CIAC 2015

An opportunistic text indexing structure based on run length encoding

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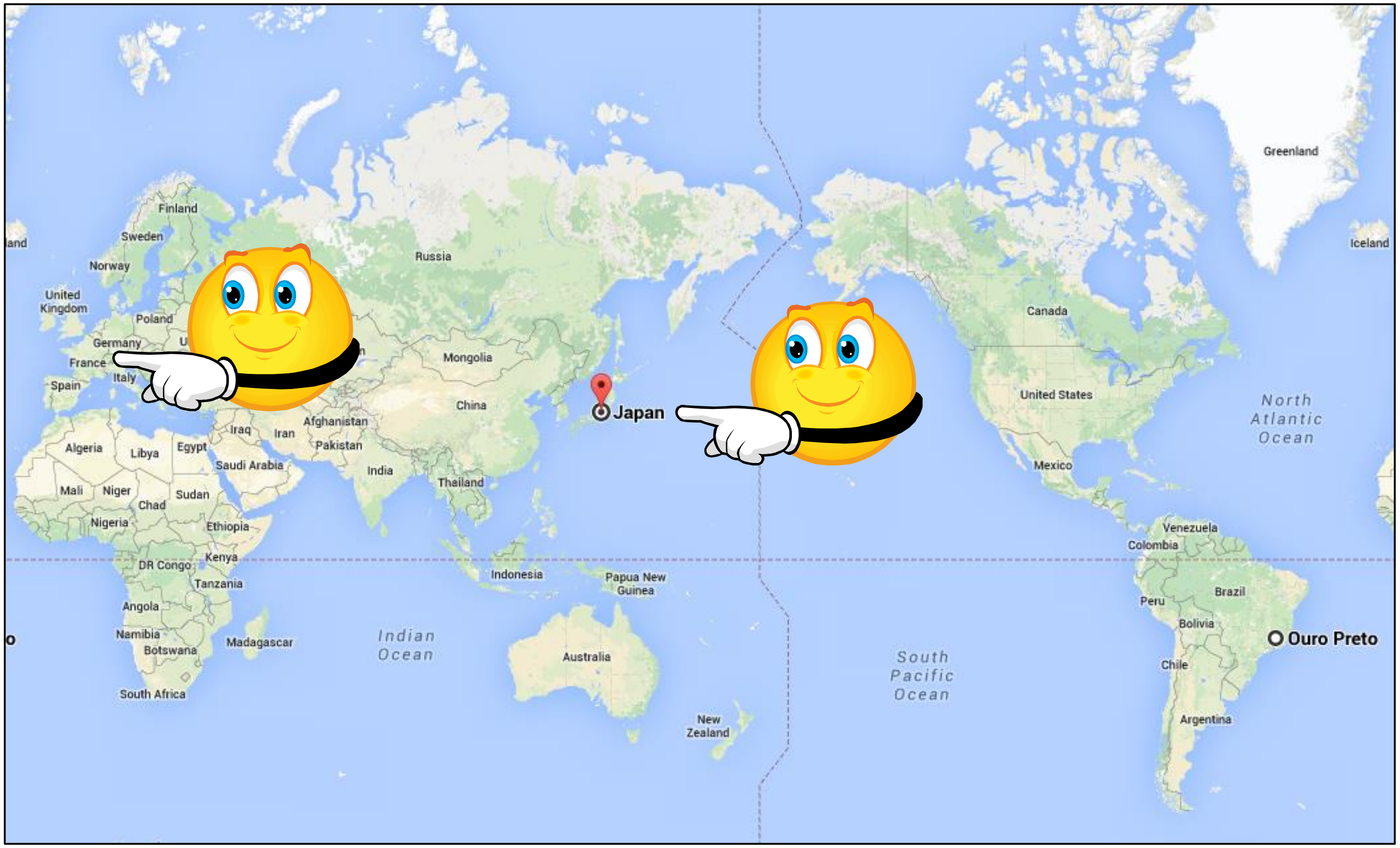

