

Tag : Calc1B.22-23.SampleTest1

Course : **Calculus 1B**

Duration : 2 hours

**Motivate all your answers.****Simplify all your answers as much as possible.****The use of electronic devices is not allowed.****The answer form**

Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored. If you need more space, you can write in the frame provided at the end of the answer form. Clearly refer to this space in the original answer.

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1. [2 pt] Given is the function

$$f(x) = \frac{x^2}{x+1}.$$

Determine which of the following eight expressions is a Riemann sum for  $f$  on the interval  $[0, 2]$  obtained by dividing  $[0, 2]$  into  $n$  equal subintervals and using the right-hand endpoint of each subinterval to evaluate  $f$ .

*Choose from the alternatives below.*

A)  $\sum_{k=1}^n \frac{k^2}{k+1} \cdot \frac{2}{n}$

E)  $\sum_{k=1}^n \frac{2k^2}{2k+n} \cdot \frac{2}{n}$

B)  $\sum_{k=1}^n \frac{k^2}{k+1} \cdot \left(\frac{2}{n}\right)^2$

F)  $\sum_{k=1}^n \frac{2k^2}{2k+n} \cdot \left(\frac{2}{n}\right)^2$

C)  $\sum_{k=1}^n \frac{2k^2}{2k+1} \cdot \frac{2}{n}$

G)  $\sum_{k=1}^n \frac{2(k-1)^2}{2(k-1)+n} \cdot \frac{2}{n}$

D)  $\sum_{k=1}^n \frac{2k^2}{2k+1} \cdot \left(\frac{2}{n}\right)^2$

H)  $\sum_{k=1}^n \frac{2(k-1)^2}{2(k-1)+n} \cdot \left(\frac{2}{n}\right)^2$

2. [2 pt] *Only write your final answer in the frame on the answer form.*

Determine

$$\frac{d}{dx} \int_2^{2x} \ln(1+t^2) dt.$$

3. [4 pt] Evaluate

$$\int_0^\infty \frac{\arctan x}{1+x^2} dx.$$

Continue on the next page.

4. [4 pt] Evaluate

$$\int_0^{\pi^2} \sin(\sqrt{x}) dx.$$

5. [4 pt] *Only write your final answers to (a) and (b) in the frames on the answer form.*

Consider the following power series:

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{3^n}.$$

a. [2pt] Compute the interval of convergence of this series.

b. [2pt] Compute the sum of this series (within its interval of convergence).

6. [5 pt] Solve the following initial value problem:

$$x^2 \frac{dy}{dx} + 2xy = \cos x, \quad x > 0, \quad y(\pi) = 1.$$

7. [4 pt] Given is the function

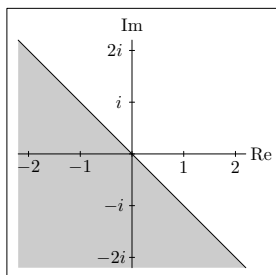
$$f(x) = \ln(1 - x).$$

Determine the Taylor polynomial of order 3 generated by  $f$  at  $x = -1$ .

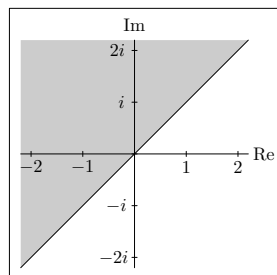
8. [2 pt] In one of the four figures below, the shaded (grey) area corresponds to the complex numbers  $z = x + iy$  that satisfy the inequality

$$|z - i| \geq |z - 1|.$$

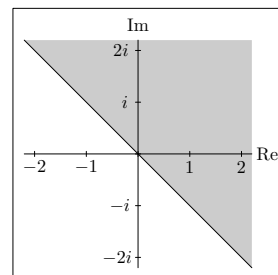
In which one? Clearly motivate your answer.



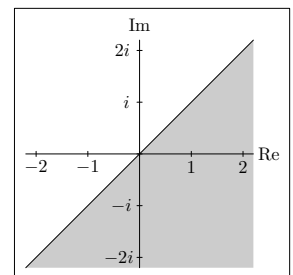
(A)



(B)



(C)



(D)

9. [3 pt] *Only write your final answer in the frame on the answer form.*

Give the six complex solutions  $z$  (in polar form) of the following equation:

$$z^6 - 2z^3 - 3 = 0.$$

10. [6 pt] Solve the following initial value problem:

$$y'' + 2y' - 3y = 4e^x, \quad y'(0) = 0, \quad y(0) = -1.$$

**The End.**

**Total: 36 points**