応用幾何 ma・pa　演習 08　解答例.　(2023.11.21)

ベクトル場 $\boldsymbol{v}(x,y,z)=(-y,x,z)$ の 流線 $\boldsymbol{x}(t)=(x(t),y(t),z(t))$ で $\boldsymbol{x}(0)=(1,1,1)$ を満たすものを求めよ.

(解答例)

$$\boldsymbol{x}'(t)=(x'(t),y'(t),z'(t))=\boldsymbol{v}(\boldsymbol{x}(t))=(-y(t),x(t),z(t))$$

$$\therefore \begin{cases} x'(t)=-y(t) \\ y'(t)=\ \ x(t) \\ z'(t)=\ \ z(t) \end{cases} \quad \therefore \begin{array}{l} x''(t)=-y'(t)=-x(t) \quad \therefore\ x(t)=a\cos t+b\sin t \\ y(t)=-x'(t)=-(-a\sin t+b\cos t)=a\sin t-b\cos t \\ z(t)=ce^t \end{array} \quad (a,b,c:\text{任意定数})$$

$$\therefore\ \boldsymbol{x}(t)=(a\cos t+b\sin t,a\sin t-b\cos t,ce^t) \qquad \boldsymbol{x}(0)=(a,-b,c)=(1,1,1)$$

$$\therefore\ \boldsymbol{x}(t)=(\cos t-\sin t,\sin t+\cos t,e^t)$$