Directed Acyclic Word Graphs and Their Efficient Implementations

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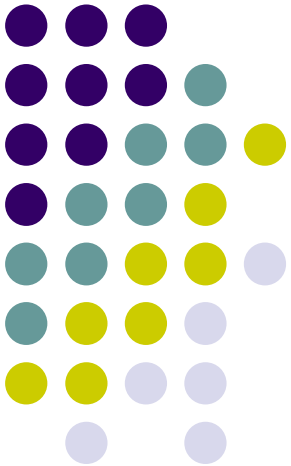
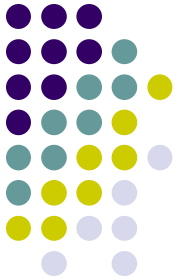


Short CV



Shunsuke Inenaga

March 2003	*PhD Kyushu University, Japan "String Processing Algorithms"*
April – Sept. 2003	*Posdoc of Japan Science Technology Agency (JST)*
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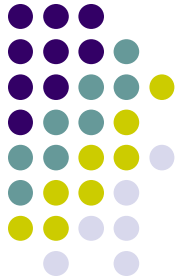
Pattern Matching Problem



Input: text string T *and pattern string* P.

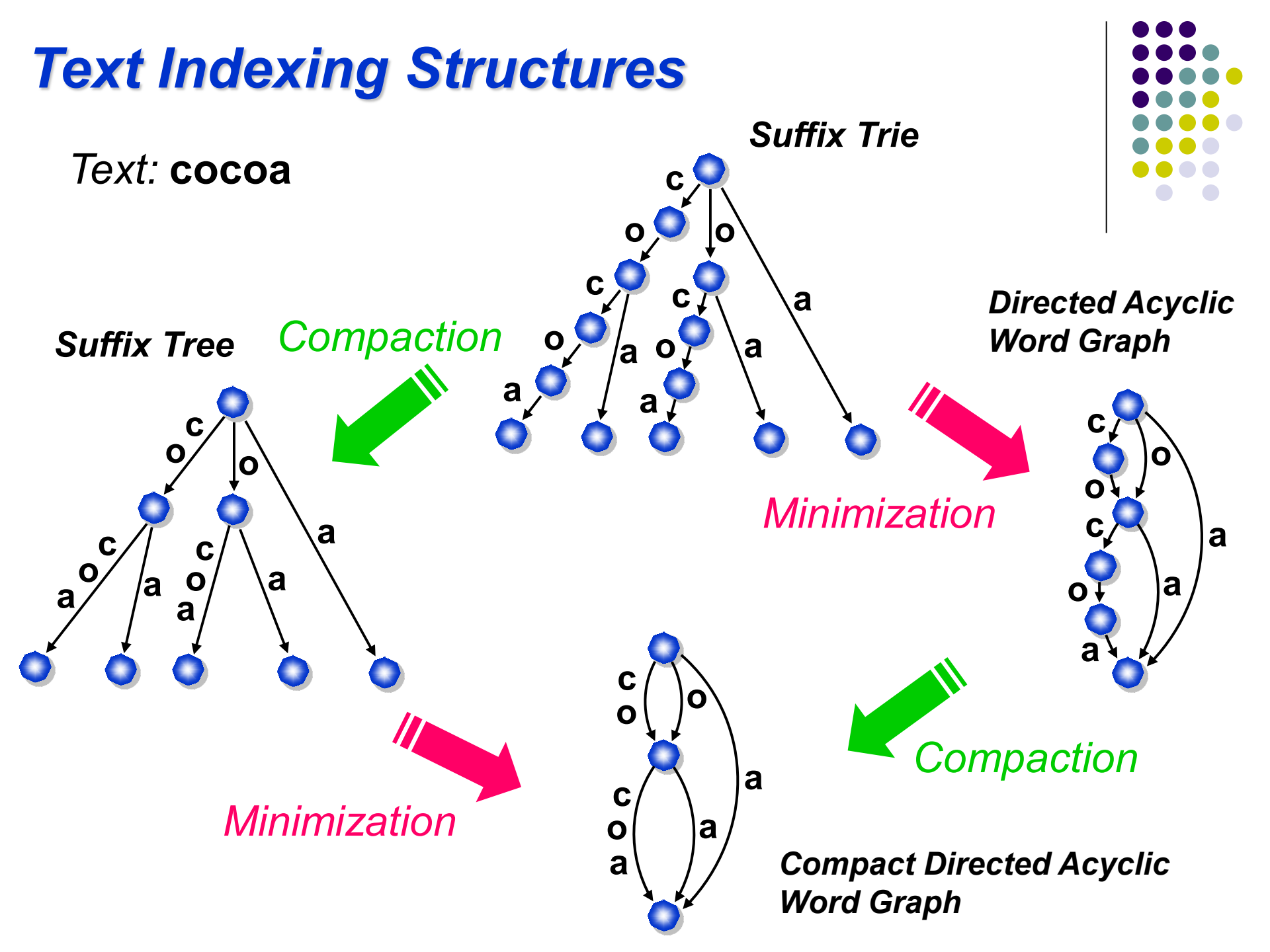
Output: all occurrences of P *in* T.

When T is fixed

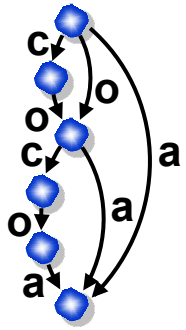
Text Indexing Structures

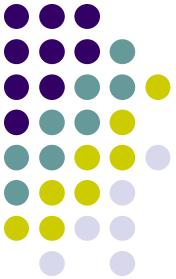
Text Indexing Structures

Text: **cocoa**



Directed Acyclic Word Graphs

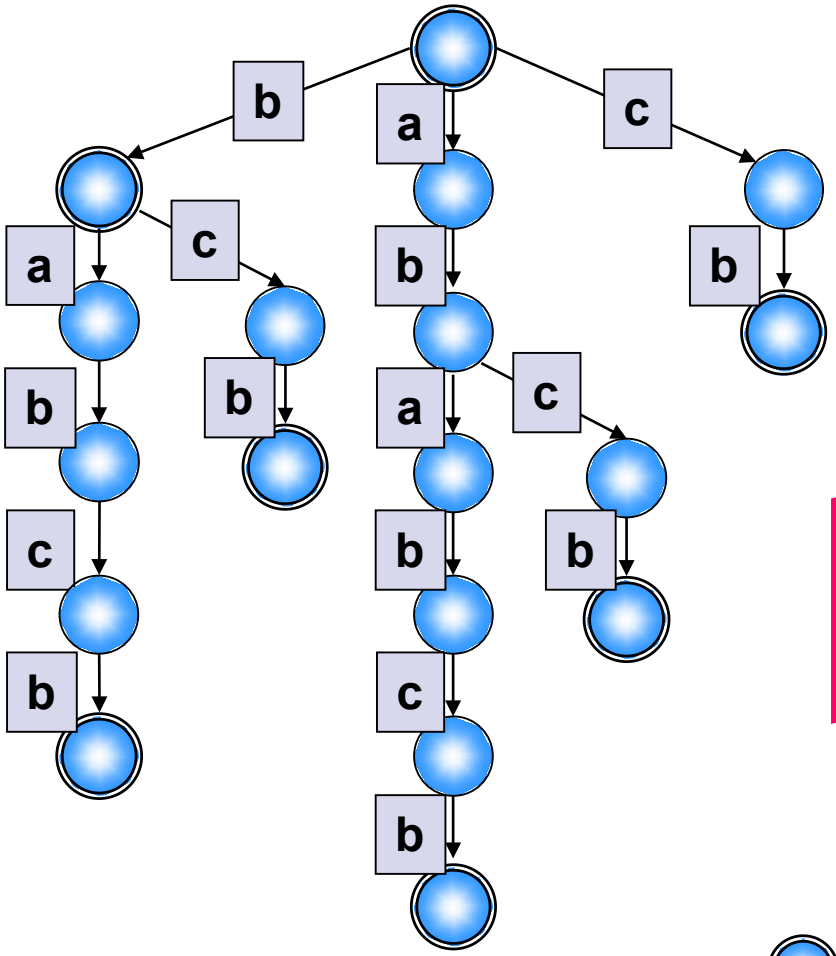




The Directed Acyclic Word Graph (DAWG) of string w *is the smallest DFA that accepts all suffixes of* w.

DAWG = Suffix Automaton

Conversion of Suffix Trie to DAWG



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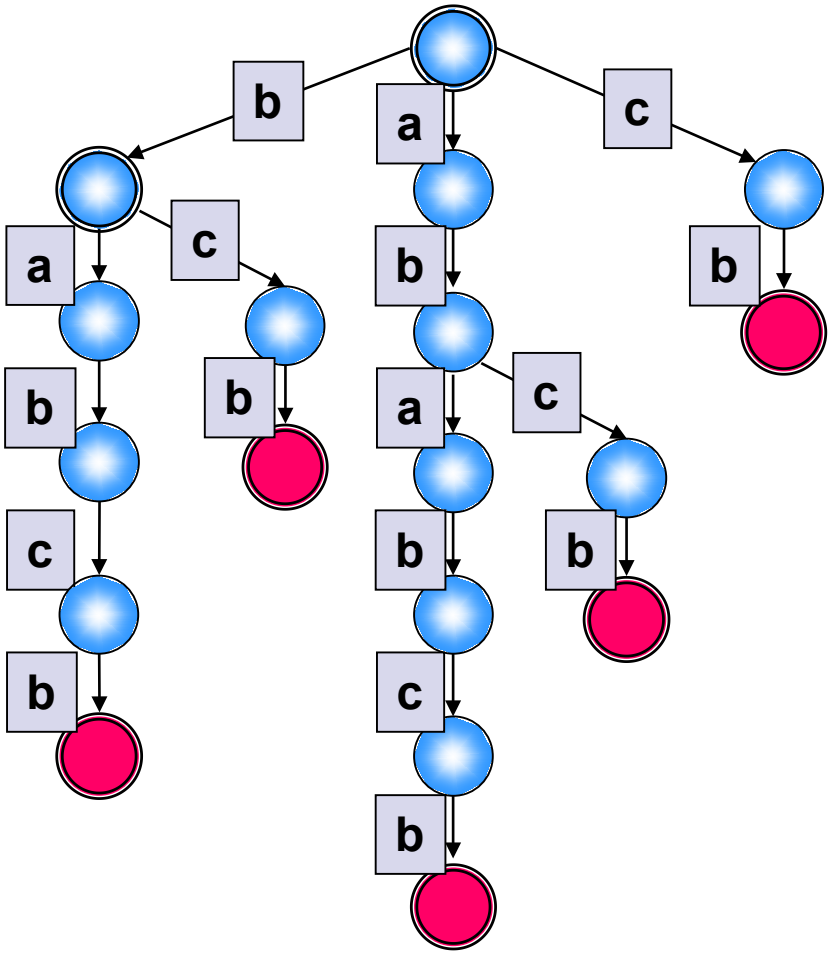
Merge isomorphic subtrees

accepting node for suffixes

Conversion to DAWG



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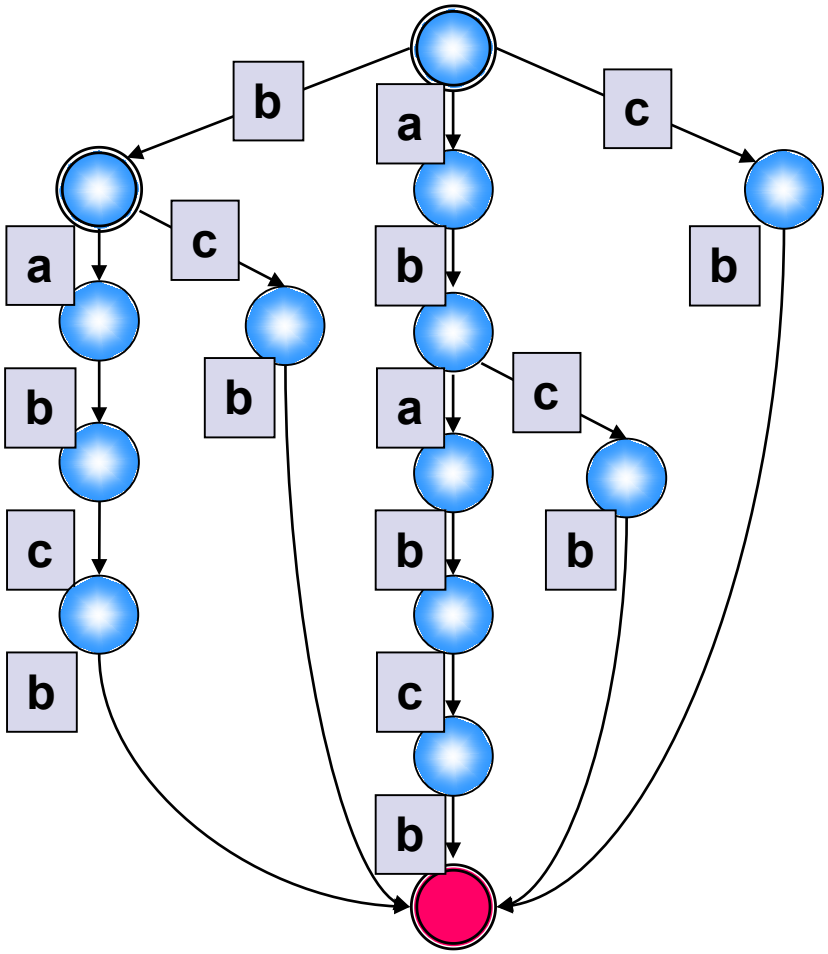
Conversion to DAWG [Cont.]



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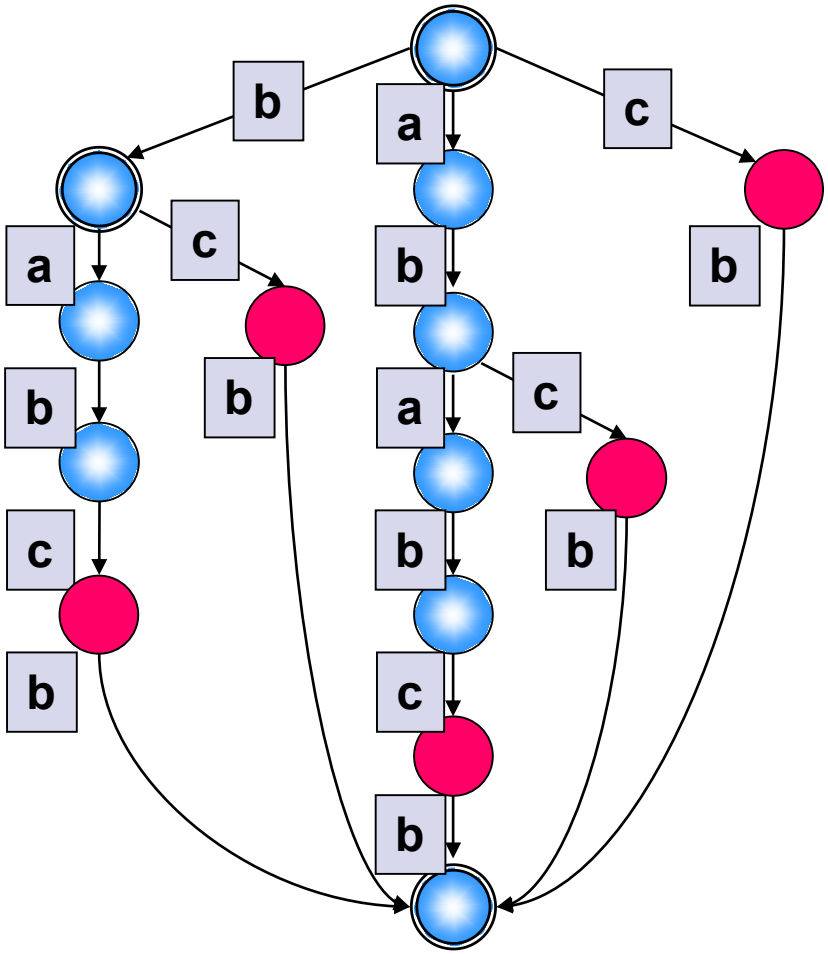
Conversion to DAWG [Cont.]



