Huffman Coding: Instructions

Help Center

Attention: You are allowed to submit **a maximum of 5 times**! for grade purposes. Once you have submitted your solution, you should see your grade and a feedback about your code on the Coursera website within 10 minutes. If you want to improve your grade, just submit an improved solution. The best of all your first 5 submissions will count as the final grade. You can still submit after the 5th time to get feedbacks on your improved solutions, however, these are for research purposes only, and will not be counted towards your final grade.

Download the patmat.zip handout archive file and extract it somewhere on your machine.

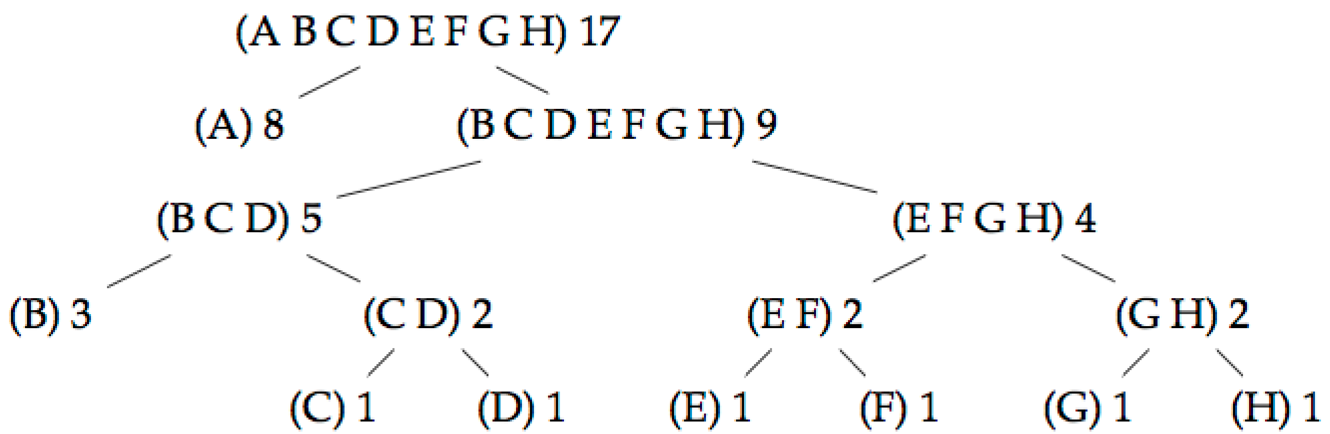
Huffman coding is a compression algorithm that can be used to compress lists of characters.

In a normal, uncompressed text, each character is represented by the same number of bits (usually eight). In Huffman coding, each character can have a bit representation of a different length, depending on how common a character is: the characters that appear often in a text are represented by a shorter bit sequence than those being used more rarely. Every huffman code defines the specific bit sequences used to represent each character.

A Huffman code can be represented by a binary tree whose leaves represent the characters that should be encoded. The code tree below can represent the characters `A` to `H`.

The leaf nodes have associated with them a weight which denotes the frequency of appearance of that character. In the example below, the character `A` has the highest weight 8, while `F` for example has weight 1.

Every branching node of the code tree can be thought of as a set containing the characters present in the leaves below it. The weight of a branching node is the total weight of the leaves below it: this information is necessary for the construction of the tree.



Note that a given encoding is only optimal if the character frequencies in the encoded text match the weights in the code tree.

Finally, observe the recursive structure of the coding tree: every sub-tree is itself a valid code tree for a smaller alphabet.

Encoding

For a given Huffman tree, one can obtain the encoded representation of a character by traversing from the root of the tree to the leaf containing the character. Along the way, when a left branch is chosen, a `0` is added to the representation, and when a right branch is chosen, `1` is added to the representation. Thus, for the Huffman tree above, the character `D` is encoded as `1011`.

Decoding

Decoding also starts at the root of the tree. Given a sequence of bits to decode, we successively read the bits, and for each 0, we choose the left branch, and for each 1 we choose the right branch. When we reach a leaf, we decode the corresponding character and then start again at the root of the tree. As an example, given the Huffman tree above, the sequence of bits, `10001010` corresponds to `BAC`.

