Homework 6: Genetic Algorithm

Shun Zhang

1 Genetic Algorithm for Sorting Networks

One iteration of genetic algorithm is run for both sorting network and the input data at each step. For sorting network,

- First generation:
- Fitness: The fitness is the linear combination of the following factors,
  - The proportion of consistent pairs. Consistent pairs are different i, j such that $(i < j \wedge d[i] \leq d[j])$, where d is an input datum. The fitness is the average of the proportion of consistent pairs on all the input data.
  - The inverse of the lengths, namely $\frac{1}{Length}$. The longer the length is, the less fitness it has.
- Selection: An individual is chosen with the probability of its fitness.
- Crossover: Two individuals (list of sorting pairs) both cut into two parts in the middle, and exchange their first halves. For example, (1, 2), (2, 3) and (3, 1), (2, 1) are crossed over to be (3, 1), (2, 3) and (1, 2), (2, 1).
- Mutation: Two types of mutation, with equal probability.
  - First generation:
  - Randomly change a number in a sorting pair. For example, (1, 5) changes to (6, 5).
  - Three randomly generated sorting pairs are appended to a randomly chosen individual. For example, (1, 5), (2, 4) is changed to be (1, 5), (2, 4), (1, 3), (2, 3), (3, 4).

For input data,

- Fitness: This is the inverse of fitness of sorting networks. Concretely, it is the average of inconsistent pairs after sorting by each sorting network. It measures how difficult the data can be sorted.
- Selection: An individual is chosen with the probability of its fitness.
- Crossover: Two individuals (list of numbers) both cut into two parts in the middle, and exchange their first halves. This is same as crossover for sorting network.
- Mutation: One number in a random individual is changed to another number. For example, 1, 4, 5, 9, 2 is changed to 1, 8, 5, 9, 2.

2 Experiments

2.1 Experiment 1

First I checked the correctness of the implementation with fixed number of sorting pairs.

Mutation rate is 0.01. The size of the input data is 8. Number of sorting pairs is 64.

2.2 Experiment 2

After the first trial, I make the lengths of sorting pairs evolve with the accuracy. The sorting network of the last generation is in Appendix.

Mutation rate is 0.01. The size of input data is still 8. The Population for sorting networks is 200, 100 for input data.

2.3 Experiment 3

The size of input data is still 8. The Population for sorting networks is 500, 200 for input data.

3 Discussion and Conclusion

Sorting in general need $O(n\log(n))$ pairwise comparisons. So $O(n\log(n))$ is also the lower bound for the number of the pairs in a sorting network. For the fitness of the sorting network, both performance on data and its

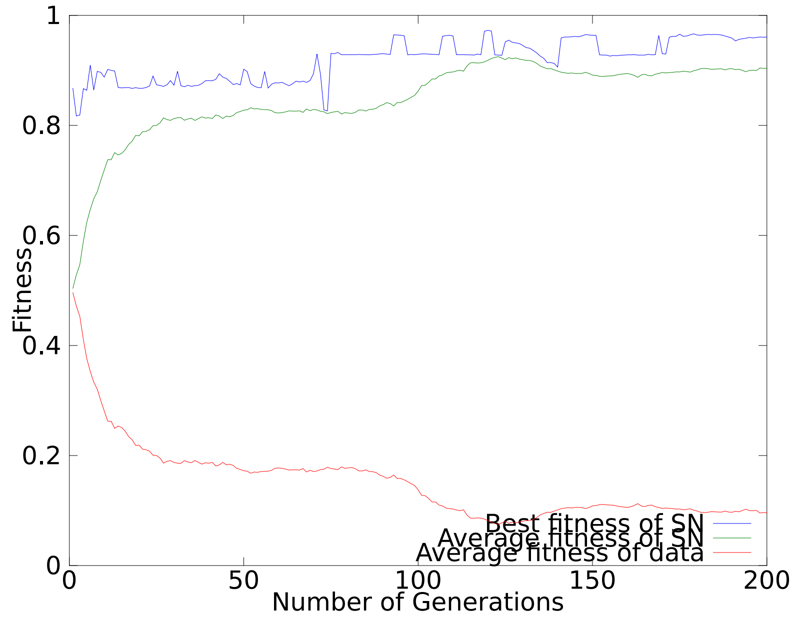
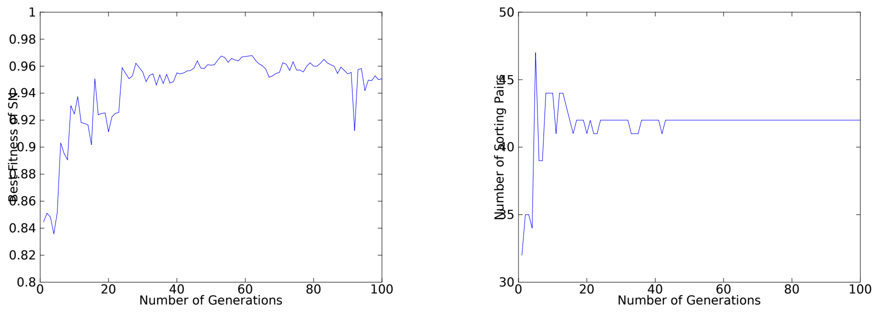


Figure 1: Experiment 1

length need to be considered. So it is hard to achieve the best performance on data, also minimize the sorting network. However, this goal should be



(a) Fitness of sorting networks v.s. Number of generations

(b) Number of sorting pairs v.s. Number of generation

Figure 2: Experiment 2

closer when more iterations are run.

