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◎ 一般的なBogoliubov変換とギャップ方程式

▨ Hamiltonian

一般的な相互作用を含むHamiltonianは（IS, TRSの破れはないとする）

$$H = \sum_{k}\sum_{\alpha} \xi_{k} c^{\dagger}_{k\alpha} c_{k\alpha} + \frac{1}{2}\sum_{k,k'}\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}(k,k') c^{\dagger}_{-k\alpha} c^{\dagger}_{k\beta} c_{k'\gamma} c_{-k'\delta}$$

このとき $V_{\alpha\beta\gamma\delta}(k,k')$ は演算子 $\hat{V}$ の行列要素：

$$V_{\alpha\beta\gamma\delta}(k,k') = \langle -k,\alpha ; k,\beta | \hat{V} | k',\gamma ; -k',\delta \rangle$$

fermionの交換に対する反対称性から

$$V_{\alpha\beta\gamma\delta}(k,k') = -V_{\beta\alpha\gamma\delta}(-k,k') = -V_{\alpha\beta\delta\gamma}(k,-k') \overset{\text{Hermite性}}{=} V_{\delta\gamma\beta\alpha}(k',k)$$

平均場近似を行う：

$$\frac{1}{2}\sum_{k,k'}\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}(k,k')\left[\langle c^{\dagger}_{-k\alpha} c^{\dagger}_{k\beta}\rangle + (c^{\dagger}c^{\dagger} - \langle c^{\dagger}c^{\dagger}\rangle)\right]\left[\langle c_{k'\gamma} c_{-k'\delta}\rangle + (cc - \langle cc\rangle)\right]$$

$$\simeq \frac{1}{2}\sum_{k,k'}\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}(k,k')\left[c^{\dagger}_{-k\alpha} c^{\dagger}_{k\beta}\langle c_{k'\gamma} c_{-k'\delta}\rangle + \langle c^{\dagger}_{-k\alpha} c^{\dagger}_{k\beta}\rangle c_{k'\gamma} c_{-k'\delta}\right]$$

$$\underbrace{-\frac{1}{2}\sum_{k,k'}\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}(k,k')\langle c^{\dagger}_{-k\alpha} c^{\dagger}_{k\beta}\rangle\langle c_{k'\gamma} c_{-k'\delta}\rangle}_{\equiv E_c}$$

ここで

$$\begin{cases} \Delta_{\sigma\sigma'}(k) = -\sum_{k'}\sum_{\gamma\delta} V_{\sigma'\sigma\gamma\delta}(k,k')\langle c_{k'\gamma} c_{-k'\delta}\rangle \longrightarrow \text{ポテンシャルの対称性から } \Delta(k) = -\Delta^{T}(-k) \\ \Delta^{*}_{\sigma\sigma'}(-k) = \sum_{k'}\sum_{\alpha\beta} V_{\alpha\beta\sigma'\sigma}(k',k)\langle c^{\dagger}_{-k'\alpha} c^{\dagger}_{k'\beta}\rangle \end{cases}$$

とすると

$$H_{MF} = \sum_{k}\sum_{\alpha}\xi_{k} c^{\dagger}_{k\alpha} c_{k\alpha} + \frac{1}{2}\sum_{k}\sum_{\alpha\beta}\Delta_{\alpha\beta}(k) c^{\dagger}_{k\alpha} c^{\dagger}_{-k\beta} + \frac{1}{2}\underbrace{\sum_{k}\sum_{\gamma\delta}\Delta^{*}_{\delta\gamma}(-k) c_{k\gamma} c_{-k\delta}}_{=\sum_{\alpha\beta}\Delta^{*}_{\alpha\beta}(k) c_{-k\beta} c_{k\alpha}} + E_c$$

$$= \sum_{k}\sum_{\alpha}\xi_{k} c^{\dagger}_{k\alpha} c_{k\alpha} + \frac{1}{2}\sum_{k}\sum_{\alpha\beta}\left[\Delta_{\alpha\beta}(k) c^{\dagger}_{k\alpha} c^{\dagger}_{-k\beta} + h.c.\right] + E_c$$