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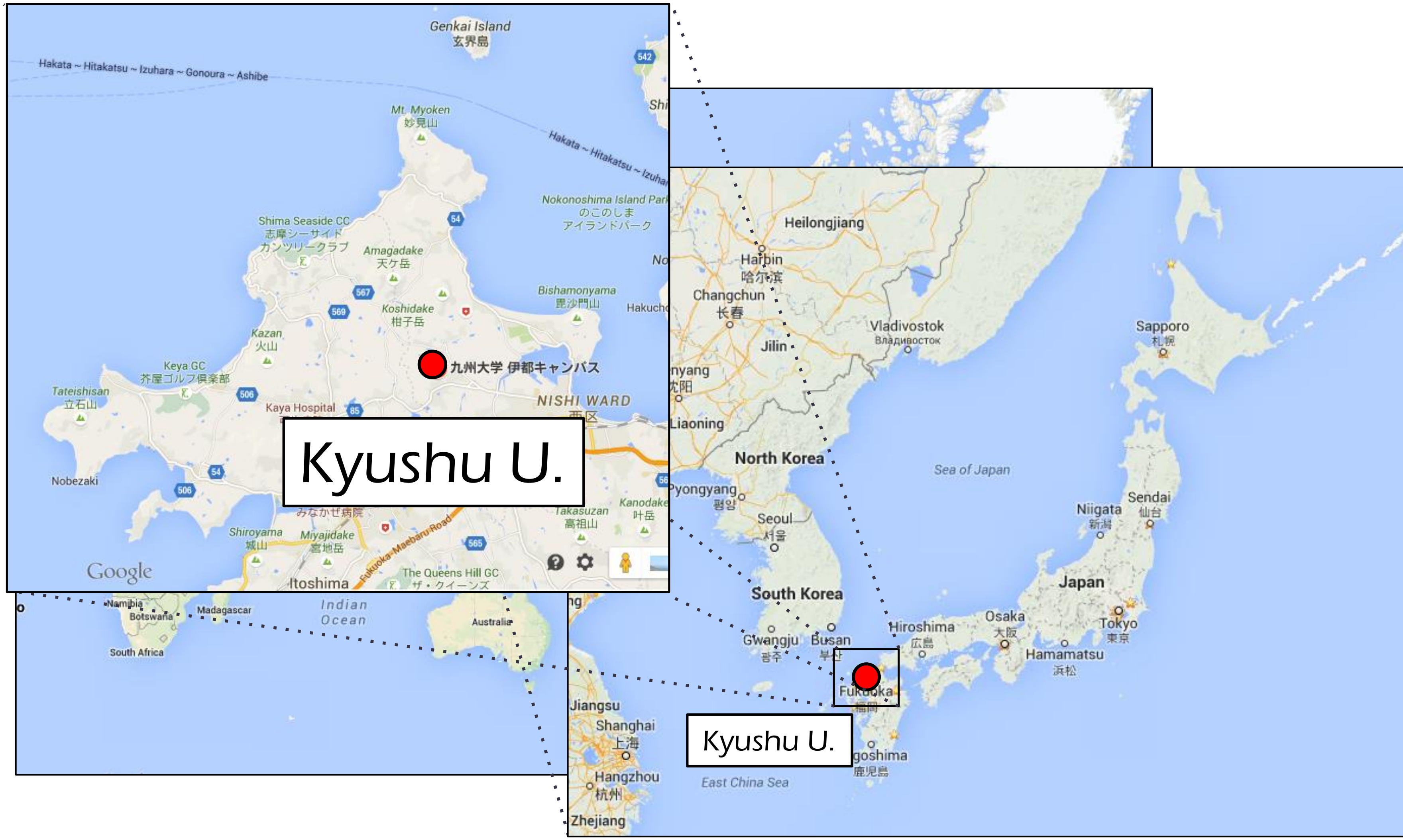

Kyushu University, Japan

Itoshima Peninsula

糸島

String Island

String matching

Input: text string T and pattern string P

Output: all occurrences of P in T

String matching

Input: text string T and pattern string P

Output: all occurrences of P in T

text T

We introduce a general framework which is suitable to capture an essence of compressed pattern matching according to various dictionary based compressions. The goal is to find all occurrences of a pattern in a text without decompression, which is one of the most active topics in string matching. Our framework includes such compression methods as Lempel-Ziv family, (LZ77, LZSS, LZ78, LZW), byte-pair encoding, and the static dictionary based method.

pattern P

compress

String matching

Input: text string T and pattern string P

Output: all occurrences of P in T

- ✓ String matching is fundamental to areas such as
  - Information Retrieval
  - Bioinformatics, etc.

Indexed string matching

Preprocess: build index on fixed text T

Query: pattern string P

Answer: all occurrences of P in T

- ✓ Goal is to construct a space-efficient index on T which quickly answers to string matching query.
  - ➢ Text T can be very long (e.g., DNA sequences).
  - ➢ We may receive many different query patterns.

Classical text index: Suffix Array

The suffix array SA of text T is an array which stores the beginning positions of the suffixes of T in lexicographic order [Manber & Myers, 1991].

T = cococacao$

$ is an end-marker which appears only at the end of any string.

Classical text index: Suffix Array

The suffix array *SA* of text *T* is an array which stores the beginning positions of the suffixes of *T* in lexicographic order [Manber & Myers, 1991].

```
 1 cococacao$
 2  ococacao$
 3   cocacao$
 4    ocacao$
 5     cacao$
 6      acao$
 7       cao$
 8        ao$
 9         o$
10          $
```

Classical text index: Suffix Array

The <u>suffix array</u> *SA* of text *T* is an array which stores the beginning positions of the suffixes of *T* in <u>lexicographic order</u> [Manber & Myers, 1991].