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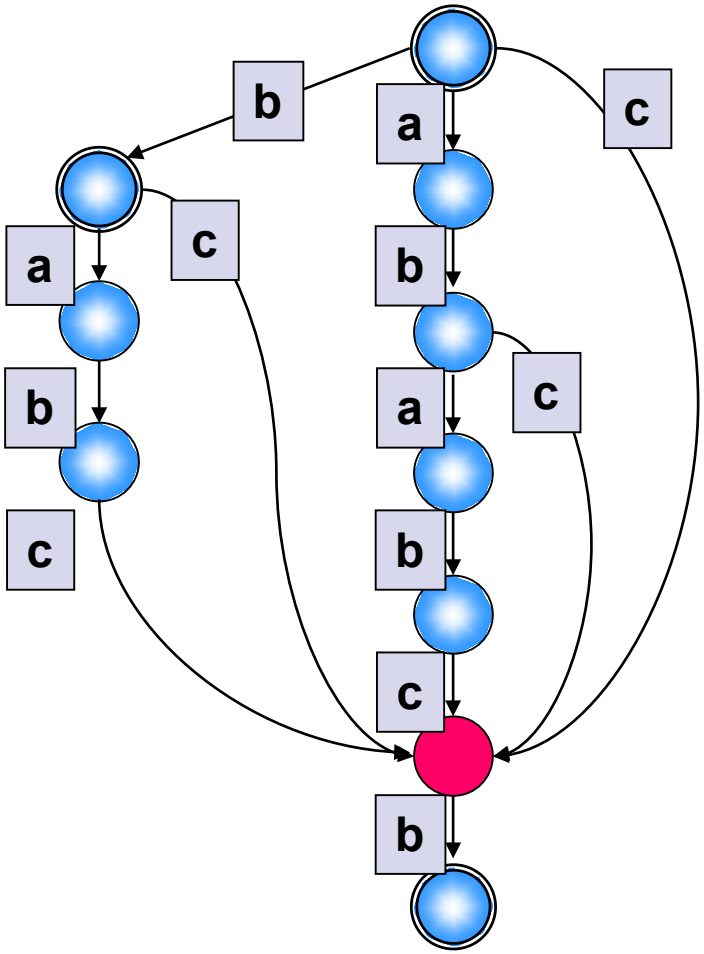
Conversion to DAWG [Cont.]



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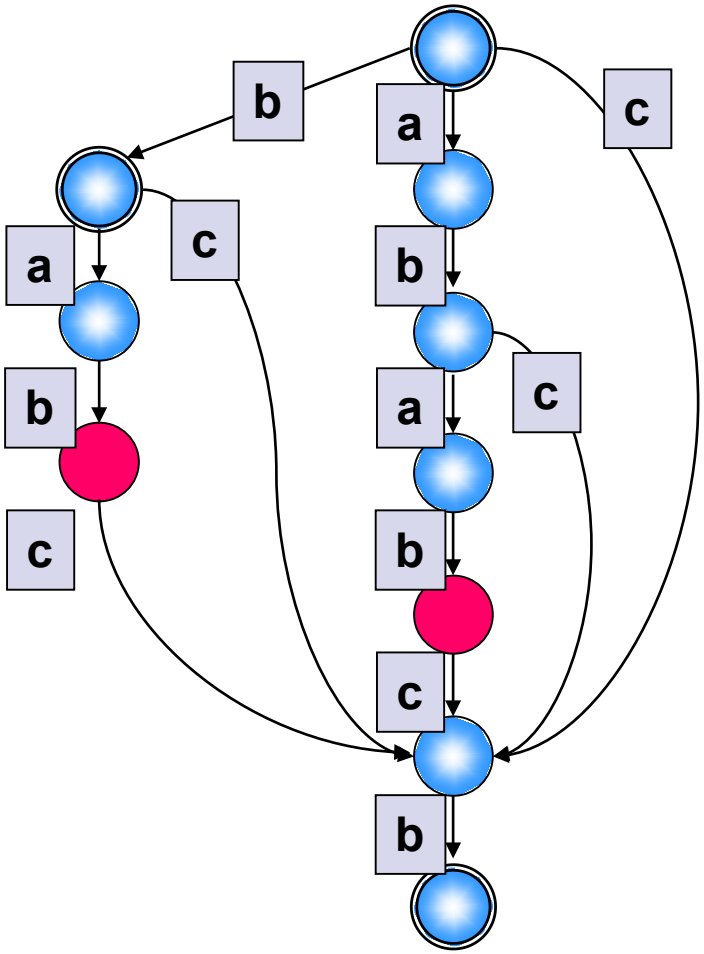
Conversion to DAWG [Cont.]



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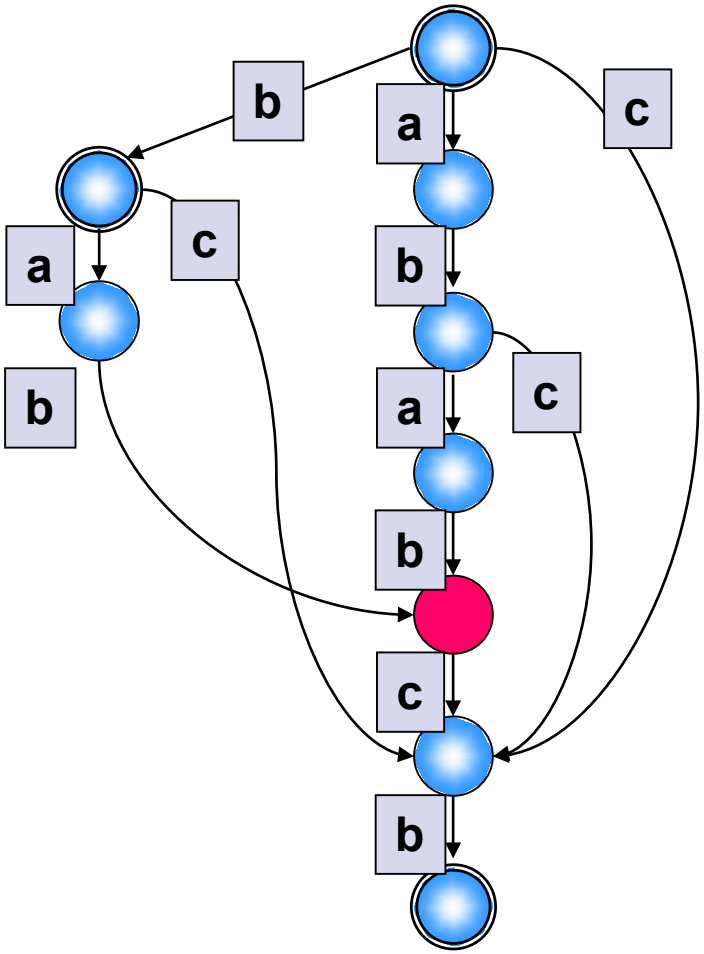
Conversion to DAWG [Cont.]



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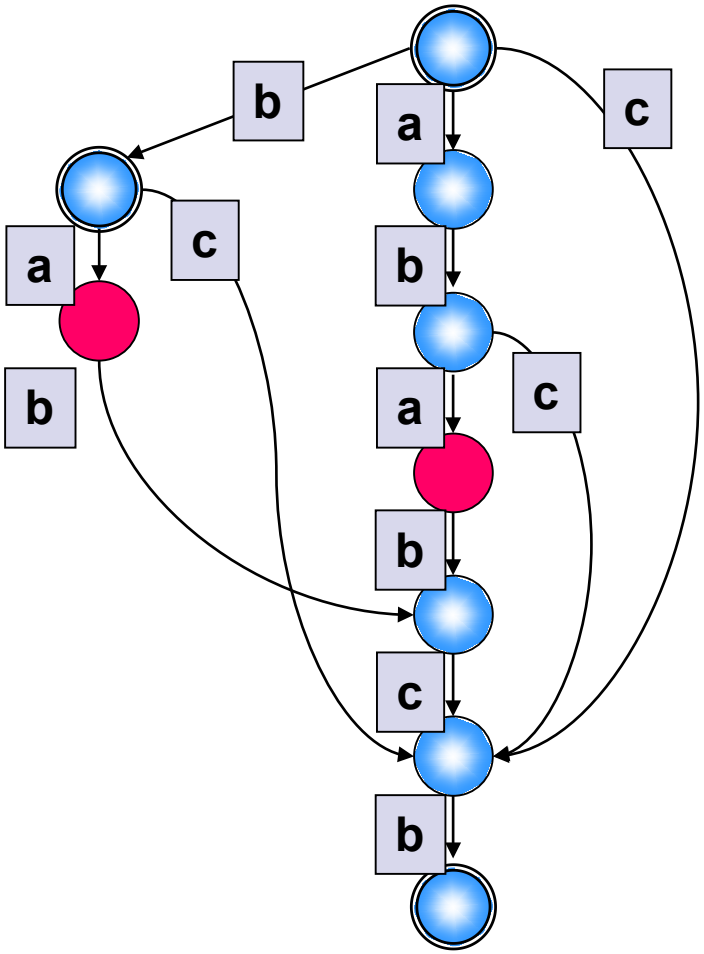
Conversion to DAWG [Cont.]



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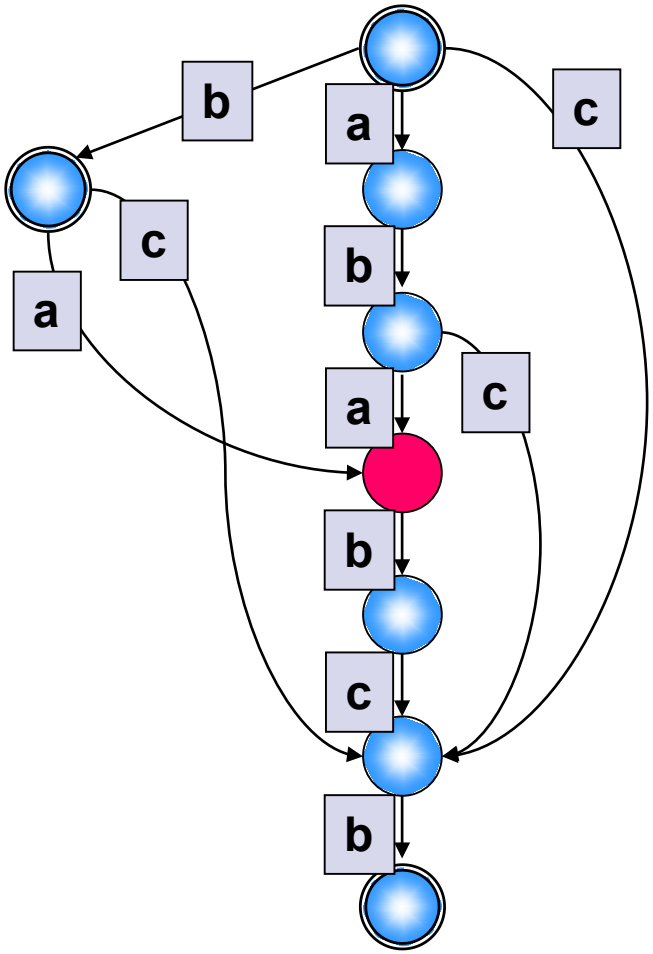
Conversion to DAWG [Cont.]



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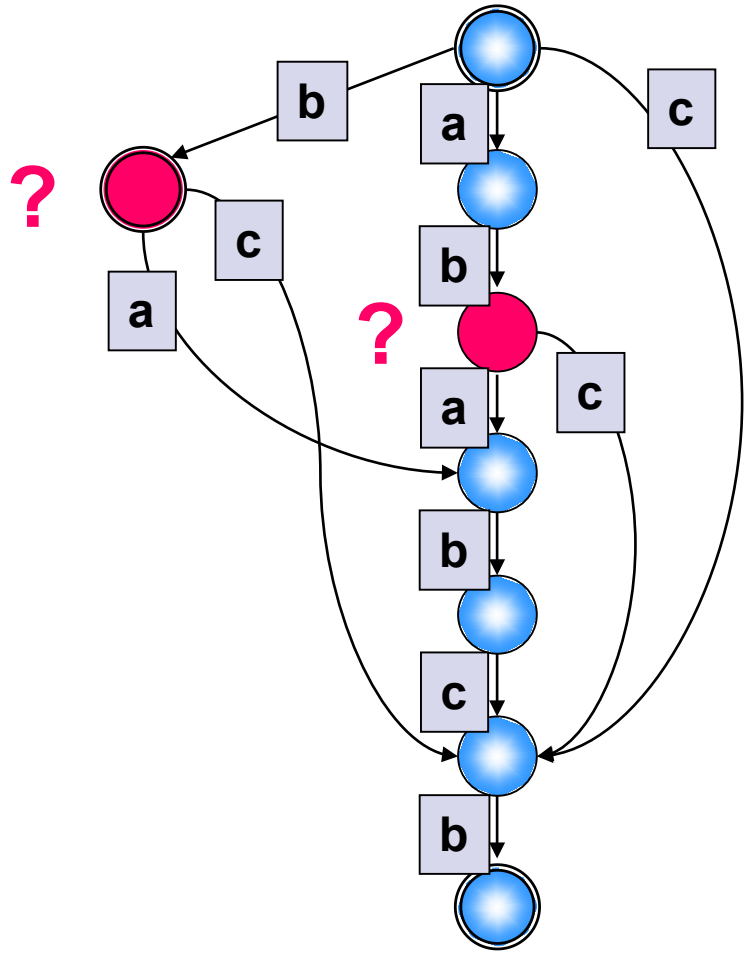
Conversion to DAWG [Cont.]