行列表示する.

$$\mathbb{C}_k^\dagger = (c_{k\uparrow}^\dagger, c_{k\downarrow}^\dagger, c_{-k\uparrow}, c_{-k\downarrow})$$

と定義すると

$$H_{MF} = \frac{1}{2}\sum_k \mathbb{C}_k^\dagger \underbrace{\begin{pmatrix} \xi_k\sigma_0 & \Delta(k) \\ \Delta^\dagger(k) & -\xi_k\sigma_0 \end{pmatrix}}_{\equiv H_{BdG}(k)} \mathbb{C}_k + E_c + 2\sum_k \xi_k$$

IS の破れなし $\xi_{-k} = \xi_k$

☑ Bogoliubov 変換

$$E(k) = U^\dagger(k) H_{BdG}(k) U(k)$$ と対角化できるとする. $\left(E(k) = \mathrm{diag}[E_{k1}, E_{k2}, -E_{-k1}, -E_{-k2}]\right)$

このとき $U(k)$ は

$$U(k) = \begin{pmatrix} u_k & v_k \\ v_{-k}^* & u_{-k}^* \end{pmatrix} \quad \text{with} \quad U(k)U^\dagger(k) = 1_4$$

$$\Updownarrow$$

$$\begin{cases} u_k u_k^\dagger + v_k v_k^\dagger = \sigma_0 \\ u_{-k}^* v_k^\dagger + v_{-k}^* u_k^\dagger = 0 \end{cases}$$

とする. 新しい基底 $\Gamma_k^\dagger = (\gamma_{k1}^\dagger, \gamma_{k2}^\dagger, \gamma_{-k1}, \gamma_{-k2}) \equiv \mathbb{C}_k^\dagger U(k)$ と定ぎすると

$$H = \sum_k \Gamma_k^\dagger E(k) \Gamma_k$$

$$c_{k\alpha} = [\mathbb{C}_k]_\alpha = [U(k)\Gamma_k]_\alpha = \sum_{\alpha'}\left[(u_k)_{\alpha\alpha'}\gamma_{k\alpha'} + (v_k)_{\alpha\alpha'}\gamma_{-k\alpha'}^\dagger\right]$$

$$c_{-k\alpha}^\dagger = [\mathbb{C}_k]_{\alpha+2} = \sum_{\alpha'}\left[(v_{-k}^*)_{\alpha\alpha'}\gamma_{k\alpha'} + (u_{-k}^*)_{\alpha\alpha'}\gamma_{-k\alpha'}^\dagger\right]$$

となる.

◦ Bogoliubov変換の導出

$$\mathcal{U}(k)E(k) = H_{BdG}(k)\mathcal{U}(k)$$ より

$$\begin{pmatrix} u_k & v_k \\ v^*_{-k} & u^*_{-k} \end{pmatrix}\begin{pmatrix} E_2(k) & 0 \\ 0 & -E_2(-k) \end{pmatrix} = \begin{pmatrix} \xi_k\sigma_0 & \Delta(k) \\ \Delta^\dagger(k) & -\xi_k\sigma_0 \end{pmatrix}\begin{pmatrix} u_k & v_k \\ v^*_{-k} & u^*_{-k} \end{pmatrix}$$

$$\longrightarrow \begin{cases} u_k(E_2(k)-\xi_k\sigma_0) = \Delta(k)v^*_{-k} & \cdots ① \\ -v_k(E_2(-k)+\xi_k\sigma_0) = \Delta(k)u^*_{-k} & \cdots ② \\ v^*_{-k}(E_2(k)+\xi_k\sigma_0) = \Delta^\dagger(k)u_k & \cdots ③ \\ -u^*_{-k}(E_2(-k)-\xi_k\sigma_0) = \Delta^\dagger(k)v_k & \cdots ④ \end{cases}$$