1	cococacao$
2	ococacao$
3	cocacao$
4	ocacao$
5	cacao$
6	acao$
7	cao$
8	ao$
9	o$
10	$

Sort

SA	
10	$
6	acao$
8	ao$
5	cacao$
7	cao$
3	cocacao$
1	cococacao$
9	o$
4	ocacao$
2	ococacao$

String matching with suffix array

Binary search a given pattern P on SA

$P =$ `coc`

SA	
10	`$`
6	`acao$`
8	`ao$`
5	`cacao$`
7	`cao$`
3	`cocacao$`
1	`cococacao$`
9	`o$`
4	`ocacao$`
2	`ococacao$`

String matching with suffix array

Binary search a given pattern P on SA

P = coc
>
cao$

SA

10	$
6	acao$
8	ao$
5	cacao$
7	cao$
3	cocacao$
1	cococacao$
9	o$
4	ocacao$
2	ococacao$

String matching with suffix array

Binary search a given pattern *P* on *SA*

P = coc
∧
o$

SA	
10	$
6	acao$
8	ao$
5	cacao$
7	cao$
3	cocacao$
1	cococacao$
9	o$
4	ocacao$
2	ococacao$

String matching with suffix array

Binary search a given pattern P on SA

P = coc
||
cocacao$

SA	
10	$
6	acao$
8	ao$
5	cacao$
7	cao$
✓3	cocacao$
1	cococacao$
9	o$
4	ocacao$
2	ococacao$

String matching with suffix array

Binary search a given pattern P on SA

P = <u>coc</u>

||

<u>coc</u>ocacao$

SA	
10	$
6	acao$
8	ao$
5	cacao$
7	cao$
✓3	<u>coc</u>acao$
✓1	<u>coc</u>ocacao$
9	o$
4	ocacao$
2	ococacao$

String matching with suffix array

Binary search a given pattern P on SA

1 2 3 4 5 6 7 8 9 10

T = cococacao$

P = coc

SA

10	$
6	acao$
8	ao$
5	cacao$
7	cao$
3	cocacao$
1	cococacao$
9	o$
4	ocacao$
2	ococacao$

String matching with suffix array

All occurrences of P in T can be found in $O(m \log u + occ)$ time using SA.

The search time can be improved to $O(m + \log u + occ)$ using the LCP array.

SA	
10	`$`
6	`acao$`
8	`ao$`
5	`cacao$`
7	`cao$`
3	`cocacao$`
1	`cococacao$`
9	`o$`
4	`ocacao$`
2	`ococacao$`

$u = |T|$

$m = |P|$

occ = # occ. of P in T

SA+LCP

Theorem [Manber & Myers, 1991]

There is an index (SA+LCP) which reports all occ occurrences of P in T in $O(m+\log u+occ)$ time, and requires $\underbrace{2u\log u}_{\text{SA \& LCP}} + \underbrace{u\log\sigma}_{\text{Text } T} + \underbrace{O(u)}_{\text{Auxiliary data structure}}$ bits of space.

SA & LCP

Text T

Auxiliary data structure

$u = |T|$
$m = |P|$
$\sigma = |\Sigma|$

✓ This can take too much space for large text T (i.e., for large u).

Compressed index

- ✓ There are a number of compressed indexes which occupy only compressed size of text.
  - FM-index [Ferragina & Mancini, 2000], Compressed Suffix Array [Grossi & Vitter, 2000], Lempel-Ziv index [Gagie et al., 2014], etc.
- ✓ Most of them are **slower** than SA+LCP.

Our proposal

New compressed index based on run length encoding (RLE) of text which is **smaller** & **faster** than SA+LCP.

Run Length Encoding (RLE)

The run length encoding of text T, denoted $RLE(T)$, is a compressed representation of T in which each maximal run $a\ldots a$ of characters is encoded by a^p, where p denotes the length of the maximal run.

T = aaaabbbaacccccccbbbbbaaaaa$

$RLE(T) = \mathtt{a}^4\mathtt{b}^3\mathtt{a}^2\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$

✓ Applications to RLE include:
- black-white fax messages
- image format (PackBits, TIFF)
- music format (MIDI)

RLE suffixes

Let $n = |RLE(T)|$. For any $1 \le i \le n$, $RLEsuf(i)$ is the suffix of $RLE(T)$ starting with the i-th run.

$RLE(T)$: $\mathtt{a}^4\mathtt{b}^3\mathtt{a}^2\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$

$n = 7$

$RLEsuf(1)$: $\mathtt{a}^4\mathtt{b}^3\mathtt{a}^2\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$

$RLEsuf(2)$: $\mathtt{b}^3\mathtt{a}^2\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$

$RLEsuf(3)$: $\mathtt{a}^2\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$

$RLEsuf(4)$: $\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$

$RLEsuf(5)$: $\mathtt{b}^5\mathtt{a}^5\$$

$RLEsuf(6)$: $\mathtt{a}^5\$$

$RLEsuf(7)$: $\$$

Difficulty in indexing RLE suffixes

✓ We want to index only RLE suffixes of the text, but simply sorted RLE suffixes don't work!

sorted RLE suffixes of text

$$\begin{array}{l}
a^5b\cdots \\
a^5b\cdots \\
a^5c\cdots \\
a^4b\cdots \\
a^4c\cdots \\
a^4c\cdots \\
a^4c\cdots \\
a^3b\cdots \\
a^3b\cdots
\end{array}$$

Difficulty in indexing RLE suffixes

- ✓ We want to index only RLE suffixes of the text, but simply sorted RLE suffixes don't work!

sorted RLE suffixes of text

```
aaaaab···
aaaaab···
aaaaac···
aaaab···
aaaac···
aaaac···
aaaac···
aaab···
aaab···
```

Difficulty in indexing RLE suffixes

- ✓ We want to index only RLE suffixes of the text, but simply sorted RLE suffixes don't work!