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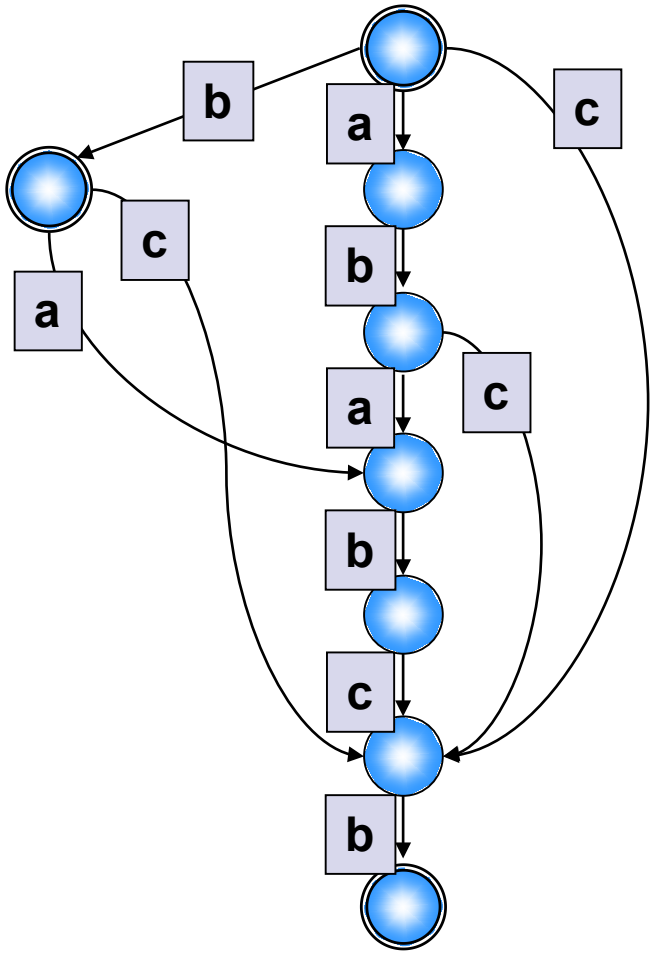
DAWG Completed



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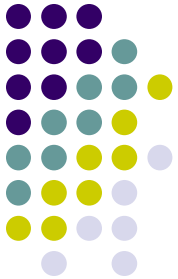
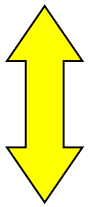
Equivalence Class on DAWG



substrings x,y *of* w *are recognized by the same node of* $DAWG(w)$

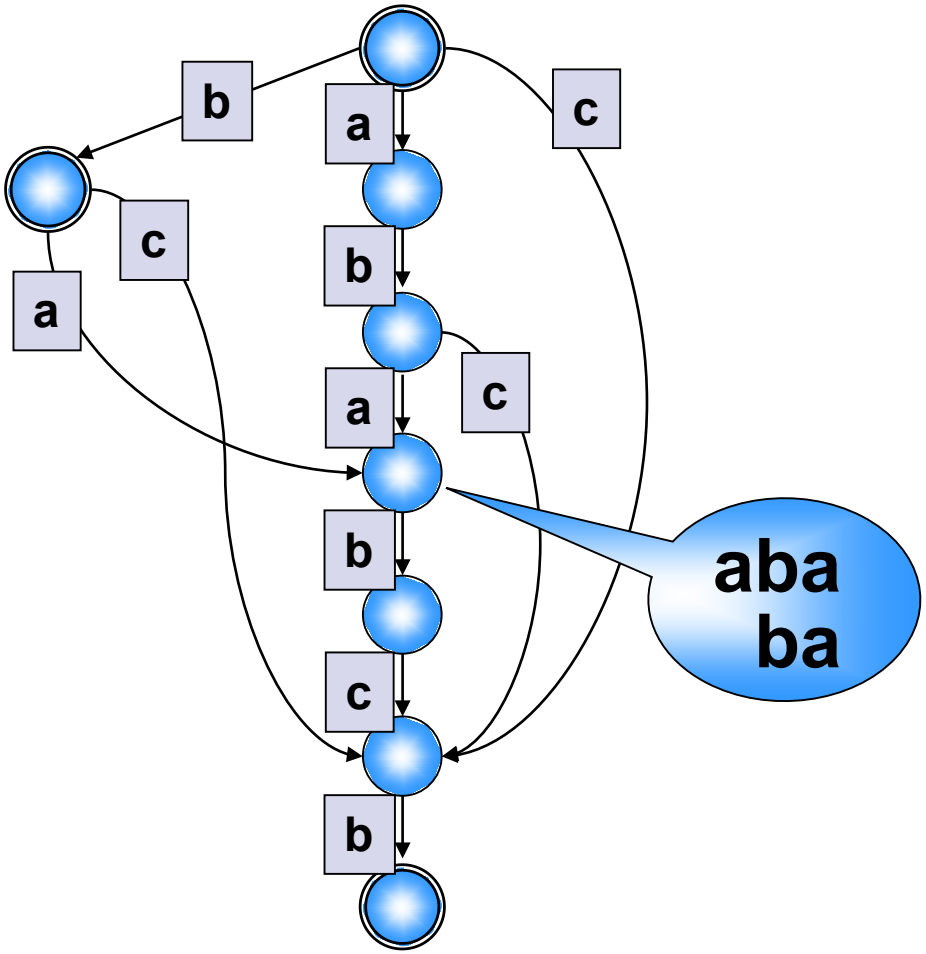


iff

$$EndPos_w(x) = EndPos_w(y)$$

$EndPos_w(x)$: *the set of all positions of* w *where* x *ends*

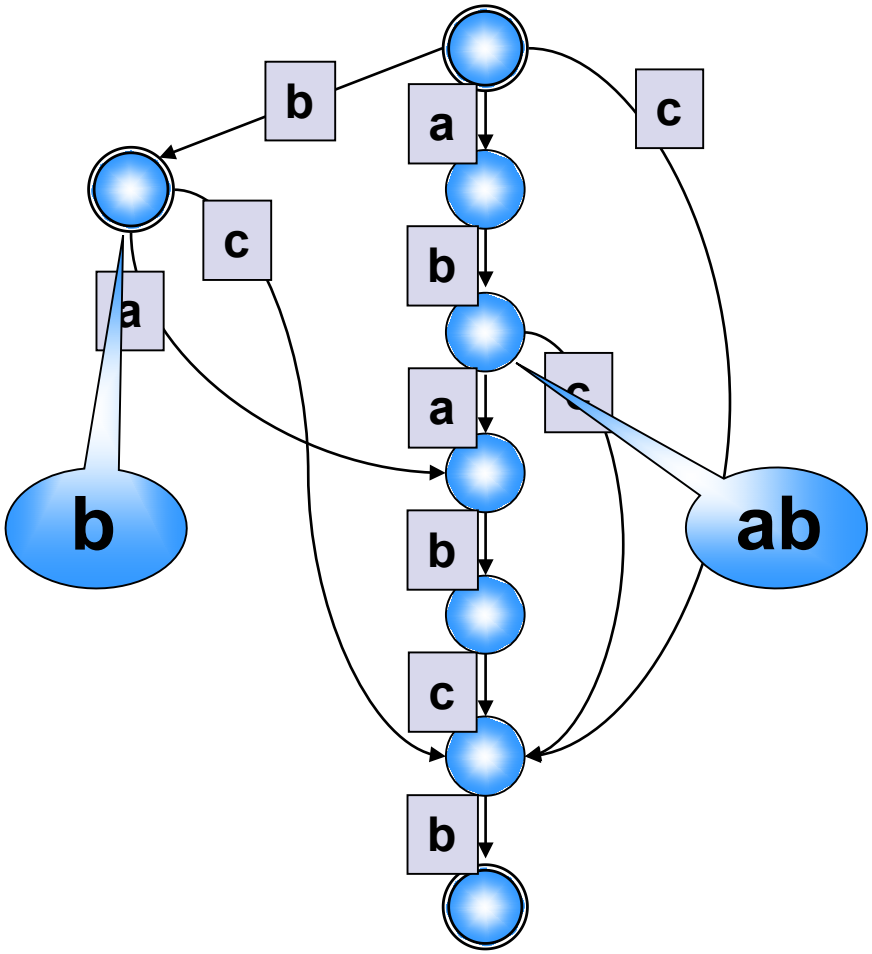
Equivalence Class on DAWG [Cont.]



ababcb
1 2 3 4 5 6

$EndPos_w(\text{aba}) = \{3\}$
$EndPos_w(\text{ba}) = \{3\}$

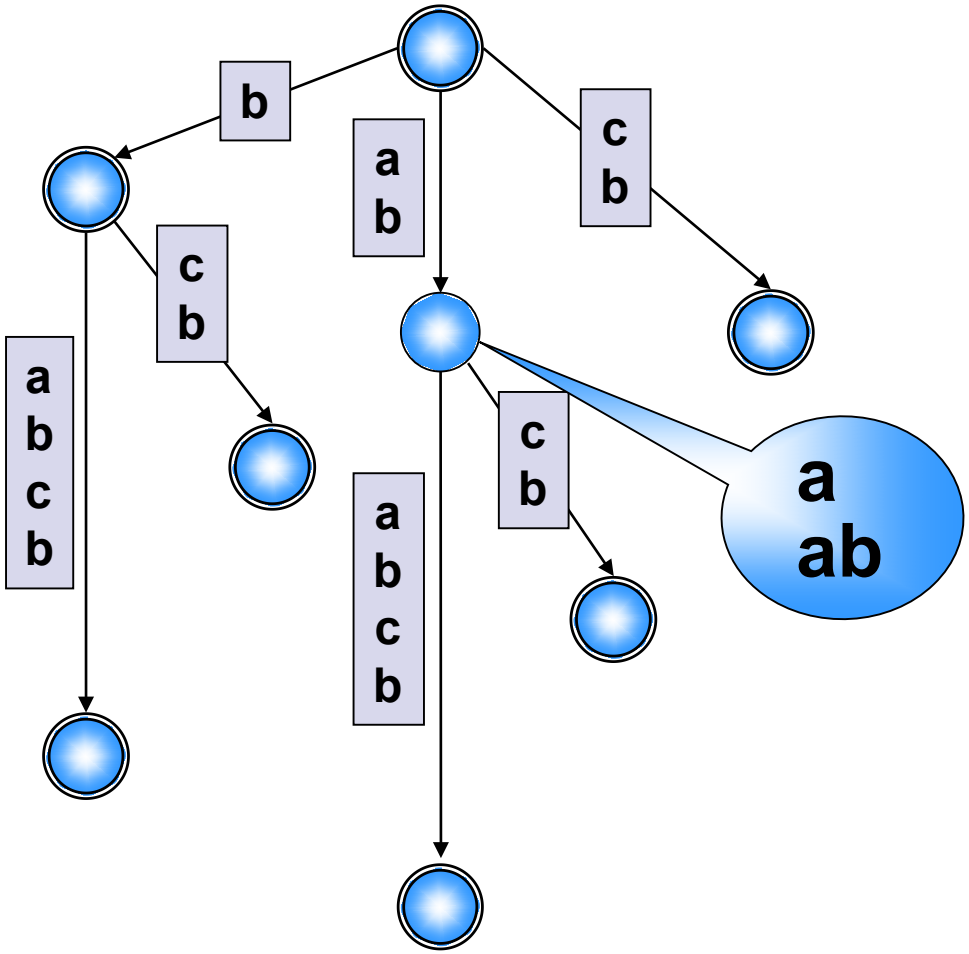
Equivalence Class on DAWG [Cont.]



ababcb
1 2 3 4 5 6

$EndPos_w(\mathbf{ab}) = \{2,4\}$
$EndPos_w(\mathbf{b}) = \{2,4,6\}$

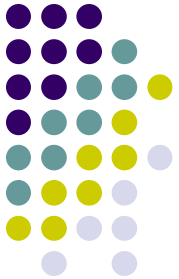
Equivalence Class on Suffix Tree



ababcb
1 2 3 4 5 6

$BegPos_w(\mathbf{a}) = \{1,2\}$
$BegPos_w(\mathbf{ab}) = \{1,2\}$

Size of DAWG

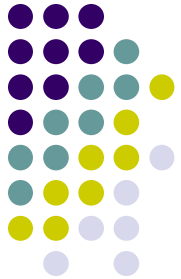


Theorem. (Blumer et al. 1985)

Let $n = |w|$.
If $n > 1$, $DAWG(w)$ *has at most* $2n-1$ *nodes and* $3n-3$ *edges.*

If $n = 1$, $DAWG(w)$ *has exactly 2 nodes and 1 edge.*

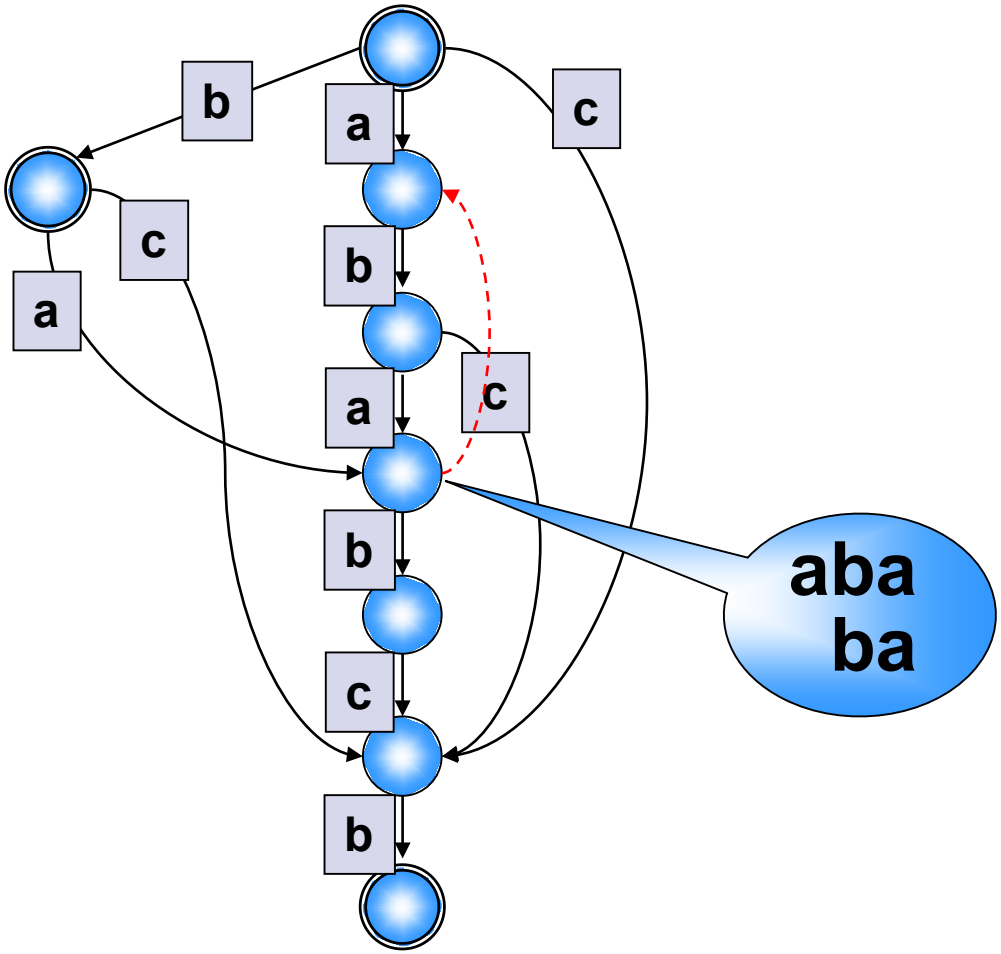
Construction of DAWG



Theorem. (Blumer et al. 1985)

