応用幾何 ma・pa 演習 02 解答例. (2023.10.06)

曲線 $\boldsymbol{x}(t) = \big(t^2, \cos 2t, \sin 2t\big)$ $(t \in \mathbb{R})$ に対して，次の量を求めよ.

(1) (i) 速度ベクトル $\boldsymbol{x}'(t)$，速度 $\|\boldsymbol{x}'(t)\|$ (ii) 加速度ベクトル $\boldsymbol{x}''(t)$，加速度 $\|\boldsymbol{x}''(t)\|$

(2) (i) $\left(\dfrac{d}{dt}(t\boldsymbol{x}(t))\right)_{t=0}$ (ii) $\left(\dfrac{d}{dt}\boldsymbol{x}(t^2)\right)_{t=1}$ (Hint : 積の微分・合成関数の微分 の公式を用いよ)

(3) 積分 $\displaystyle\int_0^t \boldsymbol{x}(s)\,ds$

(解答例)

(1) (i) $\boldsymbol{x}'(t) = \big(2t, -2\sin 2t, 2\cos 2t\big) = 2\big(t, -\sin 2t, \cos 2t\big)$

$\|\boldsymbol{x}'(t)\| = 2\|\big(t, -\sin 2t, \cos 2t\big)\| = 2\sqrt{t^2 + \sin^2 2t + \cos^2 2t} = 2\sqrt{t^2+1}$

(ii) $\boldsymbol{x}''(t) = 2\big(1, -2\cos 2t, -2\sin 2t\big)$

$\|\boldsymbol{x}''(t)\| = 2\sqrt{1 + 4\cos^2 2t + 4\sin^2 2t} = 2\sqrt{5}$

(2) (i) $\dfrac{d}{dt}(t\boldsymbol{x}(t)) = \boldsymbol{x}(t) + t\boldsymbol{x}'(t)$ $\quad\therefore\ \left(\dfrac{d}{dt}(t\boldsymbol{x}(t))\right)_{t=0} = \boldsymbol{x}(0) = (0,1,0)$

(ii) $\dfrac{d}{dt}\boldsymbol{x}(t^2) = 2t\,\boldsymbol{x}'(t^2)$ $\quad\therefore\ \left(\dfrac{d}{dt}\boldsymbol{x}(t^2)\right)_{t=1} = 2\,\boldsymbol{x}'(1) = 2\cdot 2\big(1, -\sin 2, \cos 2\big) = 4\big(1, -\sin 2, \cos 2\big)$

(3) 積分 $\displaystyle\int_0^t \boldsymbol{x}(s)\,ds = \left[\frac{1}{3}s^3, \frac{1}{2}\sin 2s, -\frac{1}{2}\cos 2s\right]_0^t = \left(\frac{1}{3}t^3, \frac{1}{2}\sin 2t, -\frac{1}{2}(\cos 2t - 1)\right)$