$RLE(P)$: $\mathtt{a}^2\mathtt{b}^1$

sorted RLE suffixes of text

✓ aaa<u>aab</u>···
✓ aaa<u>aab</u>···
  aaaaac···
✓ aa<u>aab</u>···
  aaaac···
  aaaac···
  aaaac···
✓ a<u>aab</u>···
✓ a<u>aab</u>···

Pattern occurrences are spread out, so we cannot binary search!!

Our ideas to index RLE suffixes

- ✓ When sorting RLE suffixes, we "ignore" the exponents of the <u>first runs</u> of RLE suffixes of text T.
- ✓ To find occurrences of pattern P, we first "ignore" the exponent of the first run of $RLE(P)$, and find its corresponding range.
- ✓ We then pick up only the occurrences of $RLE(P)$ from this range.

Truncated RLE suffixes

$tRLEsuf(i)$ is the suffix of $RLEsuf(i)$ where the first exponent p_i is truncated to 1.

$RLEsuf(1)$:	$\mathtt{a}^4\mathtt{b}^3\mathtt{a}^2\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$	$tRLEsuf(1)$:	$\mathtt{a}\ \mathtt{b}^3\mathtt{a}^2\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$
$RLEsuf(2)$:	$\mathtt{b}^3\mathtt{a}^2\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$	$tRLEsuf(2)$:	$\mathtt{b}\ \mathtt{a}^2\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$
$RLEsuf(3)$:	$\mathtt{a}^2\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$	$tRLEsuf(3)$:	$\mathtt{a}\ \mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$
$RLEsuf(4)$:	$\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$	$tRLEsuf(4)$:	$\mathtt{c}\ \mathtt{b}^5\mathtt{a}^5\$$
$RLEsuf(5)$:	$\mathtt{b}^5\mathtt{a}^5\$$	$tRLEsuf(5)$:	$\mathtt{b}\ \mathtt{a}^5\$$
$RLEsuf(6)$:	$\mathtt{a}^5\$$	$tRLEsuf(6)$:	$\mathtt{a}\ \$$
$RLEsuf(7)$:	$\$$	$tRLEsuf(7)$:	$\$$

Our index: Truncated RLE Suffix Array

The tRLE suffix array *tRLESA* of text *T* is an array which stores the beginning positions of the tRLE suffixes in lexicographical order.

1	$a\ b^3a^2c^7b^5a^5\$$
2	$b\ a^2c^7b^5a^5\$$
3	$a\ c^7b^5a^5\$$
4	$c\ b^5a^5\$$
5	$b\ a^5\$$
6	$a\ \$$
7	$\$$

Sort

tRLESA	
7	$\$$
6	$a\ \$$
1	$a\ b^3a^2c^7b^5a^5\$$
3	$a\ c^7b^5a^5\$$
5	$b\ a^5\$$
2	$b\ a^2c^7b^5a^5\$$
4	$c\ b^5a^5\$$

Monotonicity on Truncated RLE Suffix Array

tRLESA	tRLE suffixes
$\vdots$	$\vdots$
47	$b^{(2)}c^5a^2b^2a^6\cdots$
99	$b^{(9)}c^5a^2b^3a^1\cdots$
11	$b^{(1)}c^5a^2b^5a^4\cdots$
40	$b^{(2)}c^5a^2b^7a^3\cdots$
55	$b^{(3)}c^5a^2b^8c^2\cdots$
72	$b^{(9)}c^5a^2b^6c^3\cdots$
19	$b^{(1)}c^5a^2b^6c^3\cdots$
26	$b^{(5)}c^5a^2b^4c^7\cdots$
4	$b^{(1)}c^5a^2b^1c^8\cdots$
$\vdots$	

Ignored exponents in parentheses

Monotonicity on Truncated RLE Suffix Array

We first look for $\mathtt{b}\mathtt{c}^5\mathtt{a}^2$

$RLE(P)$: $\mathtt{b}^3\mathtt{c}^5\mathtt{a}^2\mathtt{b}^4$

The range $\mathtt{b}\mathtt{c}^5\mathtt{a}^2$ matches

This range can be found by a binary search.

tRLESA	tRLE suffixes
$\vdots$	$\vdots$
47	$\mathtt{b}^{(2)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^2\mathtt{a}^6\cdots$
99	$\mathtt{b}^{(9)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^3\mathtt{a}^1\cdots$
11	$\mathtt{b}^{(1)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^5\mathtt{a}^4\cdots$
40	$\mathtt{b}^{(2)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^7\mathtt{a}^3\cdots$
55	$\mathtt{b}^{(3)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^8\mathtt{c}^2\cdots$
72	$\mathtt{b}^{(9)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^6\mathtt{c}^3\cdots$
19	$\mathtt{b}^{(1)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^6\mathtt{c}^3\cdots$
26	$\mathtt{b}^{(5)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^4\mathtt{c}^7\cdots$
4	$\mathtt{b}^{(1)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^1\mathtt{c}^8\cdots$
$\vdots$	$\vdots$