① × $(E_2(k)+\xi_k\sigma_0)$:

$$u_k(E_2(k)-\xi_k\sigma_0)(E_2(k)+\xi_k\sigma_0) = \Delta(k)v^*_{-k}(E_2(k)+\xi_k\sigma_0) \overset{③}{=} \Delta(k)\Delta^\dagger(k)u_k$$

$$\therefore\ u_k(E_2(k)^2-\xi_k^2\sigma_0) = \Delta(k)\Delta^\dagger(k)u_k \qquad ㊍$$

いま.

$$\Delta\Delta^\dagger = \begin{cases} |\psi|^2\sigma_0 & \Delta(k) = \psi(k)i\sigma_y \quad : \text{spin-singlet} \\ |d|^2\sigma_0 + i(d\times d^*)\cdot\sigma & \Delta(k) = d(k)\cdot\sigma i\sigma_y \quad : \text{spin-triplet} \end{cases}$$

以下. unitary な OP のみ考える. (singlet or triplet で $d\times d^* = 0$)

このとき ㊍ は

$$u_k(E_2(k)^2-\xi_k^2\sigma_0) = |\Delta|^2 u_k \qquad (|\Delta|^2 = |\psi|^2 \text{ or } |d|^2)$$

であり. u_k が逆行列をもつとすると $E_2(k) = E_k\sigma_0$ と単位行列に比例する.

このとき $E_k = [\xi_k^2+|\Delta|^2]^{1/2}$, $E_{-k} = E_k$

実際. $u_k = D(k)\sigma_0$ ($\in\mathbb{R}$ とする) とおくと ②より $v_k = -\dfrac{D(-k)}{E_k+\xi_k}\Delta(k)$

unitary condition :

$D(-k) = D(k)$ を仮定した

$$u_k u_k^\dagger + v_k v_k^\dagger = \left[D(k)^2 + \frac{D(k)^2|\Delta|^2}{(E_k+\xi_k)^2}\right]\sigma_0 = \sigma_0$$

実際 $D(-k) = D(k)$ をみたす.

$$\therefore\ D(k) = \frac{E_k+\xi_k}{[(E_k+\xi_k)^2+|\Delta|^2]^{1/2}} \qquad (+\text{の方を選ぶ})$$

したがって

$$u_{\boldsymbol{k}} = \frac{E_{\boldsymbol{k}} + \xi_{\boldsymbol{k}}}{[(E_{\boldsymbol{k}} + \xi_{\boldsymbol{k}})^2 + \Delta^2]^{1/2}}\sigma_0 , \qquad v_{\boldsymbol{k}} = -\frac{\Delta(\boldsymbol{k})}{[(E_{\boldsymbol{k}} + \xi_{\boldsymbol{k}})^2 + \Delta^2]^{1/2}}$$

(これは、もう1つの unitary condition をみたす：

$$u^*_{-\boldsymbol{k}} v^\dagger_{\boldsymbol{k}} + v^*_{-\boldsymbol{k}} u^\dagger_{\boldsymbol{k}} = u_{\boldsymbol{k}} (v^\dagger_{\boldsymbol{k}} + v^*_{-\boldsymbol{k}}) = -\frac{u_{\boldsymbol{k}}}{[\quad]^{1/2}} \big(\Delta^\dagger(\boldsymbol{k}) + \underbrace{\Delta^*(-\boldsymbol{k})}_{= -\Delta^\dagger(\boldsymbol{k})} \big) = 0 \quad //$$
)

以上より、Bogoliubov 変換を行う unitary 行列は

$$\boxed{U(\boldsymbol{k}) = \frac{1}{[(E_{\boldsymbol{k}} + \xi_{\boldsymbol{k}})^2 + |\Delta|^2]^{1/2}} \begin{pmatrix} (E_{\boldsymbol{k}} + \xi_{\boldsymbol{k}})\sigma_0 & -\Delta(\boldsymbol{k}) \\ \Delta^\dagger(\boldsymbol{k}) & (E_{\boldsymbol{k}} + \xi_{\boldsymbol{k}})\sigma_0 \end{pmatrix}}$$