Assume Σ *is fixed.*
For any string w *of length* n*,*
$DAWG(w)$ *can be constructed on-line and in* $O(n)$ *time and space.*

Suffix Links



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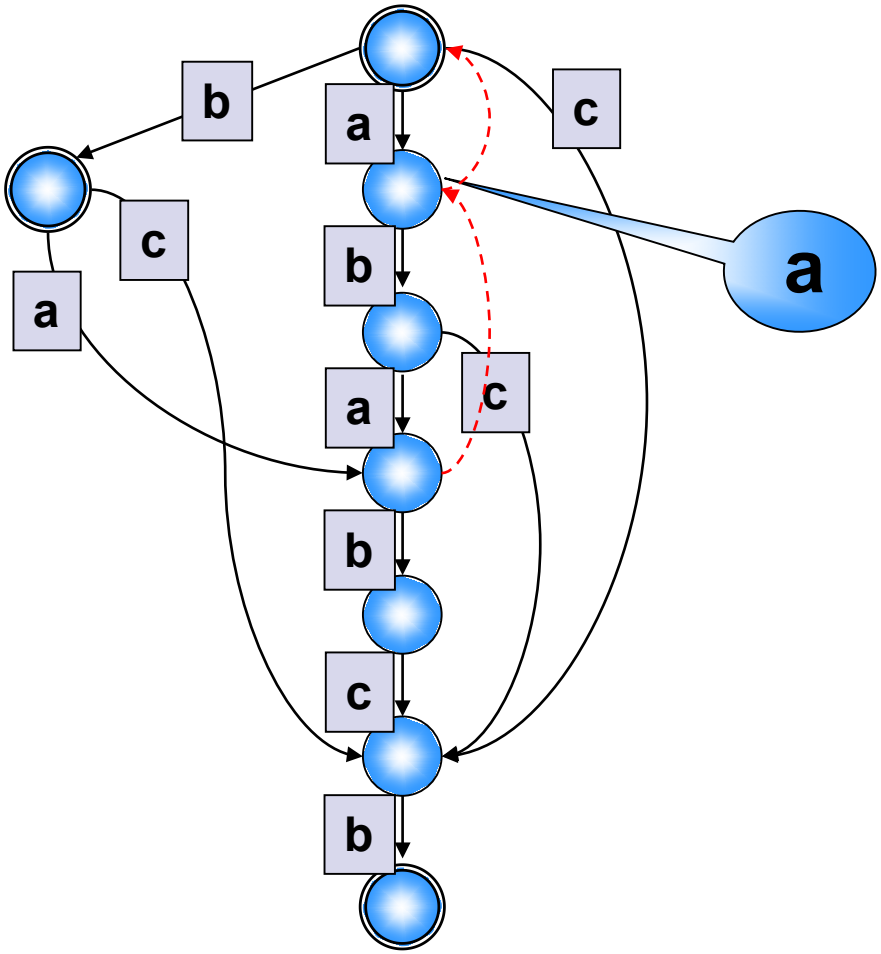
Suffix Links



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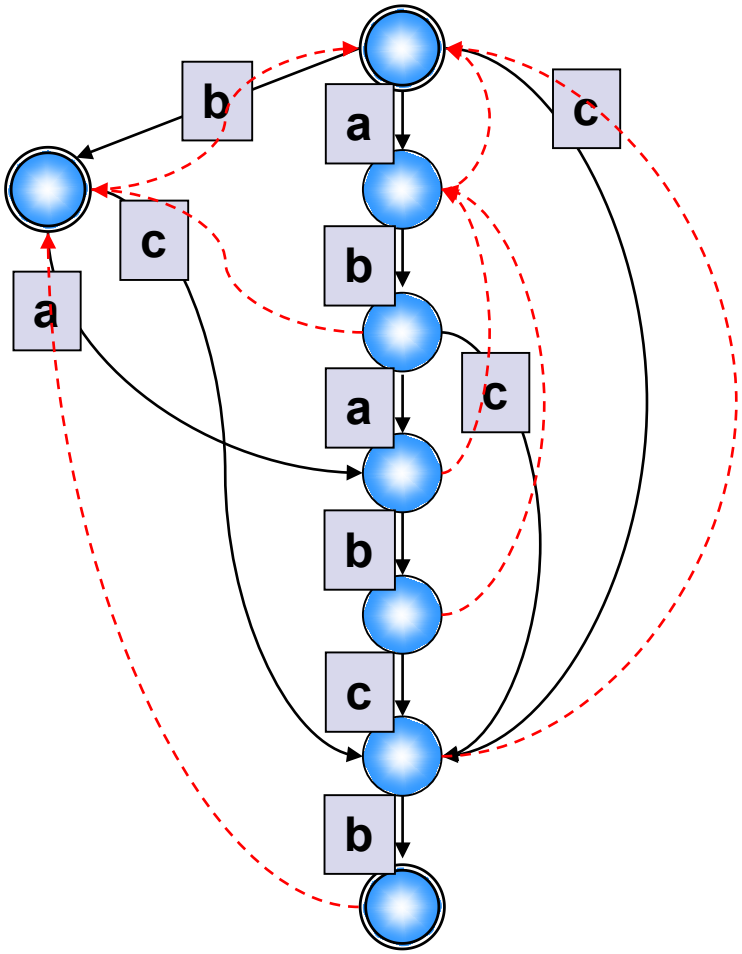
Suffix Links



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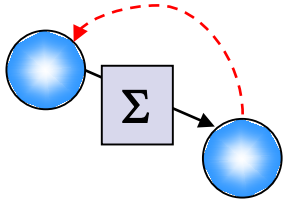
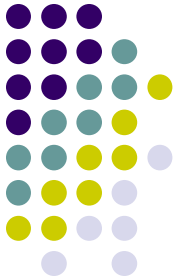
On-line Construction of DAWG





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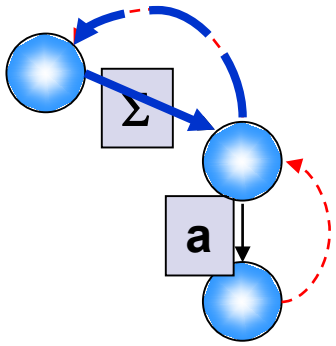
On-line Construction of DAWG



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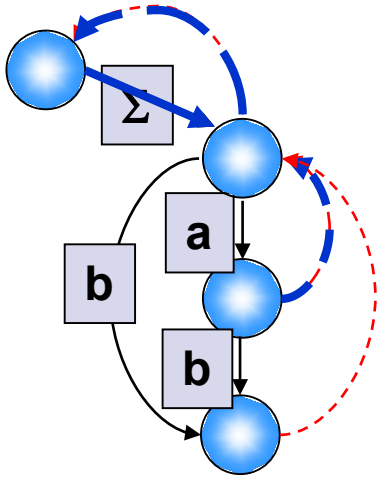
On-line Construction of DAWG



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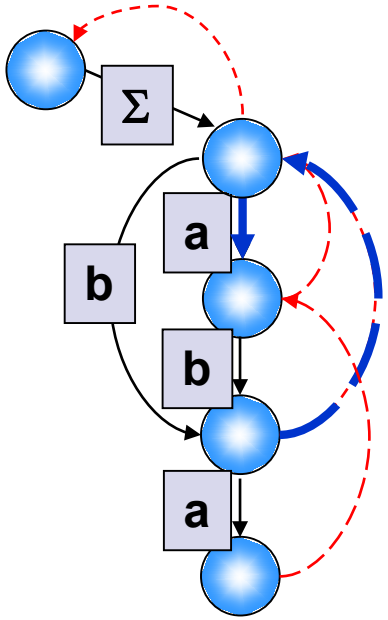
On-line Construction of DAWG



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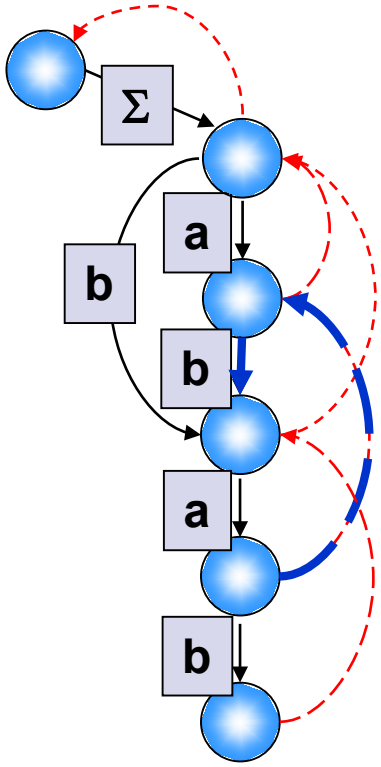
On-line Construction of DAWG



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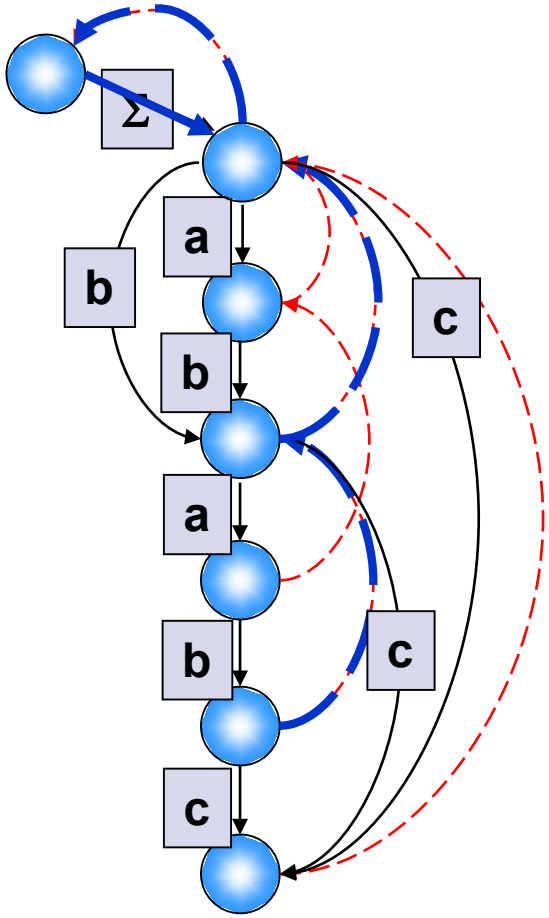
On-line Construction of DAWG



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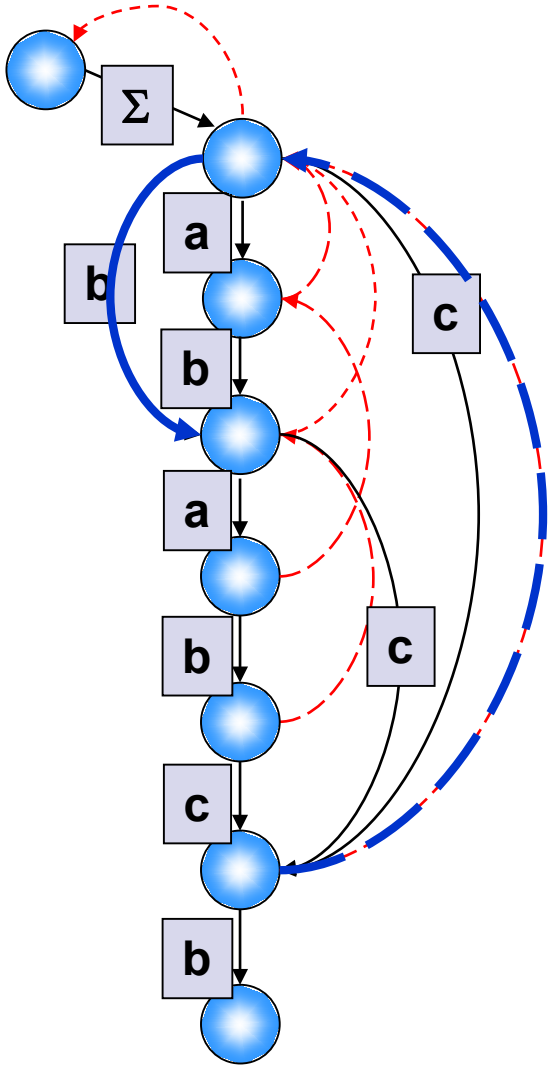
On-line Construction of DAWG



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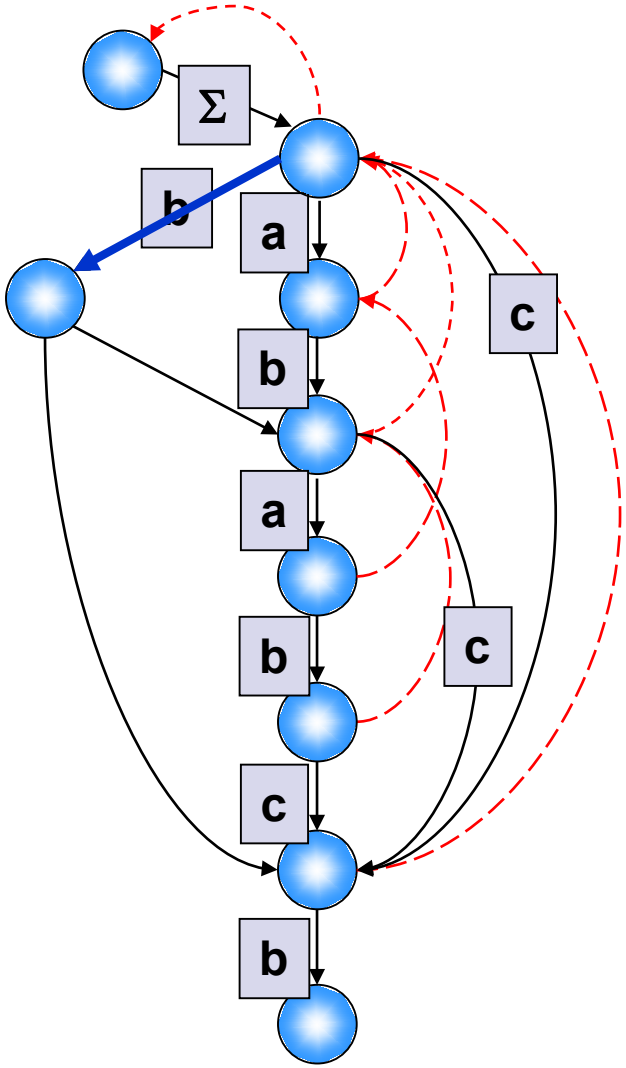
On-line Construction of DAWG