Monotonicity on Truncated RLE Suffix Array

We next look for $\mathrm{b}\mathrm{c}^5\mathrm{a}^2\underline{\mathrm{b}^4}$

$RLE(P)$: $\mathrm{b}^3\mathrm{c}^5\mathrm{a}^2\mathrm{b}^4$

The range $\boxed{\mathrm{b}\mathrm{c}^5\mathrm{a}^2}$ matches

tRLESA	tRLE suffixes
⋮	⋮
47	$\mathrm{b}^{(2)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^2\mathrm{a}^6\cdots$
99	$\mathrm{b}^{(9)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^3\mathrm{a}^1\cdots$
11	$\mathrm{b}^{(1)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^5\mathrm{a}^4\cdots$
40	$\mathrm{b}^{(2)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^7\mathrm{a}^3\cdots$
55	$\mathrm{b}^{(3)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^8\mathrm{c}^2\cdots$
72	$\mathrm{b}^{(9)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^6\mathrm{c}^3\cdots$
19	$\mathrm{b}^{(1)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^6\mathrm{c}^3\cdots$
26	$\mathrm{b}^{(5)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^4\mathrm{c}^7\cdots$
4	$\mathrm{b}^{(1)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^1\mathrm{c}^8\cdots$
⋮	⋮

Monotonicity on Truncated RLE Suffix Array

We next look for $\mathtt{b}\mathtt{c}^5\mathtt{a}^2\underline{\mathtt{b}^4}$

$RLE(P)$: $\mathtt{b}^3\mathtt{c}^5\mathtt{a}^2\mathtt{b}^4$

The range $\mathtt{b}\mathtt{c}^5\mathtt{a}^2$ matches

tRLESA	tRLE suffixes
⋮	⋮
47	$\mathtt{b}^{(2)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^{\mathbf{2}}\mathtt{a}^6\cdots$
99	$\mathtt{b}^{(9)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^{\mathbf{3}}\mathtt{a}^1\cdots$
11	$\mathtt{b}^{(1)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^{\mathbf{5}}\mathtt{a}^4\cdots$
40	$\mathtt{b}^{(2)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^{\mathbf{7}}\mathtt{a}^3\cdots$
55	$\mathtt{b}^{(3)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^{\mathbf{8}}\mathtt{c}^2\cdots$
72	$\mathtt{b}^{(9)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^{\mathbf{6}}\mathtt{c}^3\cdots$
19	$\mathtt{b}^{(1)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^{\mathbf{6}}\mathtt{c}^3\cdots$
26	$\mathtt{b}^{(5)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^{\mathbf{4}}\mathtt{c}^7\cdots$
4	$\mathtt{b}^{(1)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^{\mathbf{1}}\mathtt{c}^8\cdots$
⋮	⋮

monotonically non-decreasing

monotonically non-increasing

Matching with Truncated RLE Suffix Array

We next look for $\mathrm{b}\mathrm{c}^5\mathrm{a}^2\underline{\mathrm{b}^4}$

RLE(*P*): $\mathrm{b}^3\mathrm{c}^5\mathrm{a}^2\mathrm{b}^4$

tRLESA	tRLE suffixes
⋮	⋮
47	$\mathrm{b}^{(2)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^2\mathrm{a}^6\cdots$
99	$\mathrm{b}^{(9)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^3\mathrm{a}^1\cdots$
11	$\mathrm{b}^{(1)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^5\mathrm{a}^4\cdots$
40	$\mathrm{b}^{(2)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^7\mathrm{a}^3\cdots$
55	$\mathrm{b}^{(3)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^8\mathrm{c}^2\cdots$
72	$\mathrm{b}^{(9)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^6\mathrm{c}^3\cdots$
19	$\mathrm{b}^{(1)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^6\mathrm{c}^3\cdots$
26	$\mathrm{b}^{(5)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^4\mathrm{c}^7\cdots$
4	$\mathrm{b}^{(1)}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^1\mathrm{c}^8\cdots$
⋮	⋮

the range $\boxed{\mathrm{b}\mathrm{c}^5\mathrm{a}^2\mathrm{b}^4}$ matches

Based on the monotonicity, this range can be found by a binary search.

Matching with Truncated RLE Suffix Array

We finally look for $b^{\mathbf{3}}c^5a^2b^4$

$RLE(P)$: $b^3c^5a^2b^4$

the range $\boxed{bc^5a^2b^4}$ matches

tRLESA	tRLE suffixes
⋮	⋮
47	$b^{(2)}c^5a^2b^2a^6\cdots$
99	$b^{(9)}c^5a^2b^3a^1\cdots$
11	$b^{(1)}c^5a^2b^5a^4\cdots$
40	$b^{(2)}c^5a^2b^7a^3\cdots$
55	$b^{(3)}c^5a^2b^8c^2\cdots$
72	$b^{(9)}c^5a^2b^6c^3\cdots$
19	$b^{(1)}c^5a^2b^6c^3\cdots$
26	$b^{(5)}c^5a^2b^4c^7\cdots$
4	$b^{(1)}c^5a^2b^1c^8\cdots$
⋮	⋮

Matching with Truncated RLE Suffix Array

We finally look for $\mathtt{b}^{\mathbf{3}}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^4$

