Homework 2: Independent Component Analysis

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1 Independent Component Analysis

In this report, I applied Independent Component Analysis on Blind Source Separation problem. Let U be the orignal sources, which is a $n \times t$ matrix. n is the number of sources and t is the number of data. Let A be mixing operation, and $X = AU$. The goal is to find a matrix W such that $U = W \times X$, which recover the sources from the observations.

I used the batch update method described on the webpage (with the mistake fixed),

$$\Delta W = \eta(tI + (1 - 2Z)Y^T)W \tag{1}$$

This is slightly different from the update eqation proved in Ng's note. That equation uses one sample to update

$$\Delta W = \eta((1 - 2Z)x^T + (W^T)^{-1}) \tag{2}$$

where x stands for an observation at one time. I first convert it to the batch version. Data are added as columns, which make a matrix X. W^T is simply added t times. So,

$$\Delta W = \eta((1 - 2Z)X^T + t(W^T)^{-1}) \tag{3}$$

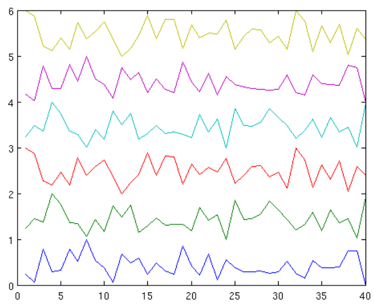
Equation 1 is more efficient, as there is no inverse of matrix. I tried to derive Equation 1 from Equation 3,

$$\begin{aligned}
\Delta W &= \eta((1 - 2Z)X^T + t(W^T)^{-1}) & (4)\\
&= \eta((1 - 2Z)X^T + t(W^T)^{-1})W^T(W^T)^{-1} & (5)\\
&= \eta((1 - 2Z)X^T W^T + tI)(W^T)^{-1} & (6)\\
&= \eta((1 - 2Z)(WX)^T + tI)(W^T)^{-1} & (7)\\
&= \eta((1 - 2Z)Y^T + tI)W(W^{-1})(W^T)^{-1} & (8)\\
&= \eta((1 - 2Z)Y^T + tI)W(W^T W)^{-1} & (9)
\end{aligned}$$

So the different is the term, $W^T W$. If W is othognal, then it's $W^T W = I$. We know this is not a valid assumption. However, I used Equation 1 in the experiments.

2 Experiment

2.1 Experiments on Small Set



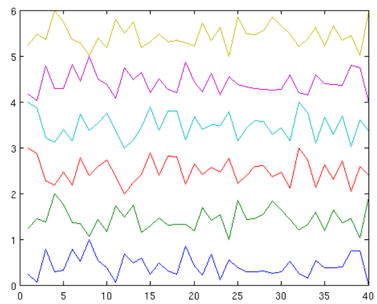


Figure 1: The bottom 3 lines are original signals from `icaTest.mat`. The top 3 lines are reconstructed signals with $\eta = 0.01$ and 1000 iterations. Two independent experiments are shown.

	Recon. 1	Recon. 2	Recon. 3
Source 1	-0.4900	**0.9905**	-0.4248
Source 2	**0.9918**	-0.3957	-0.5454
Source 3	-0.4829	-0.5073	**0.9924**

Table 1: Linear correlation between source and reconstructed signals, for the left figure in Figure 1. The maximum correlation is highlighted.

	Recon. 1	Recon. 2	Recon. 3
Source 1	-0.4248	**0.9905**	-0.4900
Source 2	-0.5454	-0.3957	**0.9918**
Source 3	**0.9924**	-0.5073	-0.4829

Table 2: Linear correlation between source and reconstructed signals, for the right figure in Figure 1. The maximum correlation is highlighted.

First I did experiment on `icaTest.mat`. $\eta = 0.01$ and 1000 iterations. The results are in shown in Figure 1.

The results are scaled into $[0, 1]$ interval. This experiment is repeated twice. Actually, the result can be permutation of the original sources. Scaling is also possible (as the results are scaled into $[0, 1]$, the signal could be flipped in this case, though I didn't observe this in the experiment). The correlation between source and reconstructed signals are shown in Table 1 and 2. By defintion, they are highly correlated if the correlavance is close to 1 or -1. We can tell that, for the left figure, the mapping is 0-4, 1-5, 2-3. While in the right one, the mapping is 0-5, 1-3, 2-4. This result can be also verified visually. We can also observe that the algorithm overall generate highly close signal, with has more than 0.99 correlavance with its source signal.

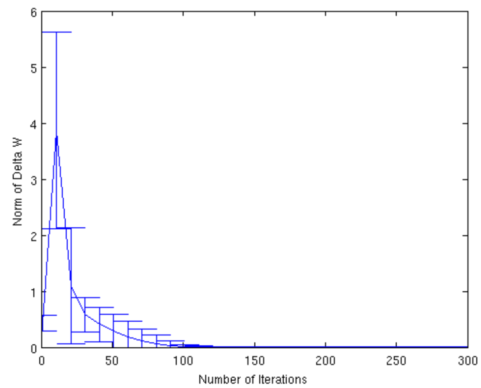


Figure 2: $||\Delta W||$ over number of iterations. Average of 100 runs. The length of vertical bars is σ assmuing Gaussian distribution of data points at each iteration.

I also examine how progress is made in each iteration in the learning process. As the algorithm uses gradient descent, the update on W each step, which is ΔW, is useful. For the convenience of visualization, I need the magtitude of this matrix. I use the largest singular value here, which can be got using `norm` function when applied to a matrix in Matlab.

The result is shown in Figure 2. Not surprisingly, steps become smaller after more iterations. The algorithm converges after 2×10^5 iterations.

2.2 Experiments on Sound

	Recon. 1	Recon. 2	Recon. 3	Recon. 4	Recon. 5
Source 1	-0.0032	0.0030	-0.0076	-0.0035	**1.0000**
Source 2	-0.0014	-0.0003	**0.9999**	-0.0126	0.0016
Source 3	-0.0026	-0.0095	0.0126	**0.9999**	0.0007
Source 4	0.0020	**0.9999**	0.0083	-0.0102	0.0041
Source 5	**0.9999**	0.0051	0.0020	0.0125	-0.0014

Table 3: Linear correlation between source and reconstructed signals.

The data are from `sounds.mat`. $\eta = 10^{-6}$ as t is large in one step. It runs 1000 iterations. Result on sound are shown in Figure 3. The correlation result is shown in Table 3.

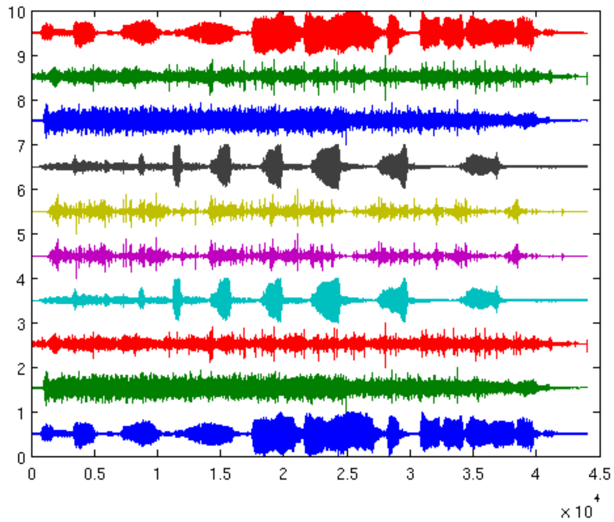


Figure 3

3 Discussion

A batch version.

4 Conclusion