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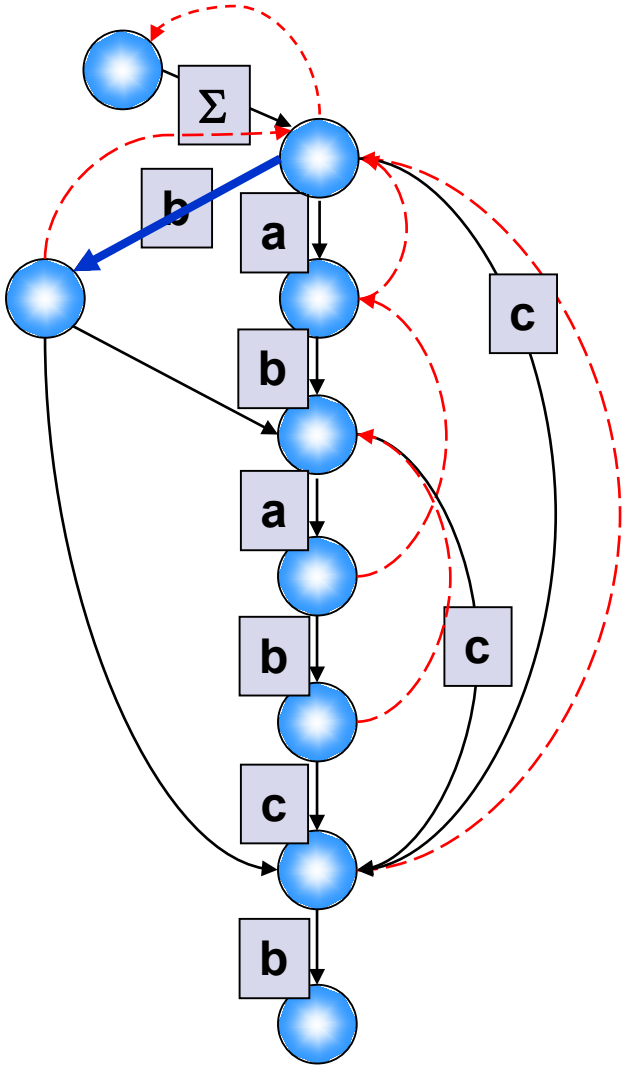
On-line Construction of DAWG



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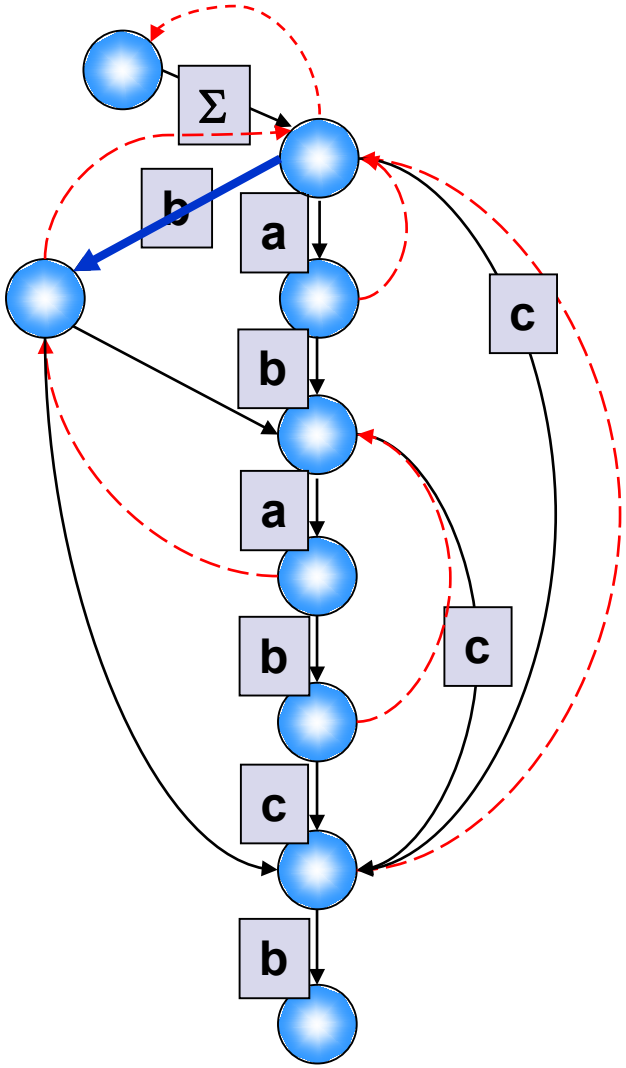
On-line Construction of DAWG



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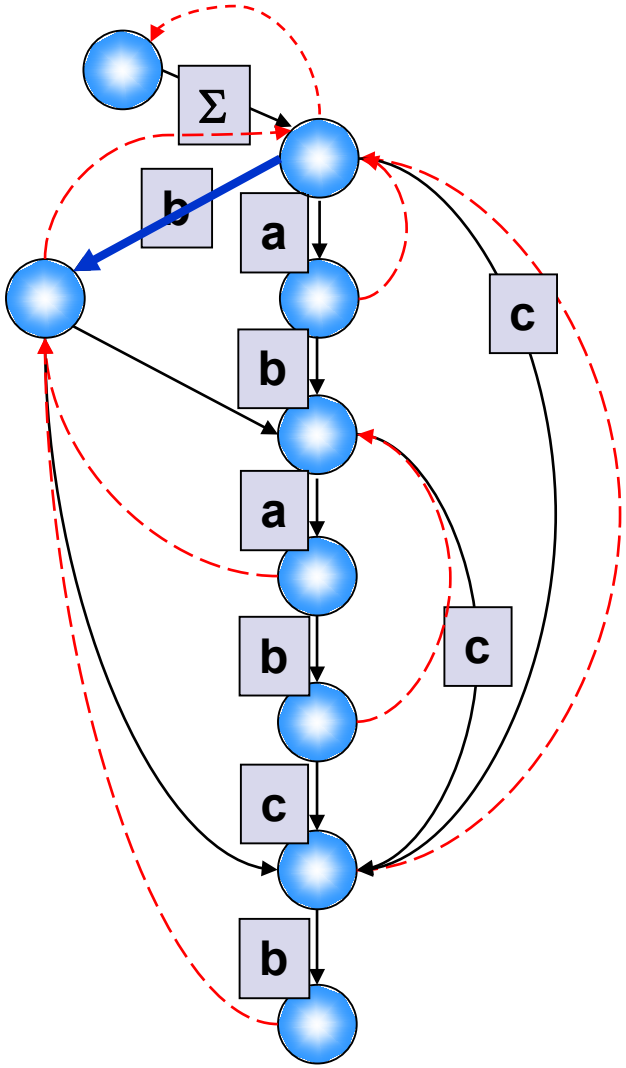
On-line Construction of DAWG



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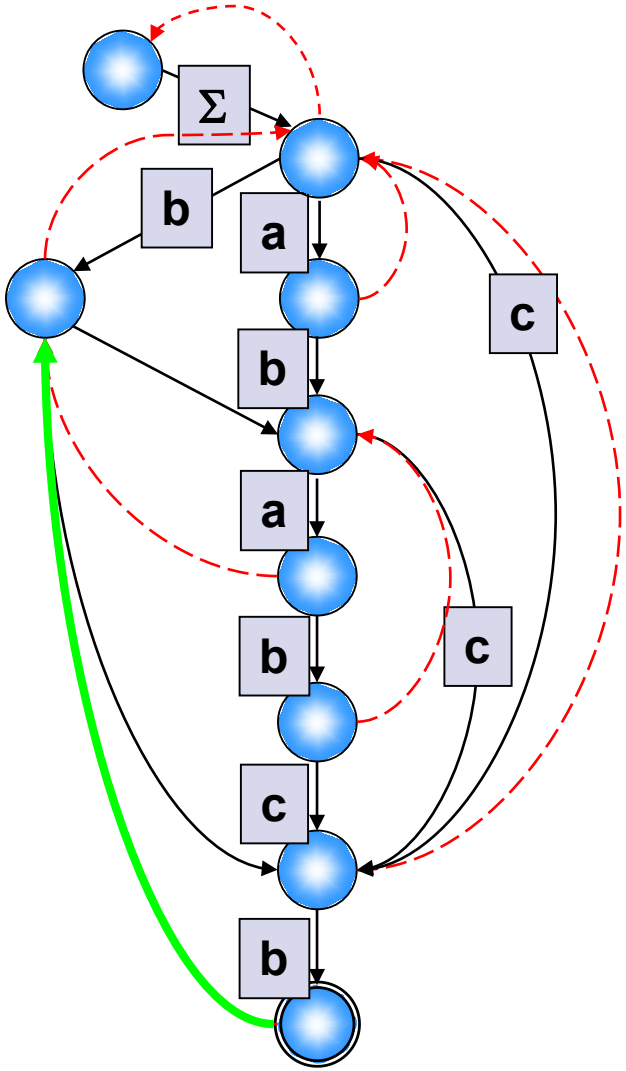
Marking Accepting Nodes



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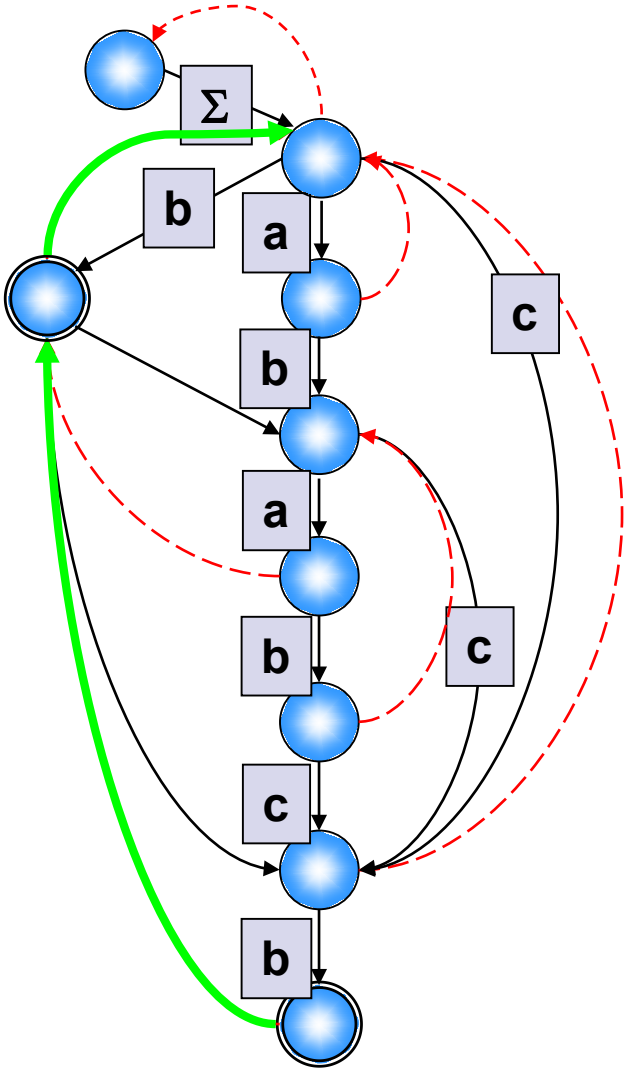
Marking Accepting Nodes



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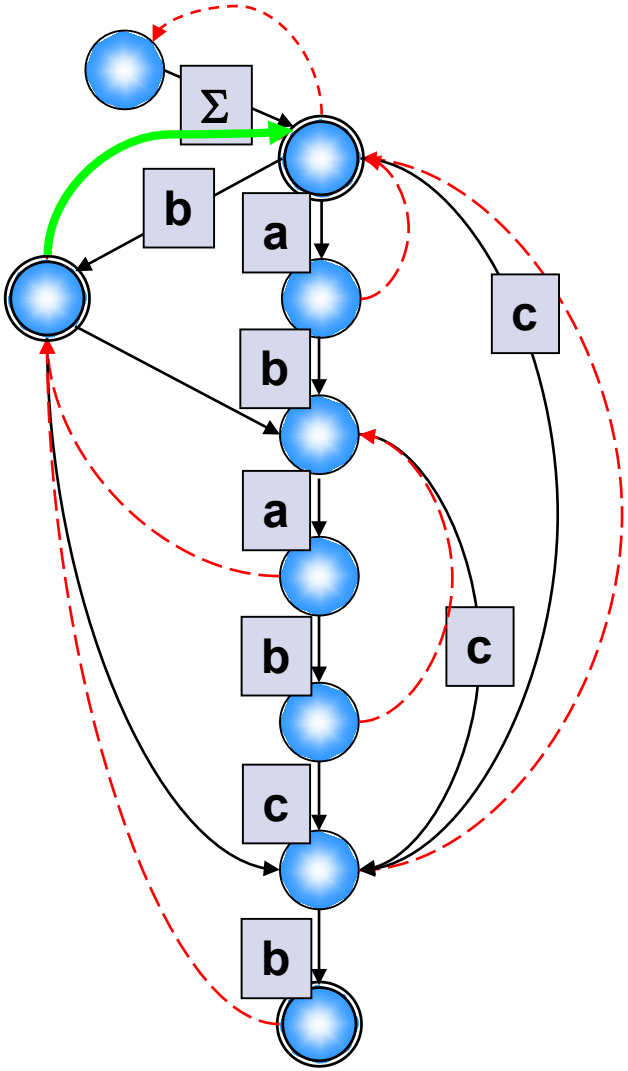
Marking Accepting Nodes