$RLE(P)$: $\mathtt{b}^3\mathtt{c}^5\mathtt{a}^2\mathtt{b}^4$

tRLESA	tRLE suffixes
⋮	⋮
47	$\mathtt{b}^{(2)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^2\mathtt{a}^6\cdots$
99	$\mathtt{b}^{(9)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^3\mathtt{a}^1\cdots$
11	$\mathtt{b}^{(1)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^5\mathtt{a}^4\cdots$
40	$\mathtt{b}^{(2)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^7\mathtt{a}^3\cdots$
55	$\mathtt{b}^{(\mathbf{3})}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^8\mathtt{c}^2\cdots$
72	$\mathtt{b}^{(\mathbf{9})}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^6\mathtt{c}^3\cdots$
19	$\mathtt{b}^{(1)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^6\mathtt{c}^3\cdots$
26	$\mathtt{b}^{(\mathbf{5})}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^4\mathtt{c}^7$
4	$\mathtt{b}^{(1)}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^1\mathtt{c}$
⋮	⋮

the range $\boxed{\mathtt{b}\mathtt{c}^5\mathtt{a}^2\mathtt{b}^4}$ matches

We want only those whose 1st exponents are at least **3**

Matching with Truncated RLE Suffix Array

RLE(*P*): $b^3c^5a^2b^4$

exponents	*tRLESA*	tRLE suffixes
⋮	⋮	⋮
2	47	$b^{(2)}c^5a^2\,b^2a^6\cdots$
9	99	$b^{(9)}c^5a^2\,b^3a^1\cdots$
1	11	$b^{(1)}c^5a^2\,b^5a^4\cdots$
2	40	$b^{(2)}c^5a^2\,b^7a^3\cdots$
3	55	$b^{(3)}c^5a^2\,b^8c^2\cdots$
9	72	$b^{(9)}c^5a^2\,b^6c^3\cdots$
1	19	$b^{(1)}c^5a^2\,b^6c^3\cdots$
5	26	$b^{(5)}c^5a^2\,b^4c^7\cdots$
1	4	$b^{(1)}c^5a^2\,b^1c^8\cdots$
⋮	⋮	⋮

We use an array of ignored exponents of the first runs.

Matching with Truncated RLE Suffix Array

We finally look for $b^{3}c^{5}a^{2}b^{4}$

$RLE(P)$: $b^3c^5a^2b^4$

the range $bc^5a^2b^4$ matches

exponents	*tRLESA*	tRLE suffixes
⋮	⋮	⋮
2	47	$b^{(2)}c^5a^2b^2a^6\ldots$
9	99	$b^{(9)}c^5a^2b^3a^1\ldots$
1	11	$b^{(1)}c^5a^2b^5a^4\ldots$
2	40	$b^{(2)}c^5a^2b^7a^3\ldots$
3	55	$b^{(3)}c^5a^2b^8c^2\ldots$
9	72	$b^{(9)}c^5a^2b^6c^3\ldots$
1	19	$b^{(1)}c^5a^2b^6c^3\ldots$
5	26	$b^{(5)}c^5a^2b^4c^7\ldots$
1	4	$b^{(1)}c^5a^2b^1c^8\ldots$
⋮	⋮	⋮

Matching with Truncated RLE Suffix Array

We finally look for $b^{3}c^{5}a^{2}b^{4}$

$RLE(P)$: $b^{3}c^{5}a^{2}b^{4}$

Range Maximum Query (RMQ)

exponents		*tRLESA*	tRLE suffixes
$\vdots$		$\vdots$	$\vdots$
2		47	$b^{(2)}c^{5}a^{2}b^{2}a^{6}\ldots$
9		99	$b^{(9)}c^{5}a^{2}b^{3}a^{1}\ldots$
1		11	$b^{(1)}c^{5}a^{2}b^{5}a^{4}\ldots$
2		40	$b^{(2)}c^{5}a^{2}b^{7}a^{3}\ldots$
3		55	$b^{(3)}c^{5}a^{2}b^{8}c^{2}\ldots$
9	✓	72	$b^{(9)}c^{5}a^{2}b^{6}c^{3}\ldots$
1		19	$b^{(1)}c^{5}a^{2}b^{6}c^{3}\ldots$
5		26	$b^{(5)}c^{5}a^{2}b^{4}c^{7}\ldots$
1		4	$b^{(1)}c^{5}a^{2}b^{1}c^{8}\ldots$
$\vdots$		$\vdots$	$\vdots$

Matching with Truncated RLE Suffix Array

We finally look for $b^{\mathbf{3}}c^5a^2b^4$

expo

We perform RMQ's recursively, in the 1st & 2nd halves of the range.

$RLE(P)$: $b^3c^5a^2b^4$

	[illegible]		[illegible]	[illegible]
	[illegible]		99	$b^{(9)}c^5a^2b^3a^1\ldots$
RMQ	1		11	$b^{(1)}c^5a^2b^5a^4\ldots$
	2		40	$b^{(2)}c^5a^2b^7a^3\ldots$
	3	✓	55	$b^{(\mathbf{3})}c^5a^2b^8c^2\ldots$
	9	✓	72	$b^{(\mathbf{9})}c^5a^2b^6c^3\ldots$
RMQ	1		19	$b^{(1)}c^5a^2b^6c^3\ldots$
	5	✓	26	$b^{(\mathbf{5})}c^5a^2b^4c^7\ldots$
	1		4	$b^{(1)}c^5a^2b^1c^8\ldots$
	⋮		⋮	⋮

Matching with Truncated RLE Suffix Array

We finally look for $b^{\mathbf{3}}c^5a^2b^4$

exp

Recursion ends when the range maxima is less than **3**.