で与えられる。

☑ ギャップ方程式

$$\Delta_{\alpha\beta}(k) = -\sum_{k'}\sum_{\gamma\delta} V_{\beta\alpha\gamma\delta}(k,k')\langle c_{k'\gamma}c_{-k'\delta}\rangle$$

において.

$$\begin{cases} c_{k\alpha} = \sum_{\alpha'}\left[(u_k)_{\alpha\alpha'}\gamma_{k\alpha'} + (v_k)_{\alpha\alpha'}\gamma^\dagger_{-k\alpha'}\right] \\ c_{-k\alpha} = \left[\Gamma^\dagger_k U^\dagger(k)\right]_{\alpha+2} = \sum_{\alpha'}\left[\gamma^\dagger_{k\alpha'}(v^T_{-k})_{\alpha'\alpha} + \gamma_{-k\alpha'}(u^T_{-k})_{\alpha'\alpha}\right] \\ u_k = \dfrac{E_k+\xi_k}{\left[(E_k+\xi_k)^2+|\Delta|^2\right]^{1/2}}\sigma_0 , \qquad v_k = -\dfrac{\Delta(k)}{\left[(E_k+\xi_k)^2+|\Delta|^2\right]^{1/2}} \end{cases}$$

を代入する:

$$(\text{右辺}) = -\sum_{k'}\sum_{\gamma\delta} V_{\beta\alpha\gamma\delta}(k,k')\sum_{\gamma'\delta'}\Big\langle\left[(u_{k'})_{\gamma\gamma'}\gamma_{k'\gamma'} + (v_{k'})_{\gamma\gamma'}\gamma^\dagger_{-k'\gamma'}\right]$$
$$\times\left[\gamma^\dagger_{k'\delta'}(v^T_{-k'})_{\delta'\delta} + \gamma_{-k'\delta'}(u^T_{-k'})_{\delta'\delta}\right]\Big\rangle$$

ここで $\langle\gamma\gamma\rangle = \langle\gamma^\dagger\gamma^\dagger\rangle = 0$

$\langle\gamma^\dagger_{k\alpha}\gamma_{k\alpha'}\rangle = f(E_k)\delta_{\alpha\alpha'}$, $\{\gamma_{k\alpha}, \gamma^\dagger_{k\alpha'}\} = \delta_{\alpha\alpha'}$ を用いると

$$(\text{右辺}) = -\sum_{k'}\sum_{\gamma\delta} V_{\beta\alpha\gamma\delta}(k,k')\sum_{\gamma'\delta'}\delta_{\gamma'\delta'}\Big[(v_{k'})_{\gamma\gamma'}(u^T_{-k'})_{\delta'\delta}f(E_{k'})$$
$$+(u_{k'})_{\gamma\gamma'}(v^T_{-k'})_{\delta'\delta}\{1-f(E_{k'})\}\Big]$$

$$= -\sum_{k'}\sum_{\gamma\delta}\frac{V_{\beta\alpha\gamma\delta}(k,k')}{(E_{k'}+\xi_{k'})^2+|\Delta'|^2}\sum_{\gamma'\delta'}\delta_{\gamma'\delta'}(E_{k'}+\xi_{k'})$$
$$\times\Big[(-\Delta(k'))_{\gamma\gamma'}\delta_{\delta'\delta}f(E_{k'}) + \delta_{\gamma\gamma'}(\overset{\Delta(k')}{\overset{\shortparallel}{-\Delta^T(-k')}})_{\delta'\delta}\{1-f(E_{k'})\}\Big]$$

$$(E+\xi)^2+|\Delta|^2 = E^2+2E\xi+\underbrace{\xi^2+|\Delta|^2}_{E^2} = 2E(E+\xi)$$

$$\therefore \boxed{\Delta_{\alpha\beta}(k) = -\sum_{k'}\sum_{\gamma\delta}V_{\beta\alpha\gamma\delta}(k,k')\frac{\Delta_{\gamma\delta}(k')}{2E_{k'}}\{1-2f(E_{k'})\}}$$

Gap eq.