Even though the performance is improving in the experiments, but there isn't a clear indicator that it has converged or not. Even though the average fitness of two consecutive generations are similar, they could also do a breakthrough by a good crossover or by mutation.

4 Appendix: Trained Sorting Networks

For Experiment 2,

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[[1, 7], [6, 3], [5, 6], [2, 7], [5, 2], [0, 7], [4, 3], [5, 4], [1,
4], [5, 5], [6, 4], [2, 4], [3, 6], [3, 0], [0, 6], [5, 7], [6, 6],
[4, 5], [0, 1], [4, 3], [3, 5], [7, 4], [7, 4], [6, 0], [4, 5], [2,
0], [6, 1], [1, 3], [1, 1], [0, 7], [1, 4], [1, 4], [3, 2], [2, 2],
[3, 4], [7, 4], [4, 2], [3, 0], [5, 7], [5, 2], [2, 6], [0, 3]]
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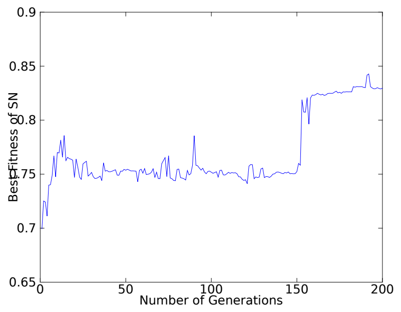
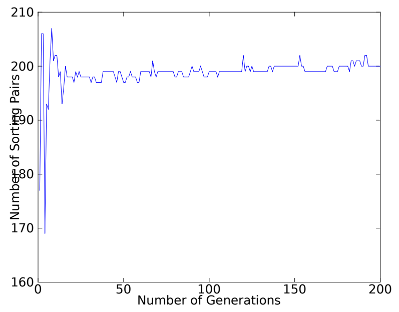
For Experiment 3,



(a) Fitness of sorting networks v.s. Number of generations



(b) Number of sorting pairs v.s. Number of generation

Figure 3: Experiment 3

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[[2, 7], [0, 0], [4, 3], [2, 13], [0, 15], [5, 5], [7, 8], [3, 8],
[15, 15], [6, 8], [6, 12], [4, 14], [6, 1], [6, 8], [15, 0], [14, 12],
[4, 9], [0, 5], [14, 8], [5, 14], [13, 12], [10, 0], [10, 11], [3, 3],
[12, 8], [12, 5], [12, 13], [3, 11], [0, 3], [3, 6], [14, 15], [2,
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3], [10, 8], [15, 15], [3, 10], [13, 5], [11, 1], [12, 4], [11, 12],
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6], [5, 3], [7, 7], [4, 10], [2, 1], [14, 8], [11, 14], [6, 2], [3,
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4], [13, 4], [0, 5], [9, 12], [5, 0], [1, 3], [13, 8], [15, 10], [7,
4], [14, 14], [11, 7], [6, 0], [1, 13], [8, 13], [11, 5], [0, 5], [5,
12], [3, 9], [10, 8], [2, 10], [6, 4], [12, 6], [5, 9], [4, 13], [4,
2], [15, 14], [13, 2], [3, 15], [3, 14], [8, 5], [15, 6], [2, 0], [7,
7], [9, 0], [14, 7], [0, 6], [8, 2], [4, 10], [0, 10], [3, 13], [8,
6], [8, 10], [4, 14], [10, 0], [2, 15], [14, 7], [13, 1], [7, 9], [1,
14], [12, 11], [3, 2], [2, 15], [6, 7], [2, 10], [7, 0], [3, 9], [9,
1], [7, 0], [3, 9], [3, 10], [11, 1], [6, 1], [0, 10], [8, 5], [8, 0],
[0, 4], [12, 12], [15, 9], [10, 7], [1, 5], [4, 12], [13, 12], [10,
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9], [0, 8], [3, 2], [8, 3], [5, 15], [7, 4], [15, 12], [8, 11], [4,
12], [2, 13], [12, 12], [15, 7], [4, 1], [2, 14], [15, 0], [2, 6], [1,
15], [5, 2], [5, 14], [3, 10], [3, 15], [11, 4], [3, 8], [7, 15], [0,
12], [2, 11]]
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