$RLE(P)$: $b^3c^5a^2b^4$

	2			$b^{(\cdot)}c^5a^2b^{\cdot}a^{\cdot}\ldots$
	9		99	$b^{(9)}c^5a^2b^3a^1\ldots$
RMQ	1		11	$b^{(1)}c^5a^2b^5a^4\ldots$
RMQ	**2**		40	$b^{(2)}c^5a^2b^7a^3\ldots$
	3	✓	55	$b^{(\mathbf{3})}c^5a^2b^8c^2\ldots$
	9	✓	72	$b^{(\mathbf{9})}c^5a^2b^6c^3\ldots$
RMQ	**1**		19	$b^{(1)}c^5a^2b^6c^3\ldots$
	5	✓	26	$b^{(\mathbf{5})}c^5a^2b^4c^7\ldots$
	1		4	$b^{(1)}c^5a^2b^1c^8\ldots$
	⋮		⋮	⋮

Matching with Truncated RLE Suffix Array

RLE(*P*): $b^{3}c^{5}a^{2}b^{4}$

exponents		*tRLESA*	tRLE suffixes
⋮		⋮	⋮
2		47	$b^{(2)}c^{5}a^{2}b^{2}a^{6}\ldots$
9		99	$b^{(9)}c^{5}a^{2}b^{3}a^{1}\ldots$
1		11	$b^{(1)}c^{5}a^{2}b^{5}a^{4}\ldots$
2		40	$b^{(2)}c^{5}a^{2}b^{7}a^{3}\ldots$
3	✓	55	$b^{(3)}c^{5}a^{2}b^{8}c^{2}\ldots$
9	✓	72	$b^{(9)}c^{5}a^{2}b^{6}c^{3}\ldots$
1		19	$b^{(1)}c^{5}a^{2}b^{6}c^{3}\ldots$
5	✓	26	$b^{(5)}c^{5}a^{2}b^{4}c^{7}\ldots$
1		4	$b^{(1)}c^{5}a^{2}b^{1}c^{8}\ldots$
⋮		⋮	⋮

# of RMQ's we perform is *O*(*occ*).

Each RMQ takes *O*(1) time [Fischer & Heum, 2011].

Our results

Theorem 1 (RLE-index)

There is an index which, given $RLE(P)$, reports all occ occurrences of P in T in $O(q+\log n+occ)$ time, and requires $2n\log u + n\log\sigma + n\log n + O(n)$ bits of space.

$u = |T|$
$n = |RLE(T)|$
$(n \leq u)$
$q = |RLE(P)|$
$\sigma = |\Sigma|$

Our results

Theorem 1 (RLE-index)

There is an index which, given $RLE(P)$, reports all occ occurrences of P in T in $\boxed{O(q+\log n+occ)}$ time, and requires $2n\log u + n\log\sigma + n\log n + O(n)$ bits of space.

- ✓ SA+LCP takes $\boxed{O(m+\log u+occ)}$ time for pattern matching ($m = |P|$).
- ✓ Since $q \leq m$ and $n \leq u$ always hold, our index is <u>faster</u> than SA+LCP.

$u = |T|$
$n = |RLE(T)|$
$(n \leq u)$
$q = |RLE(P)|$
$\sigma = |\Sigma|$

Our results

Theorem 1 (RLE-index)

There is an index which, given $RLE(P)$, reports all occ occurrences of P in T in $O(q+\log n+occ)$ time, and requires $\boxed{2n\log u + n\log\sigma + n\log n + O(n)}$ bits of space.

- ✓ SA+LCP requires $\boxed{2u\log u + u\log\sigma + O(u)}$ bits of space.
- ✓ Our RLE-index is <u>smaller</u> when text T is compressible with RLE.

$u = |T|$
$n = |RLE(T)|$
$(n \le u)$
$q = |RLE(P)|$
$\sigma = |\Sigma|$

Our results

Theorem 2 (Construction time & space)

Given $RLE(T)$ of size n, the RLE-index of T can be constructed in $O(n \log n)$ time with $O(n \log u)$ bits of working space.

$u = |T|$
$n = |RLE(T)|$

- ✓ We introduced <u>new combinatorial properties of RLE suffixes</u>.
- ✓ We also use the idea of induced-sorting [Nong et al., 2011] which was originally designed for fast suffix array construction.

Conclusions & Future work

- ✓ Our RLE-index is <u>always faster</u> than SA+LCP.
- ✓ Our RLE-index is <u>smaller</u> than SA+LCP when the text is compressible by RLE (i.e. when the $n \log n$ term is negligible).

- ◆ Comparisons to other compressed index (e.g., FM-index, compressed SA, LZ-index).

FAQ

LCP array and Range Minima

The LCP array of T stores the length of the longest common prefix of neighboring suffixes in SA of T.

SA	LCP	
10	-	`$`
6	0	`acao$`
8	1	`ao$`
5	0	`cacao$`
7	2	`cao$`
3	1	`cocacao$`
1	3	`cococacao$`
9	0	`o$`
4	1	`ocacao$`
2	2	`ococacao$`

LCP array and Range Minima

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6	0	`acao$`
8	1	`ao$`
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7	**2**	`cao$`
3	1	`cocacao$`
1	3	`cococacao$`
9	0	`o$`
4	1	`ocacao$`
2	2	`ococacao$`

LCP array and Range Minima

The LCP array of T stores the length of the longest common prefix of neighboring suffixes in SA of T.