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☑ 線形化ギャップ方程式

$T = T_c$ では $E_k = \xi_k$ とおけばよい.

$$\Delta_{\alpha\beta}(k) = -\sum_{k'}\sum_{\gamma\delta} V_{\beta\alpha\gamma\delta}(k,k')\frac{\Delta_{\gamma\delta}(k')}{2\xi_{k'}}\tanh\left(\frac{\xi_{k'}}{2T_c}\right)$$

以下 spin-singlet, spin-triplet それぞれの場合について T_c を求める.

∘ spin-singlet の場合

$$V_{\alpha\beta\gamma\delta}(k,k') = \frac{1}{2}(i\sigma_y)_{\alpha\beta}(i\sigma_y)^\dagger_{\gamma\delta}V^e(k,k')$$

$$V^e(k,k') = \overset{4\pi}{\vee}\sum_{l=\text{even}} V_l(k,k')\sum_{m=-l}^{l} Y_{lm}(\hat{k})Y^*_{lm}(\hat{k}')$$

と書けるとする. また秩序変数も球面調和関数で展開すると

$$\Delta(k) = \sum_{l=\text{even}}\sum_{m=-l}^{l} C_{lm}Y_{lm}(\hat{k})\,i\sigma_y$$

よって

$$\sum_{l_1,m_1} C_{l_1m_1}Y_{l_1m_1}(\hat{k})\cancel{(i\sigma_y)_{\alpha\beta}}$$

$$= \overset{4\pi}{\vee}+\sum_{k'}\sum_{\gamma\delta}\frac{1}{2}\cancel{(i\sigma_y)_{\alpha\beta}}(i\sigma_y)^\dagger_{\gamma\delta}\sum_{l_1}V_{l_1}(k,k')\sum_{m_1}Y_{l_1m_1}(\hat{k})Y^*_{l_1m_1}(\hat{k}')$$

$$\times\sum_{l_2,m_2}C_{l_2m_2}Y_{l_2m_2}(\hat{k}')(i\sigma_y)_{\gamma\delta}\ \frac{1}{2\xi_{k'}}\tanh\left(\frac{\xi_{k'}}{2T_c}\right)$$

ここで $\sum_{\gamma\delta}(i\sigma_y)_{\gamma\delta}(i\sigma_y)^\dagger_{\gamma\delta} = \mathrm{tr}\left[(i\sigma_y)(i\sigma_y)^*\right] = -\mathrm{tr}\,\sigma_0 = -2$

両辺に $Y^*_{lm}(\hat{k})$ をかけて $d\Omega_k$ で積分 :

$$\sum_{l_1,m_1}C_{l_1m_1}\underbrace{\int d\Omega_k\,Y^*_{lm}(\hat{k})Y_{l_1m_1}(\hat{k})}_{=\delta_{ll_1}\delta_{mm_1}}$$

$$4\pi\cdot\frac{V}{(2\pi)^3}$$

$$= -\sum_{l_1,m_1}\sum_{l_2,m_2}C_{l_2m_2}\int_0^\infty dk'\,k'^2\,V_{l_1}(k,k')\frac{1}{2\xi_{k'}}\tanh\left(\frac{\xi_{k'}}{2T_c}\right)$$

$$\times\underbrace{\int d\Omega_k\,Y^*_{lm}(\hat{k})Y_{l_1m_1}(\hat{k})}_{\delta_{ll_1}\delta_{mm_1}}\underbrace{\int d\Omega_{k'}\,Y^*_{l_1m_1}(\hat{k}')Y_{l_2m_2}(\hat{k}')}_{\delta_{l_1l_2}\delta_{m_1m_2}}$$

よって

$$1 = -\frac{V}{2\pi^2}\int_0^\infty dk'\, k'^2\, V_\ell(k,k')\, \frac{1}{2\xi_{k'}} \tanh\left(\frac{\xi_{k'}}{2T_c}\right)$$

以下、$V_\ell(k,k')$ が FS近傍しか効かないと仮定する：

$$V_\ell(k,k') = \begin{cases} -V_\ell & (|\xi_k|, |\xi_{k'}| \leq \omega_c) \\ 0 & (|\xi_k|, |\xi_{k'}| > \omega_c) \end{cases}$$