Implementation

In Scala, a Huffman tree can be represented as follows:

```
abstract class CodeTree
case class Fork (left: CodeTree, right: CodeTree, chars: List[Char], weight: Int) extend
s CodeTree
case class Leaf(char: Char, weight: Int) extends CodeTree
```

To begin, implement the following two (hint: very simple) functions using pattern matches on the code tree:

1. `weight` which returns the total weight of a given Huffman tree.
   `def weight(tree: CodeTree): Int = tree match ...`
2. `chars` which returns the list of characters defined in a given Huffman tree.
   `def chars(tree: CodeTree): List[Char] = tree match ...`

Using these functions, it's possible to define `makeCodeTree`, a function which facilitates the creation of Huffman trees by automatically calculating the list of characters and the weight when creating a node. This function is already implemented in the handout template:

```
def makeCodeTree(left: CodeTree, right: CodeTree) =
  Fork(left, right, chars(left) ::: chars(right), weight(left) + weight(right))
```

Using `makeCodeTree`, code trees can be constructed manually in the following way:

```
val sampleTree = makeCodeTree(
  makeCodeTree(Leaf('x', 1), Leaf('e', 1)),
  Leaf('t', 2)
)
```

Constructing Huffman Trees

Given a text, it's possible to calculate and build an optimal Huffman tree in the sense that the encoding of that text will be of the minimum possible length, meanwhile keeping all information (i.e., it is lossless).

To obtain an optimal tree from a list of characters, you have to define a function `createCodeTree` with the following signature:

```
def createCodeTree(chars: List[Char]): CodeTree = ...
```

Proceed with the following steps to break up this assignment into smaller parts (the handout template contains more detailed documentation):

1. Begin by writing a function `times` which calculates the frequency of each character in the text: `def times(chars: List[Char]): List[(Char, Int)] = ...`
2. Then, write a function `makeLeafList` which generates a list containing all the leaves of the Huffman tree to be constructed (the case `Leaf` of the algebraic datatype `CodeTree`). The list should be ordered by ascending weights where the weight of a leaf is the number of times (or the frequency) it appears in the given text. `def makeOrderedLeafList(freqs: List[(Char, Int)]): List[Leaf] = ...`
3. Write a simple function `singleton` which checks whether a list of code trees contains only one single tree. `def singleton(trees: List[CodeTree]): Boolean = ...`
4. Write a function `combine` which (1) removes the two trees with the lowest weight from the list constructed in the previous step, and (2) merges them by creating a new node of type `Fork`. Add this new tree to the list - which is now one element shorter - while preserving the order (by weight). `def combine(trees: List[CodeTree]): List[CodeTree] = ...`
5. Write a function `until` which calls the two functions defined above until this list contains only a single tree. This tree is the optimal coding tree. The function `until` can be used in the following way:
   `until(singleton, combine)(trees)`
   where the argument `trees` is of the type `List[CodeTree]`.
6. Finally, use the functions defined above to implement the function `createCodeTree` which respects the signature shown above.

Decoding

Define the function `decode` which decodes a list of bits (which were already encoded using a Huffman tree), given the corresponding coding tree.

```
type Bit = Int
def decode(tree: CodeTree, bits: List[Bit]): List[Char] = ...
```

Use this function and the `frenchCode` code tree to decode the bit sequence in `secret`. Store the resulting character sequence in `decodedSecret`.

Encoding

This section deals with the Huffman encoding of a sequence of characters into a sequence of bits.

...Using a Huffman Tree

Define the function `encode` which encodes a list of characters using Huffman coding, given a code tree.

```
def encode(tree: CodeTree)(text: List[Char]): List[Bit] = ...
```

Your implementation must traverse the coding tree for each character, a task that should be done using a helper function.

...Using a Coding Table

The previous function is simple, but very inefficient. You goal is now to define `quickEncode` which encodes an equivalent representation, but more efficiently.

```
def quickEncode(tree: CodeTree)(text: List[Char]): List[Bit] = ...
```

Your implementation will build a coding table once which, for each possible character, gives the list of bits of its code. The simplest way - but not the most efficient - is to encode the table of characters as a list of pairs.

```
type CodeTable = List[(Char, List[Bit])]
```

The encoding must then be done by accessing the table, via a function `codeBits`.

```
def codeBits(table: CodeTable)(char: Char): List[Bit] = ...
```

The creation of the table is defined by `convert` which traverses the coding tree and constructs the character table.

```
def convert(t: CodeTree): CodeTable = ...
```

Implement the function `convert` by using the function `mergeCodeTables` below:

```
def mergeCodeTables(a: CodeTable, b: CodeTable): CodeTable = ...
```