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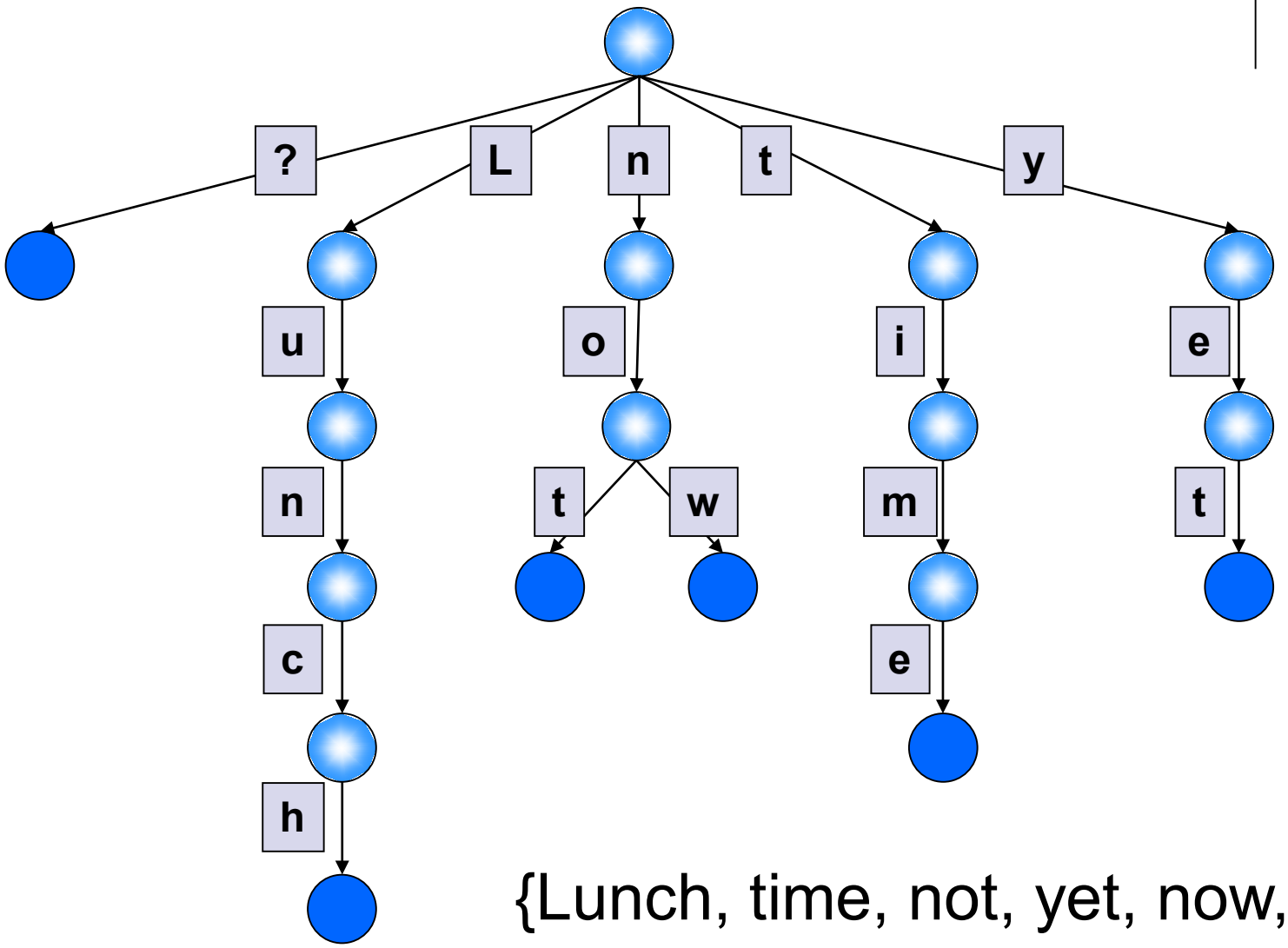
Completed!!

Search Tree of a Set of Strings



{Lunch, time, not, yet, now, ?}

Table Implementation

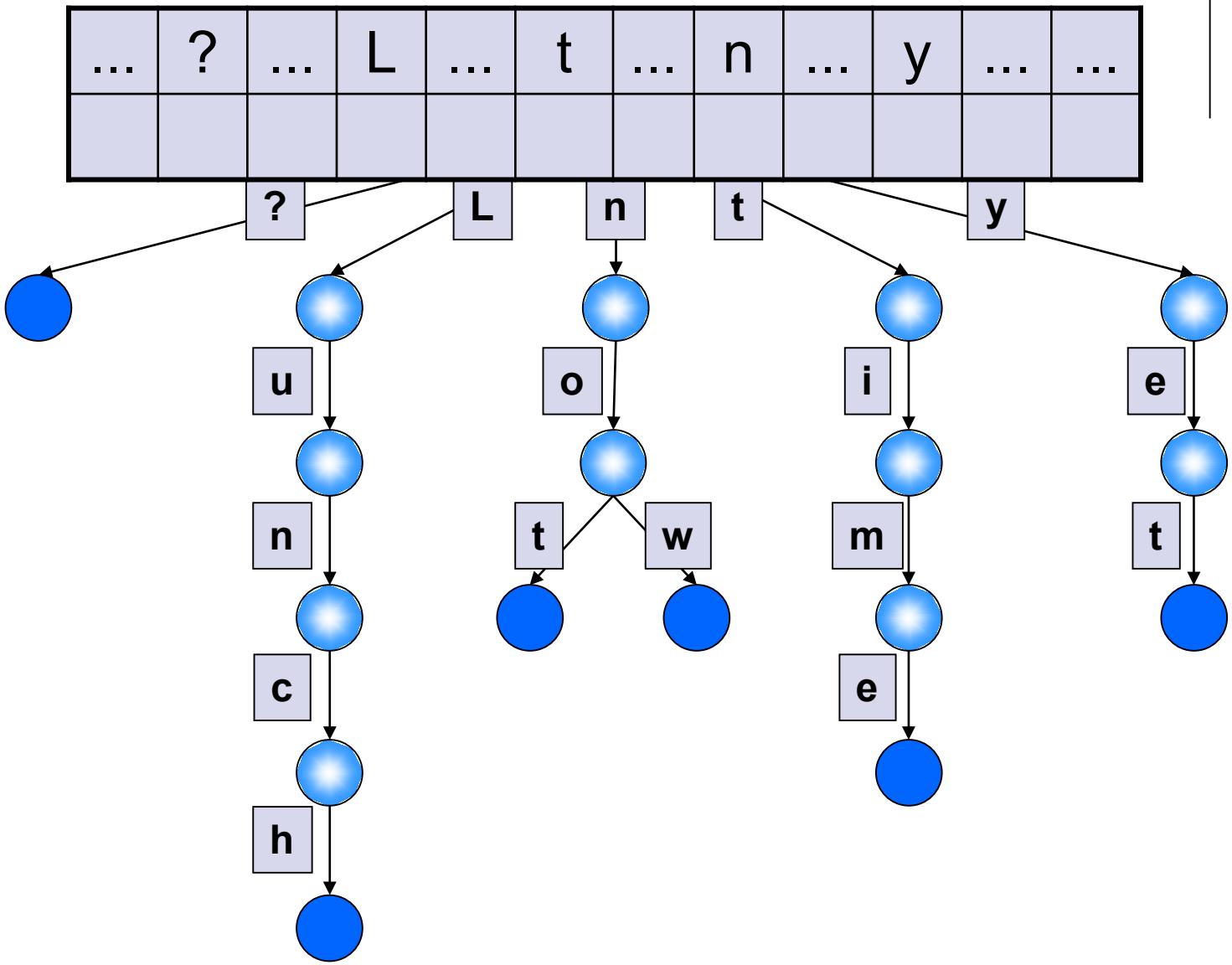
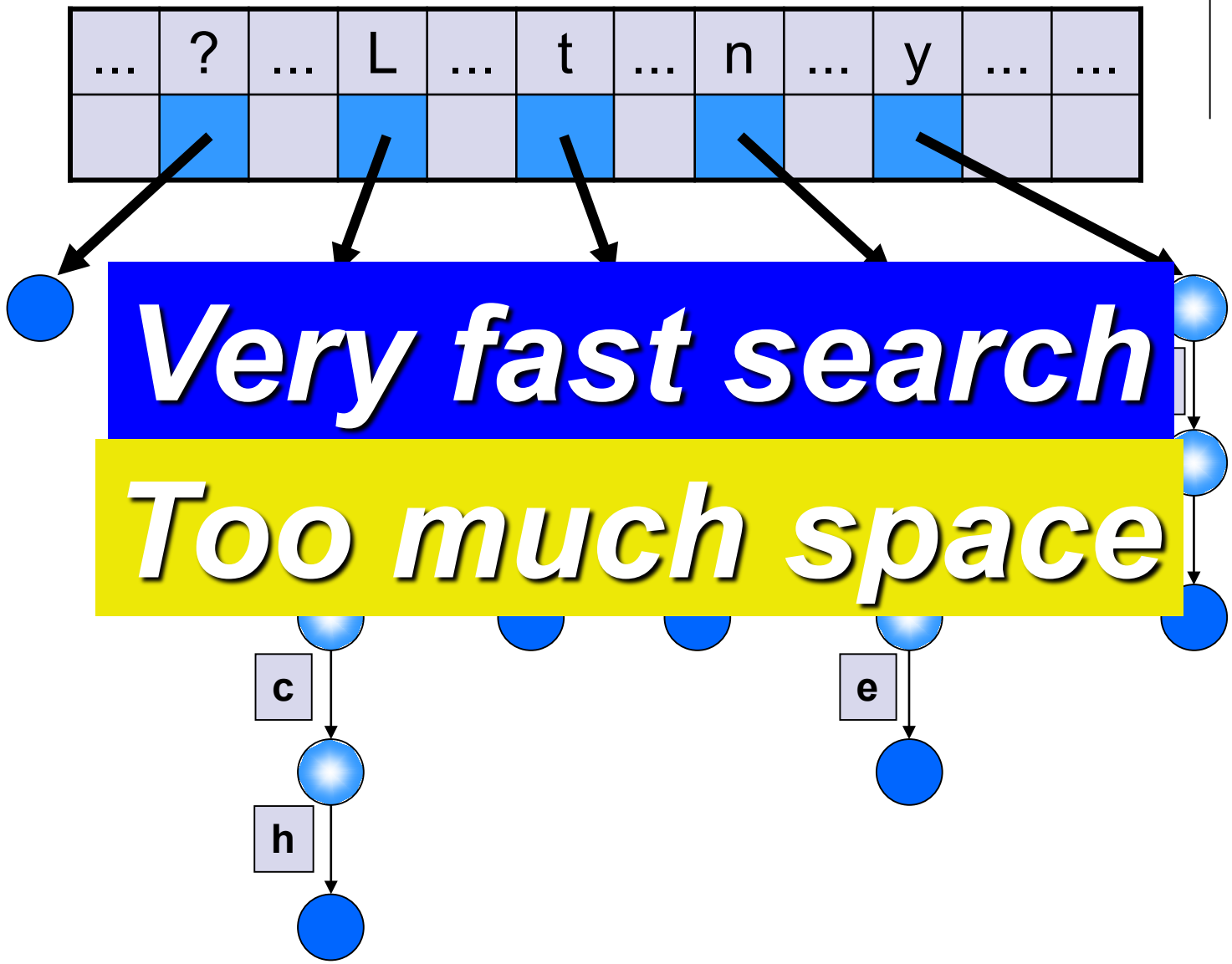
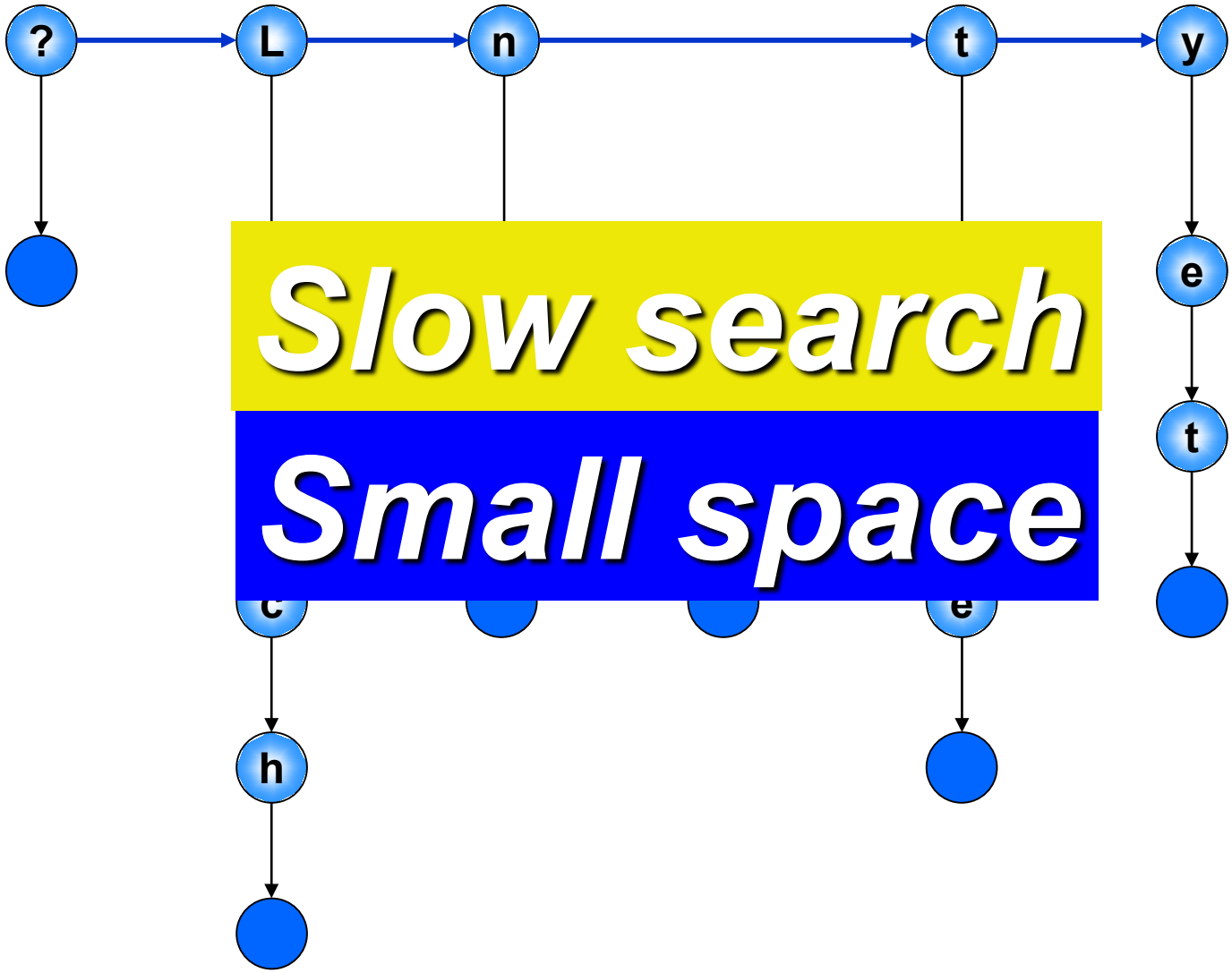