このとき

$$\xi = k^2/2m, \quad m\, d\xi = k\, dk$$

$$\longrightarrow \frac{V}{2\pi^2}\int_{k_F-k_c}^{k_F+k_c} dk\, k^2 \simeq V \underbrace{\frac{mk_F}{2\pi^2}}_{=N(0)\,:\ \text{FS の DoS}} \cdot 2\int_0^{\omega_c} d\xi$$

$$\therefore \boxed{1 = +N(0) V_\ell \int_0^{\omega_c} d\xi\, \frac{1}{\xi} \tanh\left(\frac{\xi}{2T_c}\right)} \qquad (\ell = \text{even})$$

これより、T_c は

$$T_c = 1.14\,\omega_c \exp\left(-\frac{1}{N(0)V_\ell}\right)$$

で与えられる。(*)

(*)
$$\int_0^{\omega_c} d\xi\, \frac{1}{\xi}\tanh\left(\frac{\xi}{2T_c}\right) \simeq \log\left(\frac{2e^\gamma \omega_c}{\pi T_c}\right) \simeq \log\left(\frac{1.14\,\omega_c}{T_c}\right)$$

(γ：Euler定数)

$$\rightsquigarrow 1 = N(0) V_\ell \log\left(\frac{1.14\,\omega_c}{T_c}\right)$$

$$\exp\left(\frac{1}{N(0)V_\ell}\right) = \frac{1.14\,\omega_c}{T_c}$$

$$\therefore T_c = 1.14\,\omega_c \exp\left(-\frac{1}{N(0)V_\ell}\right)$$

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- spin-triplet の場合

$$V_{\alpha\beta\gamma\delta}(\boldsymbol{k},\boldsymbol{k}') = \frac{1}{2}\sum_J \sum_{l=odd} 4\pi V_{Jl}(k,k') \sum_{M=-J}^{J} (\boldsymbol{Y}^{l}_{JM}(\hat{\boldsymbol{k}})\cdot \boldsymbol{\sigma} i\sigma_y)_{\alpha\beta} (\boldsymbol{Y}^{l}_{JM}(\hat{\boldsymbol{k}}')\cdot \boldsymbol{\sigma} i\sigma_y)^{\dagger}_{\gamma\delta}$$

$$\boldsymbol{Y}^{l}_{JM}(\hat{\boldsymbol{k}}) = \sum_{m=-l}^{l}\sum_{\mu=-1}^{1} C^{JM}_{lm1\mu} Y_{lm}(\hat{\boldsymbol{k}})\, \boldsymbol{e}_{\mu}$$

と書けるとする.

例

$$V_{\alpha\beta\gamma\delta}(\boldsymbol{k},\boldsymbol{k}') = \frac{V_0}{4}\left\{ (\hat{\boldsymbol{k}}\cdot\boldsymbol{\sigma} i\sigma_y)_{\alpha\beta} (\hat{\boldsymbol{k}}'\cdot\boldsymbol{\sigma} i\sigma_y)^{\dagger}_{\gamma\delta} - (\hat{\boldsymbol{k}}\times\boldsymbol{\sigma} i\sigma_y)_{\alpha\beta}\cdot(\hat{\boldsymbol{k}}'\times\boldsymbol{\sigma} i\sigma_y)^{\dagger}_{\gamma\delta} \right\}$$

