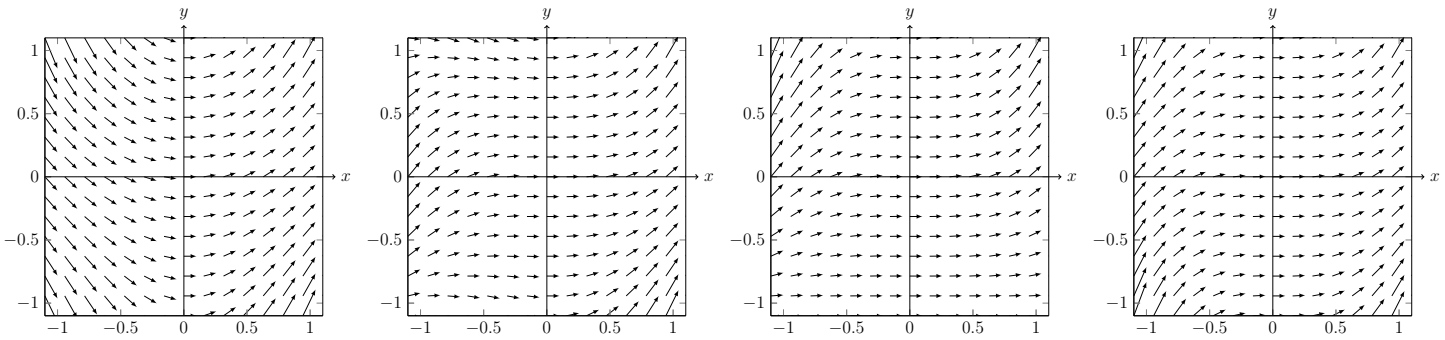

Figure 1: The four slope fields of Question 6a.

6. [5 pt] Consider the following differential equation:

$$\frac{dy}{dx} - x^2 y^2 = x^2 \,.$$

   a. [2pt] Exactly one of the four slope fields in Figure 1 is a slope field for this differential equation. Which one? Clearly motivate your answer.

   b. [3pt] Solve the differential equation subject to the initial condition $y(0) = 0$.

7. [4 pt] Given is the function

$$f(x) = \frac{1}{\pi + x} \,.$$

   Determine the Taylor polynomial of order 3 generated by f at $x = 0$.

8. [2 pt] *Only write your final answer in the frame on the answer form.*

   Let z be the complex number with $|z| = 2$ and $\arg(z) = \pi/6$.

   Determine the absolute value $|w|$ and the argument $\arg(w)$ of the complex number

$$w = \bar{z} \cdot z^3 \,.$$

9. [4 pt] Find all complex solutions z to the following equation:

$$(z - i)^3 = -27 \,.$$

   Express the solutions in Cartesian form (i.e., in the form $a + i\,b$ with $a, b \in \mathbb{R}$).

10. [6 pt] Determine the general solution $y = y(x)$ to the following differential equation:

$$y'' + 4y' + 4y = 4\sin(2x) \,, \quad y(0) = 1 \,, \quad y'(0) = 1 \,.$$

The End.

Total: 36 points