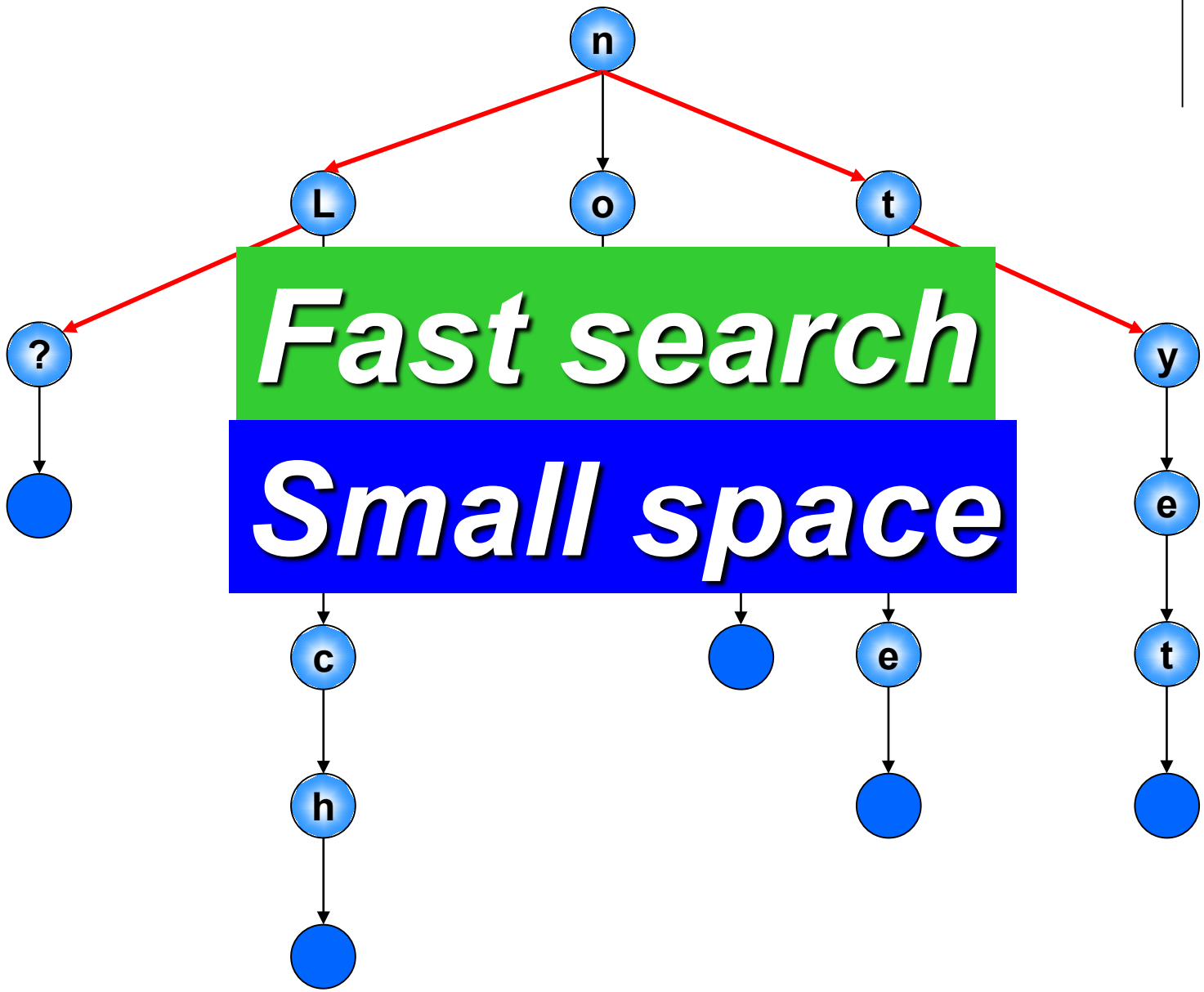
Table Implementation



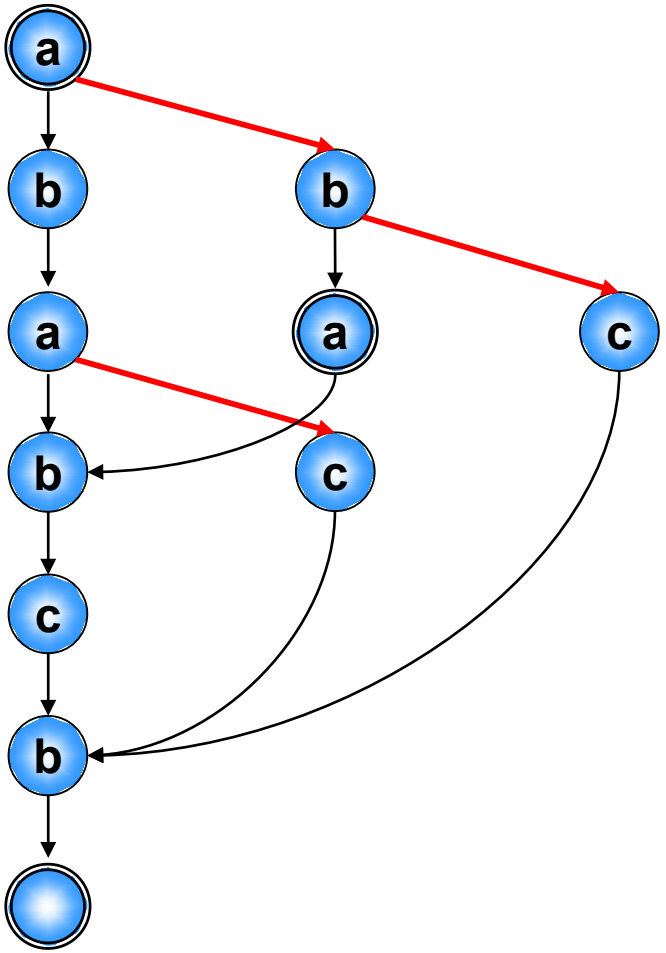
Linked List Implementation



Ternary Search Tree



Ternary Directed Acyclic Word Graph



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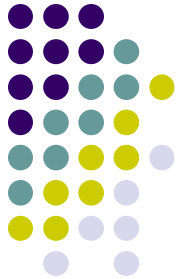
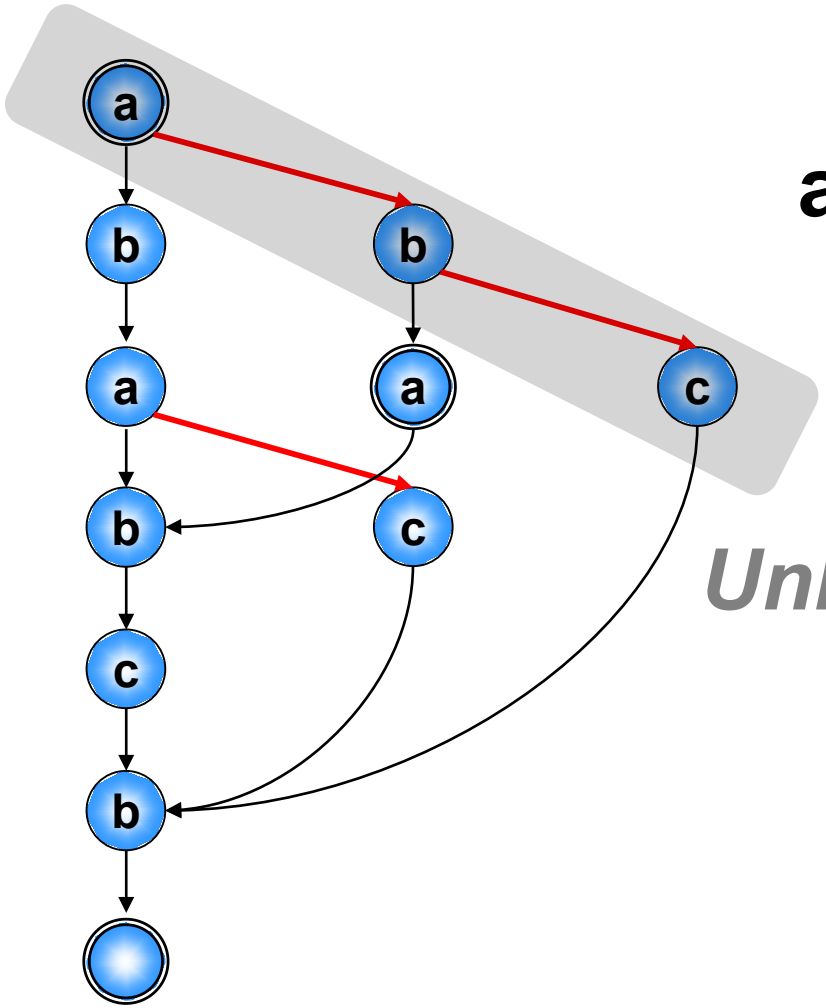
Construction of TDAWG



Theorem. (Miyamoto et al. 2003)

For any string w of length n, $TDAWG(w)$ can be constructed on-line and in $O(n|\Sigma|)$ time and $O(n)$ space.

Ternary Directed Acyclic Word Graph



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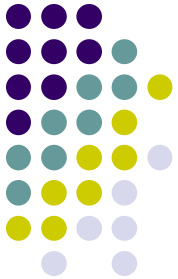
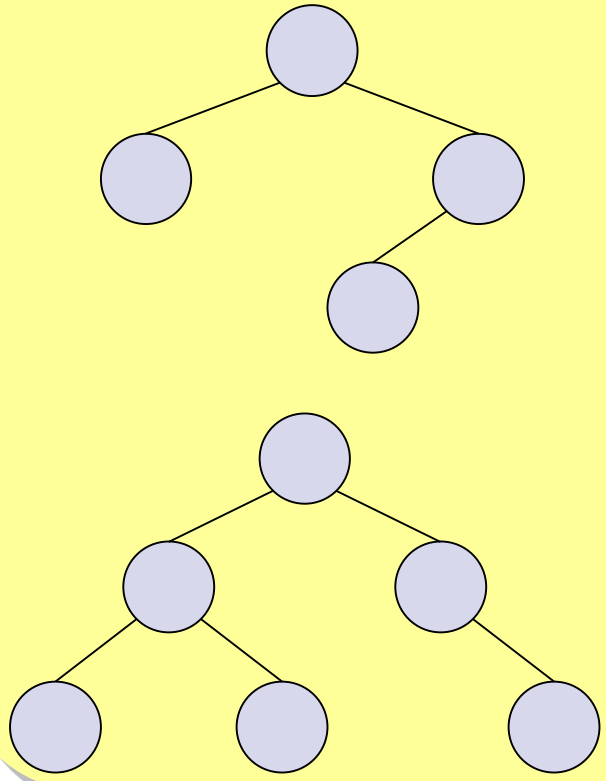
Unbalanced!

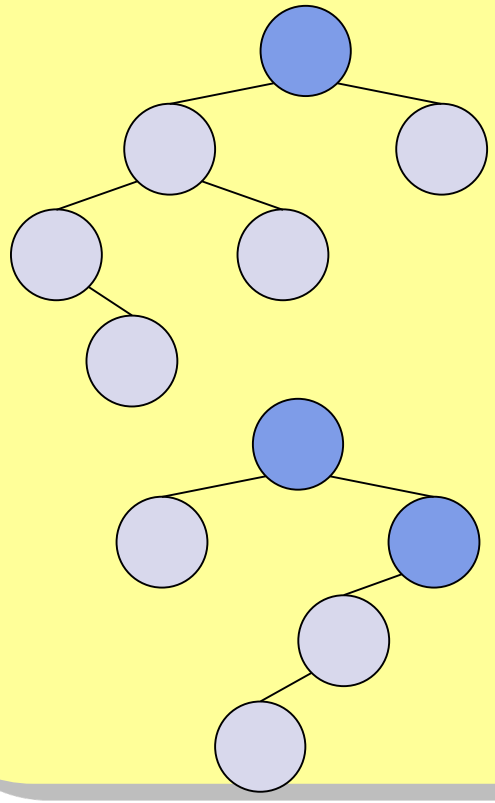
Using AVL Tree



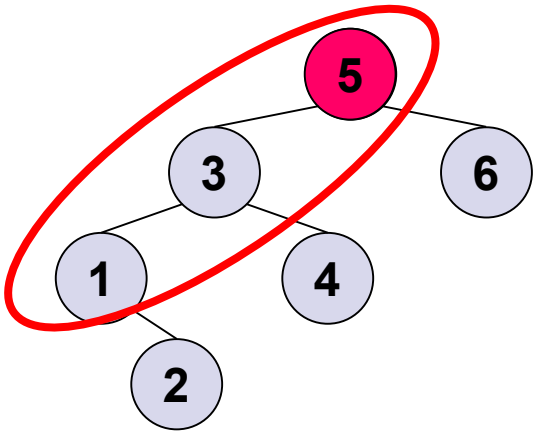
AVL Balanced

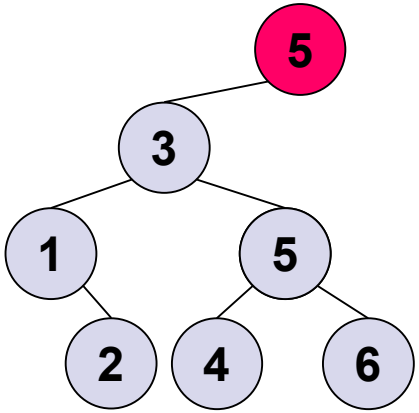


Not AVL Balanced

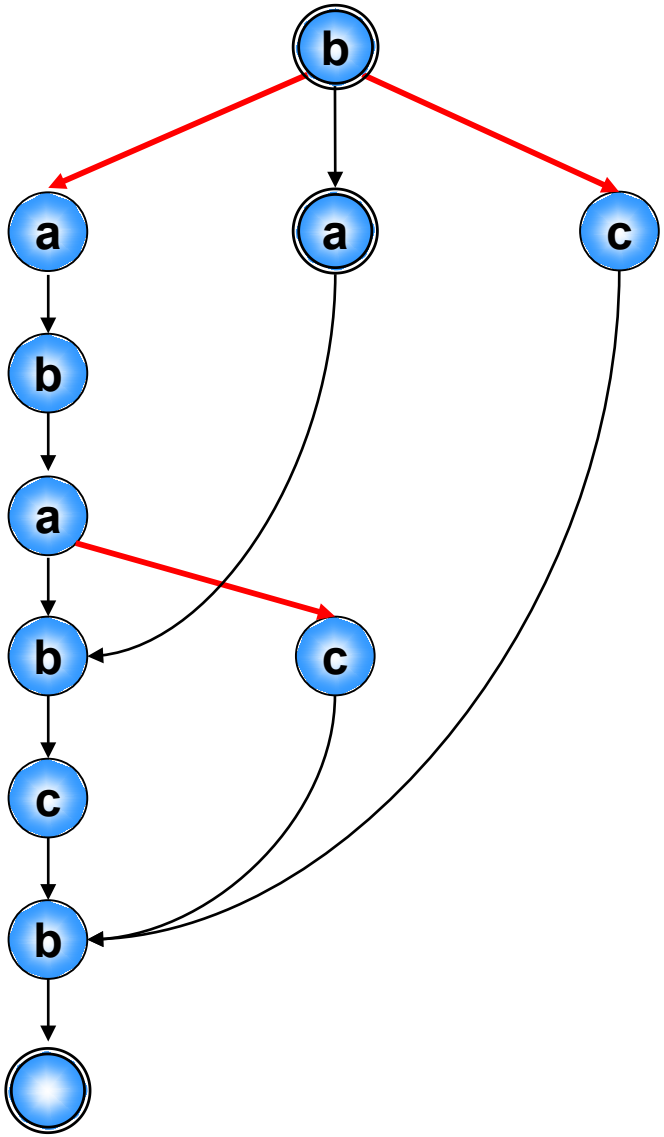


Node Rotation



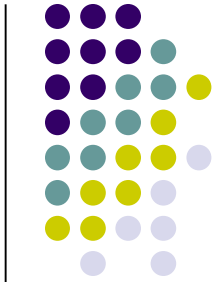


AVL DAWG



ababcb

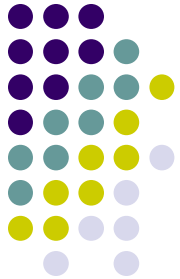
Construction of AVL TDAWG



Theorem. (Miyamoto et al. 2003)