$$\begin{cases} \boldsymbol{Y}^{1}_{00} = \frac{1}{\sqrt{3}}(Y_{11}\boldsymbol{e}_{-1} - Y_{10}\boldsymbol{e}_0 + Y_{1-1}\boldsymbol{e}_1) = -\frac{1}{\sqrt{4\pi}}(\hat{k}_x \boldsymbol{e}_x + \hat{k}_y \boldsymbol{e}_y + \hat{k}_z \boldsymbol{e}_z) \\ \boldsymbol{Y}^{1}_{11} = \frac{1}{\sqrt{2}}(Y_{11}\boldsymbol{e}_0 - Y_{10}\boldsymbol{e}_1) = \frac{1}{2}\sqrt{\frac{3}{4\pi}}\left\{(\hat{k}_x + i\hat{k}_y)\boldsymbol{e}_z - \hat{k}_z(\boldsymbol{e}_x + i\boldsymbol{e}_y)\right\} \\ \boldsymbol{Y}^{1}_{10} = \frac{1}{\sqrt{2}}(Y_{11}\boldsymbol{e}_{-1} - Y_{1-1}\boldsymbol{e}_1) = \frac{i}{\sqrt{2}}\sqrt{\frac{3}{4\pi}}(\hat{k}_y \boldsymbol{e}_x - \hat{k}_x \boldsymbol{e}_y) \\ \boldsymbol{Y}^{1}_{1-1} = \frac{1}{\sqrt{2}}(-Y_{1-1}\boldsymbol{e}_0 + Y_{10}\boldsymbol{e}_{-1}) = \frac{1}{2}\sqrt{\frac{3}{4\pi}}\left\{(\hat{k}_x - i\hat{k}_y)\boldsymbol{e}_z - \hat{k}_z(\boldsymbol{e}_x - i\boldsymbol{e}_y)\right\} \end{cases}$$ より

$$= \frac{V_0}{4}\left\{ 4\pi (\boldsymbol{Y}^{1}_{00}(\hat{\boldsymbol{k}})\cdot\boldsymbol{\sigma} i\sigma_y)_{\alpha\beta}(\boldsymbol{Y}^{1}_{00}(\hat{\boldsymbol{k}}')\cdot\boldsymbol{\sigma} i\sigma_y)^{\dagger}_{\gamma\delta} - \frac{8\pi}{3}\sum_{M=-1}^{1}(\boldsymbol{Y}^{1}_{1M}(\hat{\boldsymbol{k}})\cdot\boldsymbol{\sigma} i\sigma_y)_{\alpha\beta}(\boldsymbol{Y}^{1}_{1M}(\hat{\boldsymbol{k}}')\cdot\boldsymbol{\sigma} i\sigma_y)^{\dagger}_{\gamma\delta} \right\}$$

$$\therefore V_{01} = \frac{V_0}{2},\quad V_{11} = -\frac{V_0}{3},\quad (\text{otherwise}) = 0$$

また.秩序変数もベクトル球面調和関数で展開すると

$$\Delta(\boldsymbol{k}) = \sum_J \sum_{l=odd} \sum_{M=-J}^{J} C_{JMl} (\boldsymbol{Y}^{l}_{JM}(\hat{\boldsymbol{k}})\cdot\boldsymbol{\sigma} i\sigma_y)$$

となる.

よって

$$\sum_{J_1M_1\ell_1} C_{J_1M_1\ell_1}\left(\mathcal{Y}^{\ell_1}_{J_1M_1}(\hat{k})\cdot\sigma i\sigma_y\right)_{\alpha\beta}$$

$$= -\sum_{k'}\sum_{\gamma\delta}\frac{1}{2}\sum_{J_1\ell_1}4\pi V_{J_1\ell_1}(k,k')\sum_{M_1}\left(\mathcal{Y}^{\ell_1}_{J_1M_1}(\hat{k})\cdot\sigma i\sigma_y\right)_{\beta\alpha}\left(\mathcal{Y}^{\ell_1}_{J_1M_1}(\hat{k}')\cdot\sigma i\sigma_y\right)^\dagger_{\gamma\delta}$$

$$\times\sum_{J_2M_2\ell_2}C_{J_2M_2\ell_2}\left(\mathcal{Y}^{\ell_2}_{J_2M_2}(\hat{k}')\cdot\sigma i\sigma_y\right)_{\gamma\delta}\quad\frac{1}{2\xi_{k'}}\tanh\left(\frac{\xi_{k'}}{2T_c}\right)$$

