Homework 4: Approximate Inference in Bayesian Networks

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1 Sampling Methods

In this assignment, I implemented Rejection Sampling and Gibbs Sampling to generate samples for a Bayesian network.

For Rejection Sampling, the implementation is quite simple. Samples are generated ignoring the constraints of the evidence. Then, samples inconsistent with the evidence are dropped. In the remaining samples, proportion of samples consistent with the query is returned.

For Gibbs Sampling, in each iteration, for node x_i, it is sampled from the following probability (revised from the textbook).

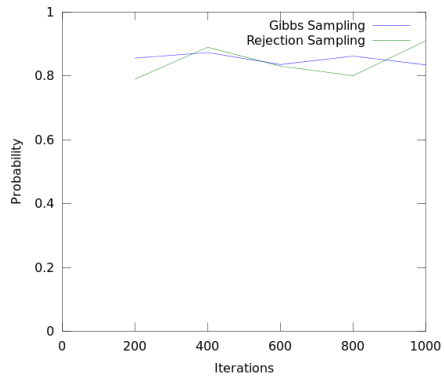
$$p(x_i|x_{\{j\neq i\}}) \propto \prod_k P(x_k|pa_k) \tag{1}$$

2 Experiments

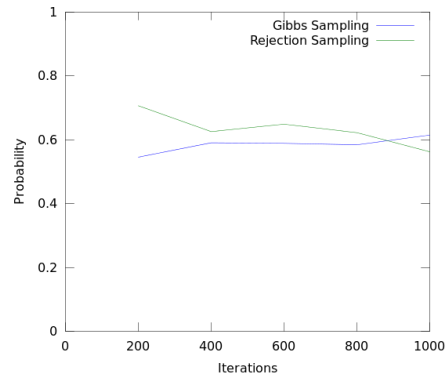
In this experiment, I used the data provided by Ian. The running iterations for both are 1000. Note that for rejection sampling, the samples collected are less than 1000. For Gibbs sampling, the parameter $T = 200$. The algorithm starts collect data after T iterations. The results generated are shown below.

In Figure 1, I show the learning process in each sampling algorithms, for each query data in the first evidence given.

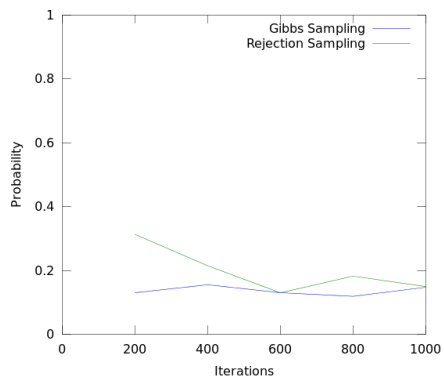
The following is the numerical results. Each line corresponds to one evidence, from the first to the last evidence given in the `data.678` file. Each number in one line is the result of one query, from $X_0 = 1$ up to $X_5 = 1$.



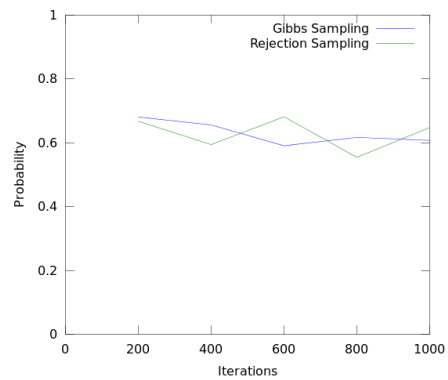
(a) $P(X_0 = 1|E_1)$



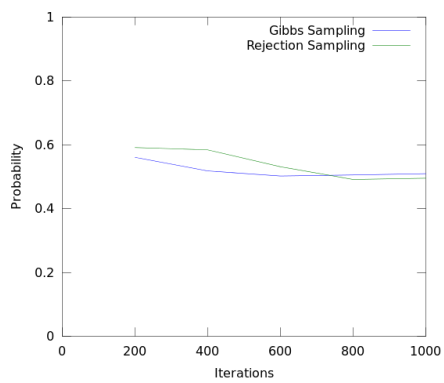
(b) $P(X_1 = 1|E_1)$



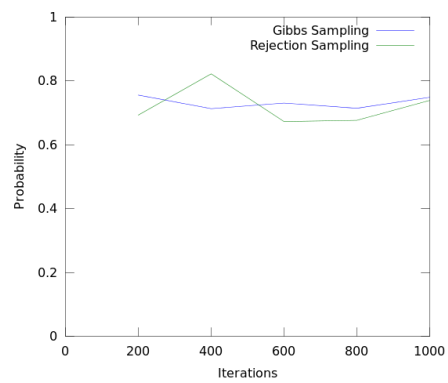
(c) $P(X_2 = 1|E_1)$



(d) $P(X_3 = 1|E_1)$



(e) $P(X_4 = 1|E_1)$



(f) $P(X_5 = 1|E_1)$

Figure 1: Sampling process for each test case. E_1 denotes $(X_6 = 1, X_7 = 0, X_8 = 0)$.

By rejection sampling,

0.848837	0.538462	0.142857	0.638298	0.607143	0.750000
0.795082	0.766129	0.132231	0.990566	0.393939	0.170213
0.862069	0.642857	0.134021	0.515789	0.450000	0.240385
0.831579	0.644689	0.130268	0.944649	0.350195	0.847059
0.930000	0.715909	0.098901	0.495050	0.403670	0.282828
0.828125	0.648649	0.094595	1.000000	0.742857	0.810811
0.849462	0.571429	0.122449	0.394737	0.554348	0.611111
0.825688	0.791304	0.148515	0.943925	0.333333	0.111111
0.773585	0.696203	0.109091	0.964912	0.731707	0.745098
0.855263	0.533333	0.117155	0.519824	0.427928	0.836538

By Gibbs sampling,

0.83500	0.56800	0.13300	0.62500	0.49600	0.75200
0.81700	0.77600	0.11900	0.95500	0.38800	0.16700
0.84900	0.68200	0.12400	0.44900	0.44000	0.29100
0.81500	0.64300	0.13300	0.96700	0.35700	0.86100
0.84500	0.65900	0.12900	0.49500	0.46700	0.27800
0.82600	0.66100	0.15900	0.97600	0.76700	0.68700
0.85200	0.59900	0.13800	0.42300	0.54600	0.65200
0.78800	0.79200	0.10800	0.96300	0.35200	0.15000
0.81600	0.66100	0.13100	0.97800	0.76800	0.72400
0.87300	0.51800	0.12000	0.54100	0.41800	0.82000

3 Discussion

The rejection sampling is less efficient in general, because it generates samples inconsistent with the evidence. This depends on the probability of the evidence - if the evidence is very unlikely, then the algorithm needs to run many times to gather enough samples consistent with the evidence.

In this sense, Gibbs sampling beats rejection sampling because every sample counts. However, Gibbs sampling relies on the initial sample in the first few iterations. It needs some iterations to reach a stationary distribution.