1. Two solutions to the second-order differential equation

$$y'' + 5y' + 4y = 0 (1)$$

are the functions $y(x) = e^{-x}$ and $y(x) = e^{-4x}$.

Show that:

1.1 for any $c_1, c_2 \in \mathbb{R}$,

$$y(x) = c_1 e^{-x}$$
 and $y(x) = c_2 e^{-4x}$

are solutions to Equation (1) as well.

1.2 for any $c_1, c_2 \in \mathbb{R}$,

$$y(x) = c_1 e^{-x} + c_2 e^{-4x}$$

is a solution to Equation (1) as well.

2. Solve the initial value problem

$$y'' + 3y' + 2y = 0$$

with
$$y(0) = 0$$
 and $y'(0) = -1$.

3. Solve the following second-order differential equations:

3.1
$$y'' + 4y = \sin(x)$$

3.2
$$y'' = x + y$$

4. Exercise 9, Section 17.2: Solve the equation

$$y'' - y = e^x + x^2.$$

(Hint: When choosing a candidate for your particular solution y_p , pay attention to the remarks on slides 48 and 49 of this week's lecture slides, or -equivalently- the middle column of Table 17.1 in the book, Section 17.2. See also Example 5 in Section 17.2.)