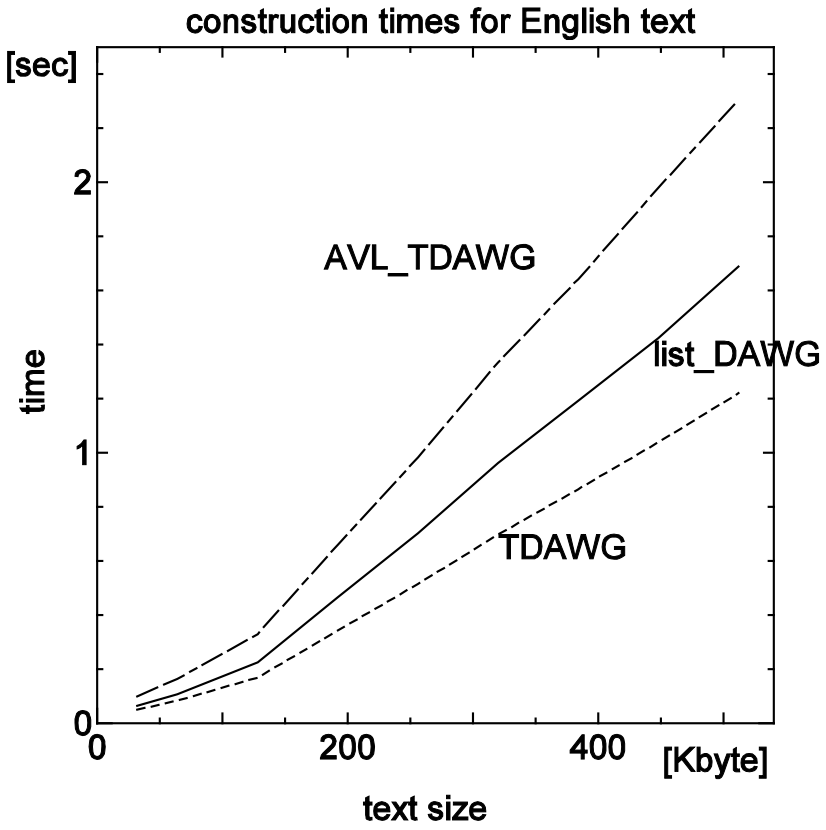
For any string w *of length* n*,*
$AVL\_TDAWG(w)$ *can be constructed on-line and in* $O(n \log |\Sigma|)$ *time and* $O(n)$ *space*.

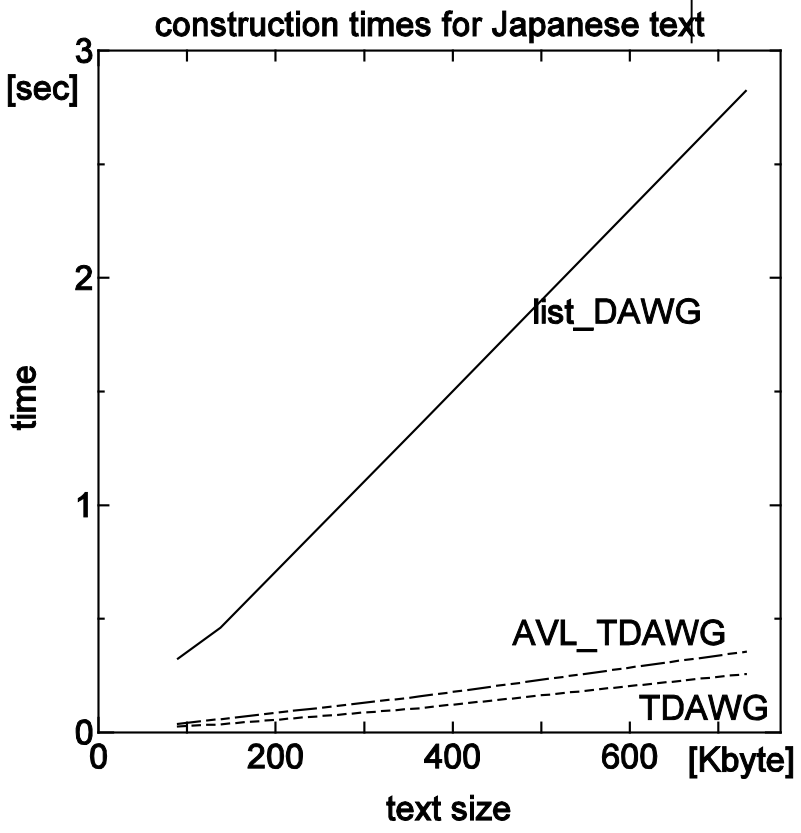
Complexities



	space	search time	
		average case	worst case
table	$O(\|w\| \cdot \|\Sigma\|)$	$O(\|p\|)$	$O(\|p\|)$
linked list	$O(\|w\|)$	$O(\|p\| \cdot \|\Sigma\|)$	$O(\|p\| \cdot \|\Sigma\|)$
TDAWG	$O(\|w\|)$	$O(\|p\| \cdot \log \|\Sigma\|)$	$O(\|p\| \cdot \|\Sigma\|)$
AVL-TDAWG	$O(\|w\|)$	$O(\|p\| \cdot \log \|\Sigma\|)$	$O(\|p\| \cdot \log \|\Sigma\|)$

Experiment – Construction Times

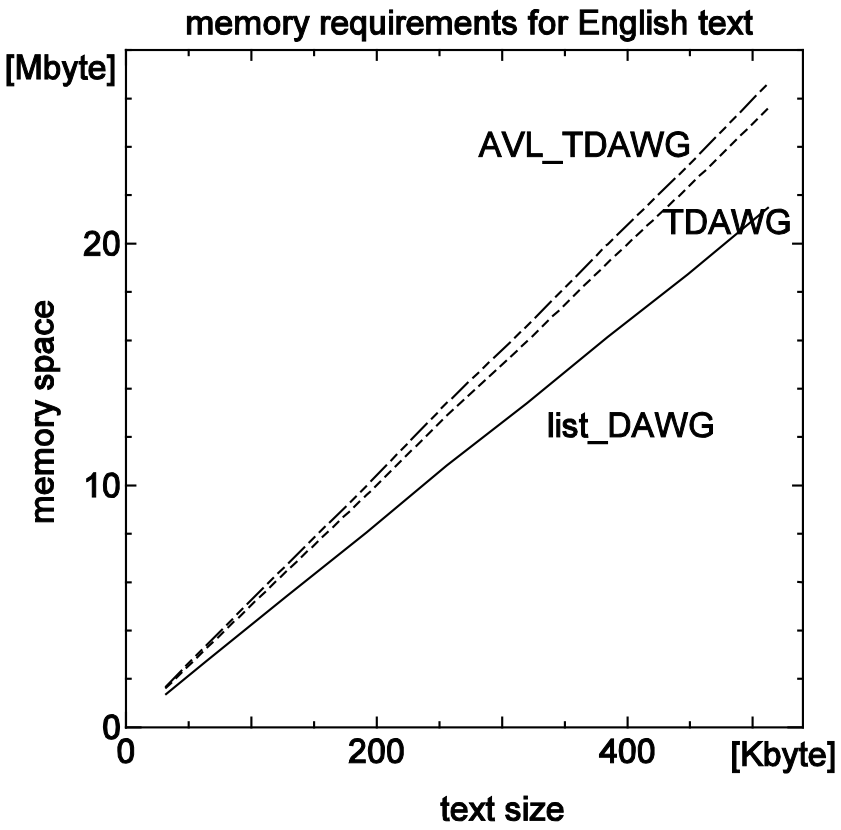


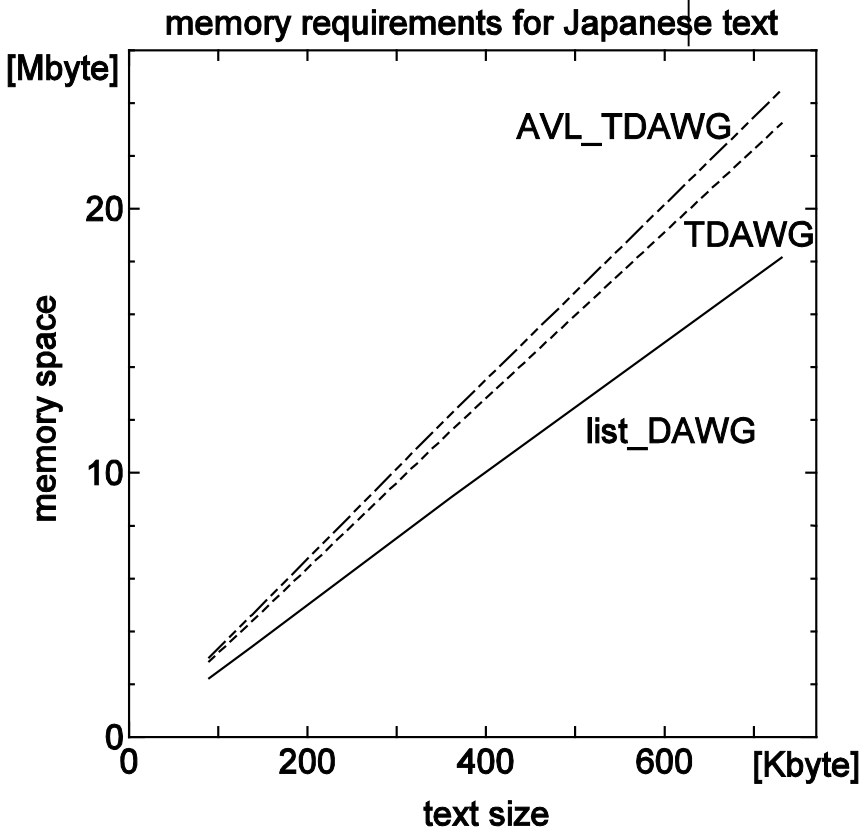


English

Japanese

Experiment – Memory Space

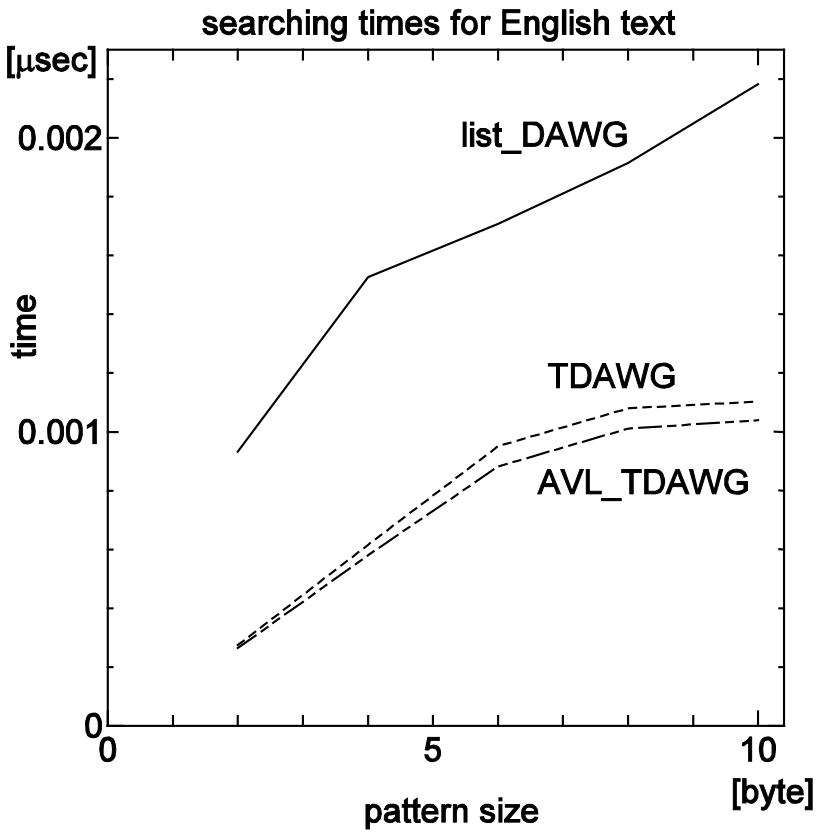


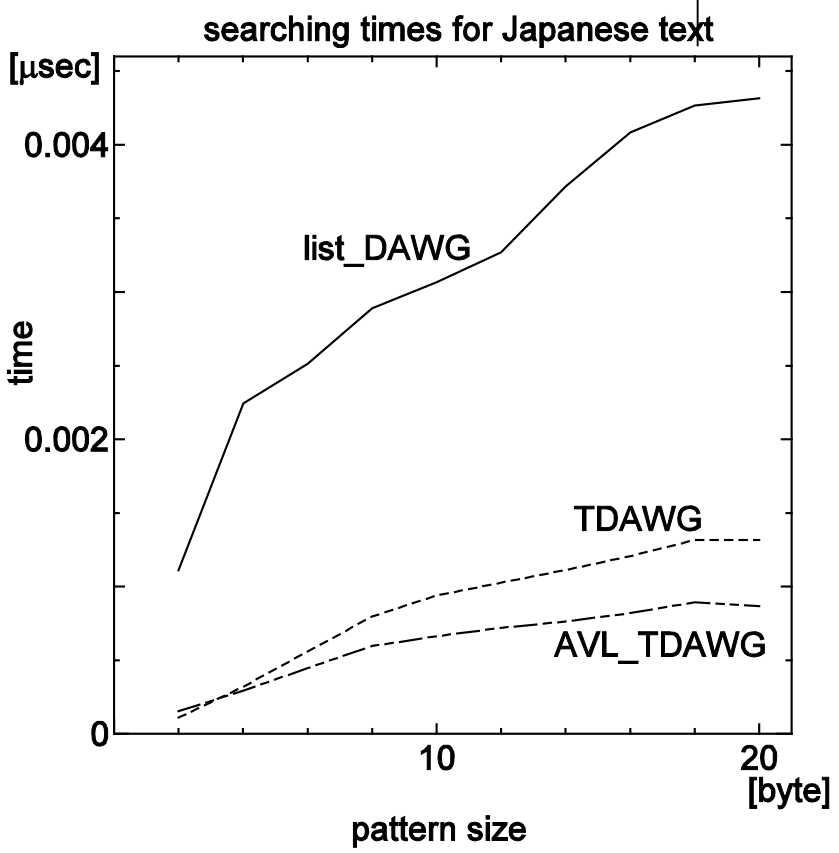


English

Japanese

Experiment – Search Times

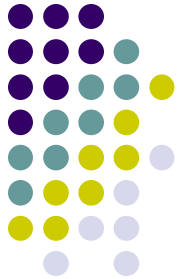




English

Japanese

Citations



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Also Accepted to Theoretical Computer Science