Calculus 1B - Supervised Self Study exercises, week 1

1. Find the most general antiderivative[1] for each of the following functions. Check your answers by differentiation.

    1.1 $f_1(x) = 2x^{-3}$

    1.2 $f_2(x) = -x^{-3} + x^3 - 1$

    1.3 $f_3(x) = \dfrac{2}{5x}$

    1.4 $f_4(x) = 1 + \dfrac{4}{3x} - \dfrac{1}{x^2}$

[1] See Theorem 8 on p. 280 of Thomas' Calculus for the definition of 'the most general antiderivative'

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2. 2.1 Graph the function $f(x) = x^3$ with $0 \leq x \leq 1$.

   2.2 Solve Exercise 6 of Section 5.1 in the book.

   2.3 Partition the interval $[0, 1]$ into n subintervals of equal width. Find a formula for the Riemann sum[2] for f on $[0, 1]$ corresponding to this partition, choosing the points c_k to be the right endpoints of the subintervals.

   2.4 Evaluate $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^3}{n^4}$.

   (Hint: See p. 309 for the sum of the first n cubes.)

   2.5 Evaluate $\int_0^1 x^3\, dx$.

[2]See the definition on p. 312 of Thomas' Calculus

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3. Study Example 2 on page 327 of Thomas' Calculus. Then calculate the following derivatives:

   3.1 $\dfrac{d}{dx}\displaystyle\int_1^x 3t^2 dt$

   3.2 $\dfrac{d}{dx}\displaystyle\int_1^{\sin x} 3t^2 dt$

   3.3 $\dfrac{d}{dx}\displaystyle\int_x^1 \frac{1}{t^4+1} dt$

   3.4 $\dfrac{d}{dx}\displaystyle\int_{\sin x}^1 \frac{1}{t^4+1} dt$

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4. Let f be the function given by $f(x) = 1 + \sqrt{1 - x^2}$.

   4.1 Evaluate the integral $\int_{-1}^{1} f(x)\,dx$.

   (Hint: You don't need to find the antiderivative of $1 + \sqrt{1 - x^2}$ to calculate this integral. Instead, start by graphing the function $f(x) = 1 + \sqrt{1 - x^2}$ and use the definition on p. 319.)

   Let g be the function given by $g(x) = -2 + \sqrt{1 - x^2}$.

   4.2 Compute the area of the region bounded by the x-axis, the lines $x = -1$ and $x = 1$, and the curve $y = g(x)$.

   4.3 Evaluate the integral $\int_{-1}^{1} g(x)\,dx$.