両辺に $Y^*_{\ell m}(\hat{k})$ をかけて $d\Omega_{\hat{k}}$ で積分：

$$\sum_{J_1M_1\ell_1}C_{J_1M_1\ell_1}\sum_{m_1\mu_1}C^{J_1M_1}_{\ell_1m_11\mu_1}(\sigma_{\mu_1}i\sigma_y)_{\alpha\beta}\underbrace{\int d\Omega_{\hat{k}}\,Y^*_{\ell m}(\hat{k})Y_{\ell_1m_1}(\hat{k})}_{=\delta_{\ell\ell_1}\delta_{mm_1}}$$

$$= -4\pi\sum_{J_1M_1\ell_1}\sum_{J_2M_2\ell_2}\sum_{m_1\mu_1}\sum_{m_2\mu_2}\sum_{m_3\mu_3}C^{J_1M_1}_{\ell_1m_11\mu_1}C^{J_1M_1}_{\ell_1m_21\mu_2}C^{J_2M_2}_{\ell_2m_31\mu_3}C_{J_2M_2\ell_2}$$

$$\times\frac{V}{(2\pi)^3}\int dk'\,k'^2\,V_{J_1\ell_1}(k,k')\frac{\tanh\left(\frac{\xi_{k'}}{2T_c}\right)}{2\xi_{k'}}\underbrace{\int d\Omega_{\hat{k}}\,Y^*_{\ell m}(\hat{k})Y_{\ell_1m_1}(\hat{k})}_{\delta_{\ell\ell_1}\delta_{mm_1}}\underbrace{\int d\Omega_{\hat{k}'}\,Y^*_{\ell_1m_2}(\hat{k}')Y_{\ell_2m_3}(\hat{k}')}_{\delta_{\ell_1\ell_2}\delta_{m_2m_3}}$$

$$\times(\sigma_{\mu_1}i\sigma_y)_{\beta\alpha}\cdot\underbrace{\frac{1}{2}\sum_{\gamma\delta}(\sigma_{\mu_2}i\sigma_y)^\dagger_{\gamma\delta}(\sigma_{\mu_3}i\sigma_y)_{\gamma\delta}}$$

$$\text{"}\ \mathrm{tr}\left[(\sigma_{\mu_2}i\sigma_y)^*(\sigma_{\mu_3}i\sigma_y)\right] = \mathrm{tr}\left[(\sigma_{\mu_2}i\sigma_y)^\dagger(\sigma_{\mu_3}i\sigma_y)\right]$$
$$= \mathrm{tr}\left[(i\sigma_y)^\dagger\sigma_{\mu_2}^\dagger\sigma_{\mu_3}(i\sigma_y)\right] = \mathrm{tr}\left[\sigma_{\mu_2}^\dagger\sigma_{\mu_3}\right] = 2\delta_{\mu_2\mu_3}$$

$V_{J\ell}(k,k')$ が FS 近傍のみ効くとすると

$$\sum_{J_1M_1}C_{J_1M_1\ell}\sum_{\mu_1}C^{J_1M_1}_{\ell m1\mu_1}(\sigma_{\mu_1}i\sigma_y)_{\alpha\beta}$$

$$= +\sum_{J_1M_1}\sum_{\mu_1}C^{J_1M_1}_{\ell m1\mu_1}(\sigma_{\mu_1}i\sigma_y)_{\alpha\beta}\sum_{J_2M_2}\underbrace{\sum_{m_2\mu_2}C^{J_1M_1}_{\ell m_21\mu_2}C^{J_2M_2}_{\ell m_21\mu_2}}_{\delta_{J_1J_2}\delta_{M_1M_2}}C_{J_2M_2\ell}$$

$$\times N(0)V_{J_1\ell}\int_0^{\omega_c}d\xi\,\frac{1}{\xi}\tanh\left(\frac{\xi}{2T_c}\right)$$

以下、同様に直交性を用いると

$$\therefore\ \boxed{1 = +N(0)V_{J\ell}\int_0^{\omega_c}d\xi\,\frac{1}{\xi}\tanh\left(\frac{\xi}{2T_c}\right)}$$