Homework 2: Independent Component Analysis

Name: Shun Zhang
Email address: jensen.zhang@utexas.edu
EID: sz4554

1 Independent Component Analysis

In this report, I applied Independent Component Analysis on Blind Source Separation problem. Let U be the orignal sources, A be mixing operation. Let $X = A \times U$. The goal is to find a matrix W such that $U = W \times X$, which recover the sources from the observations.

I used the batch update method described on the webpage,

$$\Delta W = \eta(I + (1 - 2Z)Y^T)W \tag{1}$$

This is slightly different from the update eqation proved in Ng's note,

$$\Delta W = \eta((1 - 2Z)X^T + (W^T)^{-1}) \tag{2}$$

Equation 1 is more efficient, as there is no inverse of matrix. I tried to derive Equation 1 from Equation 2,

$$\begin{aligned}
\Delta W &= \eta((1 - 2Z)X^T + (W^T)^{-1}) &(3)\\
&= \eta((1 - 2Z)X^T + (W^T)^{-1})W^T(W^T)^{-1} &(4)\\
&= \eta((1 - 2Z)X^T W^T + I)(W^T)^{-1} &(5)\\
&= \eta((1 - 2Z)(WX)^T + I)(W^T)^{-1} &(6)\\
&= \eta((1 - 2Z)Y^T + I)W(W^{-1})(W^T)^{-1} &(7)\\
&= \eta((1 - 2Z)Y^T + I)W(W^T W)^{-1} &(8)
\end{aligned}$$

So the different is the term, $W^T W$. If W is othognal, then it's $W^T W = I$. We know this is not a valid assumption. However, I used Equation 1 in the experiments.

2 Experiment

2.1 Experiments on Small Set

The results are scaled into $[0, 1]$ interval. This experiment is repeated twice. The results are in shown in Figure 1. The result can be permutation of the

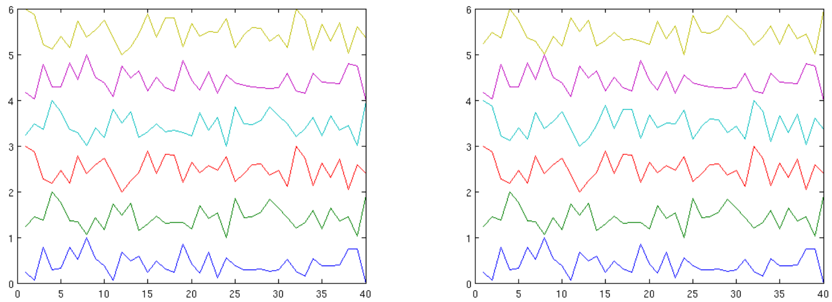

Figure 1: The bottom 3 lines are original signals from `icaTest.mat`. The top 3 lines are reconstructed signals with $\eta = 0.01$ and 1000000 iterations. Two independent experiments are shown.

	Recon. 1	Recon. 2	Recon. 3
Source 1	-0.4900	**0.9905**	-0.4248
Source 2	**0.9918**	-0.3957	-0.5454
Source 3	-0.4829	-0.5073	**0.9924**

Table 1: Linear correlation between source and reconstructed signals, for the left figure in Figure 1. The maximum correlation is highlighted.

original sources. Scaling is also possible (as the results are scaled into $[0, 1]$, the signal could be flipped in this case). The correlation between source and reconstructed signals are shown in Table 1 and 2. They are highly correlated if the correlavance is close to 1 or -1. We can tell that, for the left figure, the mapping is 0-4, 1-5, 2-3. While in the right one, the mapping is 0-5, 1-3, 2-4. This result can be verified visually. We can also observe that the algorithm overall generate highly close signal, with has more than 0.99 correlavance with its source signal.

I also examine how progress is made in each iteration in the learning process. As the algorithm uses gradient descent, the update on W each step, which is ΔW, is useful. For the convenience of visualization, I need the magtitude of this matrix. I use the largest singular value here, which can be got using `norm` function when applied to a matrix in Matlab.

	Recon. 1	Recon. 2	Recon. 3
Source 1	-0.4248	**0.9905**	-0.4900
Source 2	-0.5454	-0.3957	**0.9918**
Source 3	**0.9924**	-0.5073	-0.4829

Table 2: Linear correlation between source and reconstructed signals, for the right figure in Figure 1. The maximum correlation is highlighted.

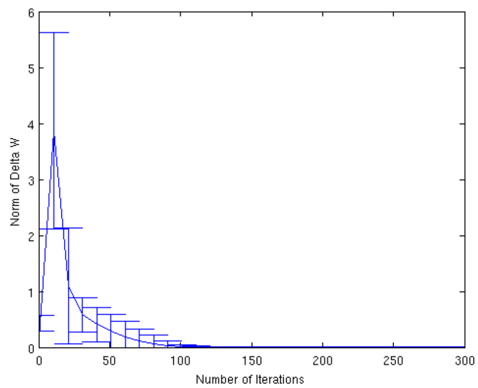

Figure 2: ΔW over number of iterations. Average of 5 runs. The length of vertical bars is σ assmuing Gaussian distribution of data points at each iteration.

	Recon. 1	Recon. 2	Recon. 3	Recon. 4	Recon. 5
Source 1	-0.0032	0.0030	-0.0076	-0.0035	**1.0000**
Source 2	-0.0014	-0.0003	**0.9999**	-0.0126	0.0016
Source 3	-0.0026	-0.0095	0.0126	**0.9999**	0.0007
Source 4	0.0020	**0.9999**	0.0083	-0.0102	0.0041
Source 5	**0.9999**	0.0051	0.0020	0.0125	-0.0014

Table 3: Linear correlation between source and reconstructed signals.

The result is shown in Figure 2. Not surprisingly, steps become smaller after more iterations. The algorithm converges after 2×10^5 iterations.

2.2 Experiments on Sound

Result on sound are shown in Figure 3. The correlation result is shown in Table **??**. We can observe that the correlation is smaller compared to that of experiment on small set.

3 Discussion

4 Conclusion

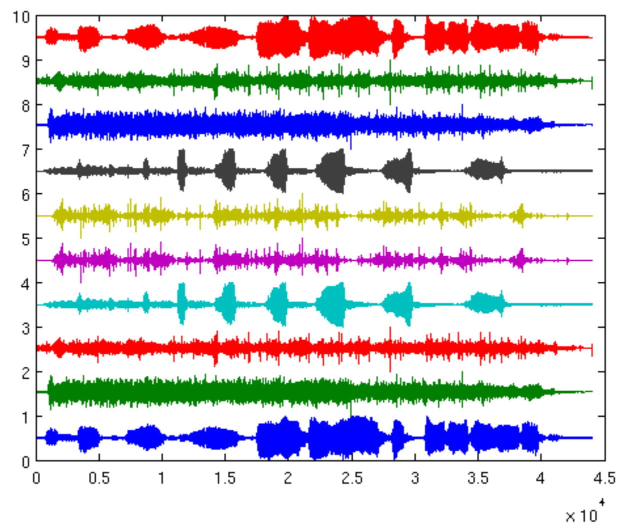

Figure 3