SA	LCP	
10	-	$
6	0	acao$
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5	0	cacao$
7	2	cao$
3	1	cocacao$
1	**3**	cococacao$
9	0	o$
4	1	ocacao$
2	2	ococacao$

LCP array and Range Minima

The length of the LCP of any suffixes can also be computed by a range minimum query.

LCP		
-		`$`
0		`acao$`
1		`ao$`
0	✓	`cacao$`
2		`cao$`
1		`cocacao$`
3	✓	`cococacao$`
0		`o$`
1		`ocacao$`
2		`ococacao$`

LCP array and Range Minima

The length of the LCP of <u>any</u> suffixes can also be computed by a range <u>minimum</u> query.

Range minimum query

LCP		
-		$
0		acao$
1		ao$
0	✓	cacao$
2		cao$
1		cocacao$
3	✓	cococacao$
0		o$
1		ocacao$
2		ococacao$

LCP array and Range Minima

For any integer array of length k, there is a data structure which supports range minimum query in $O(1)$ time, and requires $2k + o(k)$ bits of extra space [Fischer & Heum, 2011].

S-type and L-type RLE suffixes

$RLEsuf(i)$ is S-type if $RLEsuf(i) < RLEsuf(i+1)$.
$RLEsuf(i)$ is L-type if $RLEsuf(i) > RLEsuf(i+1)$.

- ✓ $a^4b^3a^2c^7b^5a^5\$$ is S-type, because
  $a^4b^3a^2c^7b^5a^5\$ < b^3a^2c^7b^5a^5\$$.
- ✓ $b^3a^2c^7b^5a^5\$$ is L-type, because
  $b^3a^2c^7b^5a^5\$ < a^2c^7b^5a^5\$$.

※ Lex. order < on RLE strings is the same as the lex. order < on decompressed strings.

Properties of lex. order of RLE suffixes

For any $1 \leq i \leq n$, let a_i, p_i be the ith character and exponent of $RLE(T)$, respectively.

Lemma

For any $RLEsuf(i)$ and $RLEsuf(j)$ with $a_i = a_j$,

1. if $RLEsuf(i)$ is L-type and $RLEsuf(j)$ is S-type, then $RLEsuf(i) < RLEsuf(j)$.
2. if $RLEsuf(i)$ and $RLEsuf(j)$ are L-type and $p_i < p_j$, then $RLEsuf(i) < RLEsuf(j)$.
3. if $RLEsuf(i)$ and $RLEsuf(j)$ are S-type and $p_i > p_j$, then $RLEsuf(i) < RLEsuf(j)$.

Properties of lex. order of RLE suffixes

Lemma (Case 1)

For any $RLEsuf(i)$ and $RLEsuf(j)$ with $a_i = a_j$,

1. if $RLEsuf(i)$ is L-type and $RLEsuf(j)$ is S-type, then $RLEsuf(i) < RLEsuf(j)$.

L-type (a > $)

S-type (a < b)

$$\mathtt{a}^5\$ \ < \ \mathtt{a}^4\mathtt{b}^3\mathtt{a}^2\mathtt{c}^7\mathtt{b}^5\mathtt{a}^5\$$$

```
aaaaa$
     ∧
aaaabbbaacccccccbbbbbaaaaa$
```

Properties of lex. order of RLE suffixes

Lemma (Case 2)

For any $RLEsuf(i)$ and $RLEsuf(j)$ with $a_i = a_j$,

2. if $RLEsuf(i)$ and $RLEsuf(j)$ are L-type and $p_i < p_j$, then $RLEsuf(i) < RLEsuf(j)$.

L-type (b > a)

L-type (b > a)

$b^{3}a^{2}c^{7}b^{5}a^{5}\$ < b^{5}a^{5}\$$

bbbaacccccccbbbbbaaaaa$

∧

bbbbbaaaaa$

Properties of lex. order of RLE suffixes

Lemma (Case 3)

For any $RLEsuf(i)$ and $RLEsuf(j)$ with $a_i = a_j$,

3. if $RLEsuf(i)$ and $RLEsuf(j)$ are S-type and $p_i > p_j$, then $RLEsuf(i) < RLEsuf(j)$.

S-type (a < b)

S-type (a < c)

$a^{4}b^{3}a^{2}c^{7}b^{5}a^{5}\$ \quad < \quad a^{2}c^{7}b^{5}a^{5}\$$

aaaabbbaaccccccccbbbbbaaaaa$

$\wedge$

aacccccccbbbbbaaaaa$

Our results

Theorem 3 (accessing *SA*)

There is an index which, given an integer $1 \le j \le u$, answers $SA[j]$ in $O(\log^2 n)$ time, and requires $n(3\log u + \log n + \log\sigma) + 2\sigma\log\frac{u}{\sigma} + O(n\log\log n)$ bits of space.

- ✓ Use a wavelet tree [Grossi et al., 2003] in place of RMQ data structure.
- ✓ Then, we can access arbitrary position of *SA*, using our RLE-index.

$u = |T|$
$n = |RLE(T)|$
$\sigma = |\Sigma|$